CHAPTER 3

Fraction Operations

Fractions are all around us. When Rebecca and Vishal were helping to clean up the kindergarten room, Rebecca dropped an armload of puzzles. How do these puzzles relate to fractions? Consider how you can use the puzzles to talk about numbers, colours, and seasons. For example:

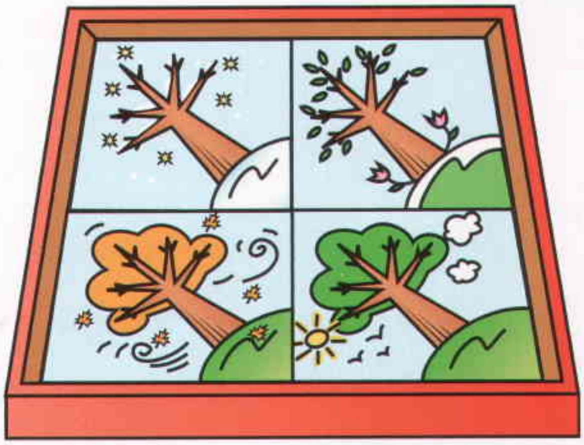
- There are four seasons in a year. What fraction of a year does each season cover?
- When you were 5 years old, what fraction of your life did 1 year make? 2 years?
- Sets of markers or paints often come in eight colours. Which colours are your favourites? What fraction of your wardrobe uses these colours?

Understanding fractions is important outside of the classroom. Describe the last time you used fractions in your life.

In this chapter, you will add and subtract fractions. You will find a common denominator and learn how to multiply a fraction by a whole number.

Chapter Problem

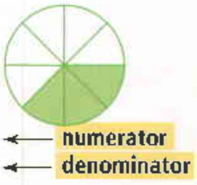
How can you use fraction puzzles to help others learn more about fractions?



Get Ready

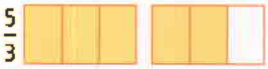
Writing Fractions

A fraction is a number that represents a part of a whole or a part of a group.

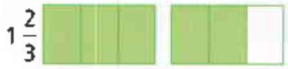


$\frac{3}{8}$ means that 3 parts out of a group of 8 equal parts are shaded.

$\frac{3}{8}$ is a **proper fraction**. Its denominator is greater than its numerator.



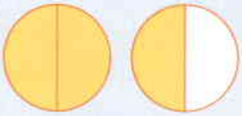
$\frac{5}{3}$ is an **improper fraction**. Its numerator is greater than its denominator.



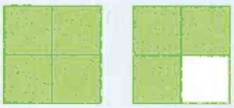
A **mixed number** is made up of a whole number and a fraction.
$\frac{5}{3}$ can be written as the mixed number $1\frac{2}{3}$.

1. Write the fraction shaded in each diagram. Show each as an improper fraction and as a mixed number.

a)



b)



c)

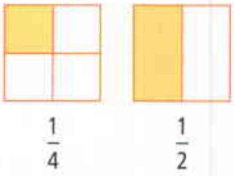
2. Draw a diagram to represent each fraction.

a) $\frac{1}{4}$ **b)** $\frac{1}{3}$ **c)** $\frac{2}{5}$

d) $\frac{8}{5}$ **e)** $1\frac{3}{4}$ **f)** $2\frac{1}{2}$

Comparing and Ordering Fractions

Which is greater, $\frac{1}{4}$ or $\frac{1}{2}$?



Use diagrams that are the same size to compare the two fractions.

$\frac{1}{2}$ is greater than $\frac{1}{4}$.

A number line can help you order fractions.
$1\frac{1}{2}$, $\frac{1}{4}$, 1, and $\frac{7}{4}$ are ordered from least to greatest on the number line.

3. Use diagrams that are the same size to compare the fractions. Which is greater?

a) $\frac{1}{4}$ and $\frac{1}{8}$ **b)** $\frac{1}{4}$ and $\frac{1}{3}$

c) $\frac{2}{3}$ and $\frac{5}{8}$

4. Use a number line to order $\frac{3}{4}$, $\frac{3}{2}$, $\frac{1}{2}$, and $1\frac{1}{4}$ from least to greatest.

Multiples

The first five **multiples** of 3 are 3, 6, 9, 12, and 15.

Each multiple is the product of 3 and a natural number.

$3 \times 1 = 3$ $3 \times 2 = 6$ $3 \times 3 = 9$ $3 \times 4 = 12$ $3 \times 5 = 15$

5. List the first five multiples of each number.

a) 2 **b)** 4 **c)** 5

6. Which of the following numbers is not a multiple of 8?

8 24 40 15 32

Equivalent Fractions

Equivalent fractions represent the same part of the whole or group.

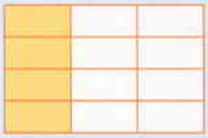
$\frac{4}{8}$ and $\frac{1}{2}$ are equivalent fractions.

$\frac{1}{2}$

$\frac{4}{8}$

7. Identify the fraction shaded in each diagram. Show the fraction in another way using equivalent fractions.

a)



b)

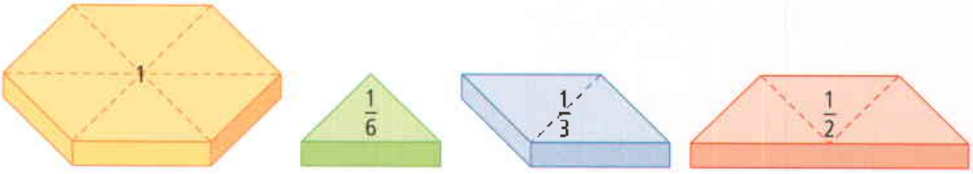
8. Draw a diagram to show an equivalent fraction for each fraction.

a) $\frac{3}{4}$ **b)** $\frac{4}{6}$ **c)** $\frac{10}{15}$

3.1

Add Fractions Using Manipulatives

Focus on...
- representing fractions using concrete materials
- using patterns to add fractions



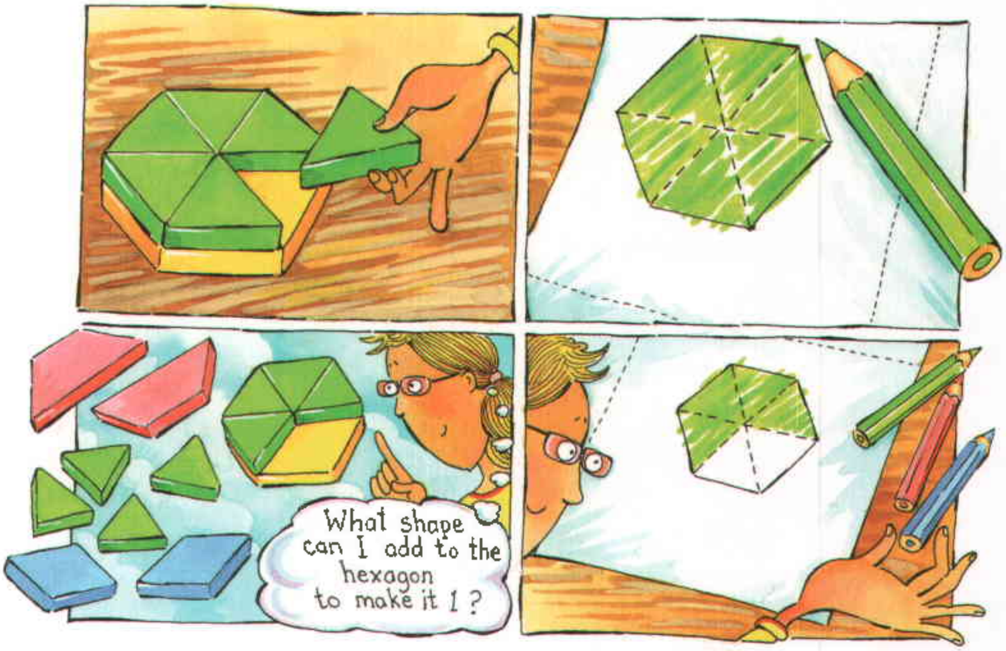
The blue rhombus is one third of the yellow hexagon. What other pattern block relationships are there?

Discover the Math

Materials
- pattern blocks
- pencil crayons
- BLM 3.1A Pattern Block Worksheet

How can you add fractions using manipulatives?

1. Allison used pattern blocks to make 1. Here is how she started.



2. There are at least 8 different ways to make 1 whole hexagon using yellow hexagon, red trapezoid, blue rhombus, and green triangle pattern blocks. How many ways can you find?

3. **Reflect** How can concrete materials and diagrams help you represent and add fractions?

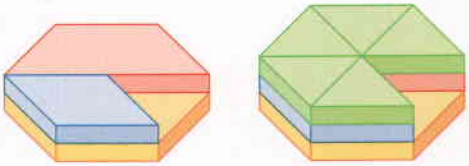
Example: Add Fractions Using Manipulatives

Add $\frac{1}{2} + \frac{1}{3}$.

Solution

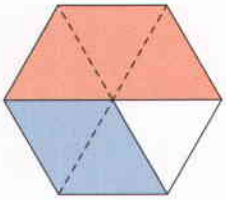
Method 1: Use Concrete Materials

$\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$



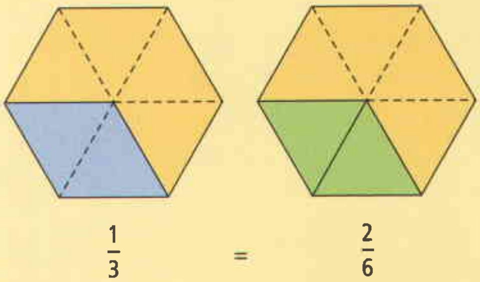
Method 2: Use a Diagram

$\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$

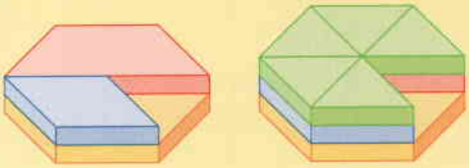


Key Ideas

- Concrete materials or diagrams can be used to represent fractions.
- Concrete materials or diagrams can be used to show equivalent fractions.



- To add fractions using concrete materials or diagrams, each fraction can be shown using parts of equal size.



Communicate the Ideas

1. Show why $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6}$.

2. Use words or diagrams to show that $\frac{3}{6} = \frac{1}{2}$.

3. Describe how you would explain your answers to questions 1 and 2 to a friend.

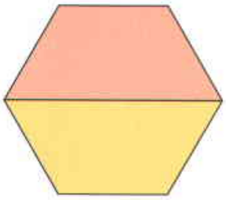
Check Your Understanding

Practise

For help with questions 4 to 9, refer to the Example.

4. What fraction of each hexagon is covered?

a)



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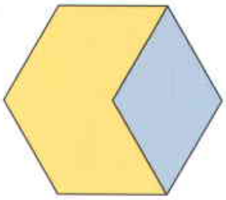
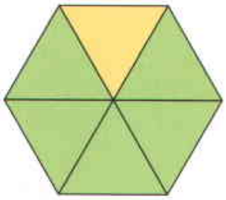


5. What fraction of each hexagon is covered?

a)



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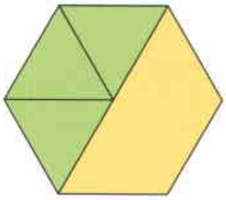
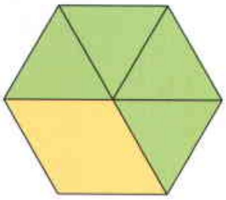


6. Write an addition sentence to represent the total fraction of each hexagon that is covered. State the total fraction covered.

a)



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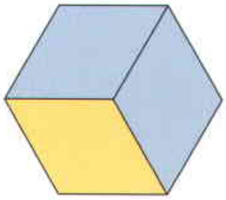
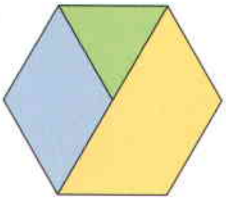


7. Write an addition sentence to represent the total fraction of each hexagon that is covered. State the total fraction covered.

a)



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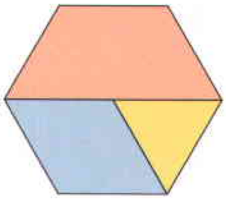

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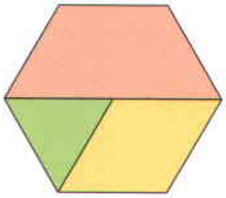
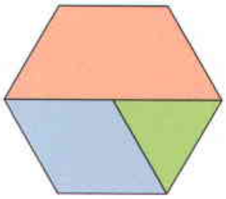


d)



8. Add. Use concrete materials or diagrams.

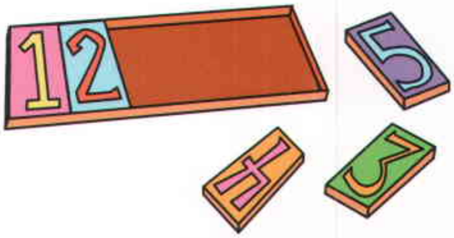
a) $\frac{1}{3} + \frac{1}{3}$ **b)** $\frac{1}{6} + \frac{1}{6}$ **c)** $\frac{2}{6} + \frac{3}{6}$

9. Add. Use concrete materials or diagrams.

a) $\frac{4}{6} + \frac{1}{3}$ **b)** $\frac{1}{2} + \frac{3}{6}$ **c)** $\frac{2}{3} + \frac{1}{6}$

Chapter Problem

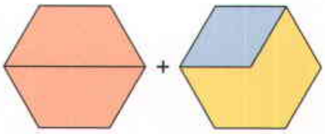
10. Write an addition sentence to describe this puzzle.



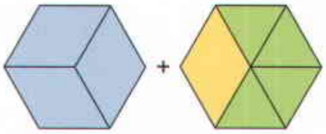
Apply

11. Write an addition sentence to describe how many hexagons are covered in each of the following.

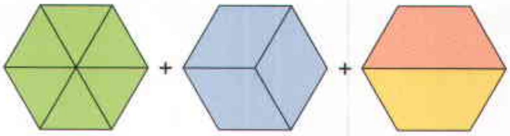
a)



b)

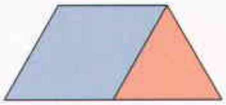


c)

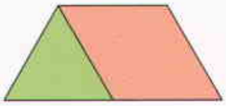


12. Suppose 1 red trapezoid = 1 whole.

a) What fraction of the trapezoid is represented by this diagram?



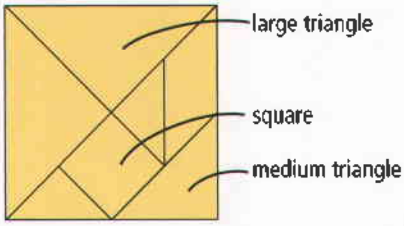
b) What fraction of the trapezoid is represented by this diagram?



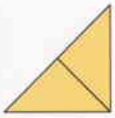
c) Write an addition statement to show the sum of the fractions in parts a) and b).

d) Find the sum.

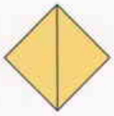
13. In Chapter 2, you used a tangram to classify geometric shapes.



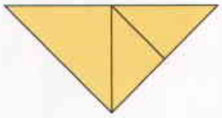
a) Suppose 1 medium triangle = 1 whole. Write an addition statement to represent this diagram.



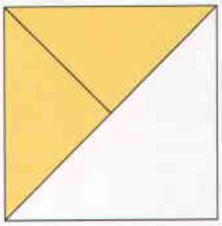
b) Suppose 1 square = 1 whole. Write an addition statement to represent this diagram.



c) Suppose 1 large triangle = 1 whole. Write an addition statement to represent this diagram.



14. Some of the tangram pieces are placed in the tangram square. Write an addition statement to show the fraction of the completed tangram that is placed in the square.



15. Go to www.mcgrawhill.ca/links/math7 and follow the links to a Web site where you can build pattern block diagrams on-screen.

a) Create three addition diagrams using yellow, red, green, and blue pattern blocks.

b) Ask a classmate to write an addition sentence to represent your diagrams.

16. Use concrete materials of your own choice to make up two addition questions. Draw diagrams to show your questions. Challenge a classmate to solve your questions using the materials you chose.

Extend

17. Use pattern blocks to simplify $\frac{1}{3} + \frac{1}{3} + \frac{1}{6}$. Show your answer visually and using a number sentence.

18. Suppose 2 hexagons = 1 whole. You have 1 triangle, 2 rhombuses, and 2 trapezoids.

a) Draw a diagram to represent this situation.

b) What fraction of 1 whole is covered?

19. Suppose 1 hexagon = 1 whole. You have 2 trapezoids, 1 rhombus, and 3 triangles. How many hexagons can you make? Show how you found your answer.

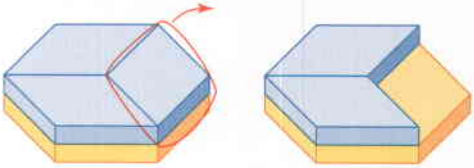
3.2

Subtract Fractions Using Manipulatives

Focus on...

- representing fractions using concrete materials
- using patterns to subtract fractions

The hexagon represents 1. What fraction of the hexagon is still covered?



Discover the Math

How can you use pattern blocks to represent subtracting fractions?

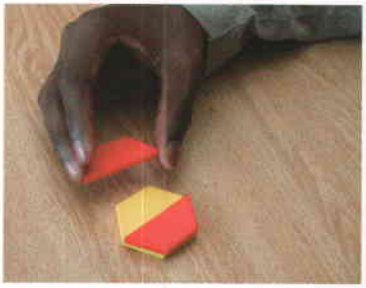
Materials

- pattern blocks
- pencil crayons
- BLM 3.1A Pattern Block Worksheet

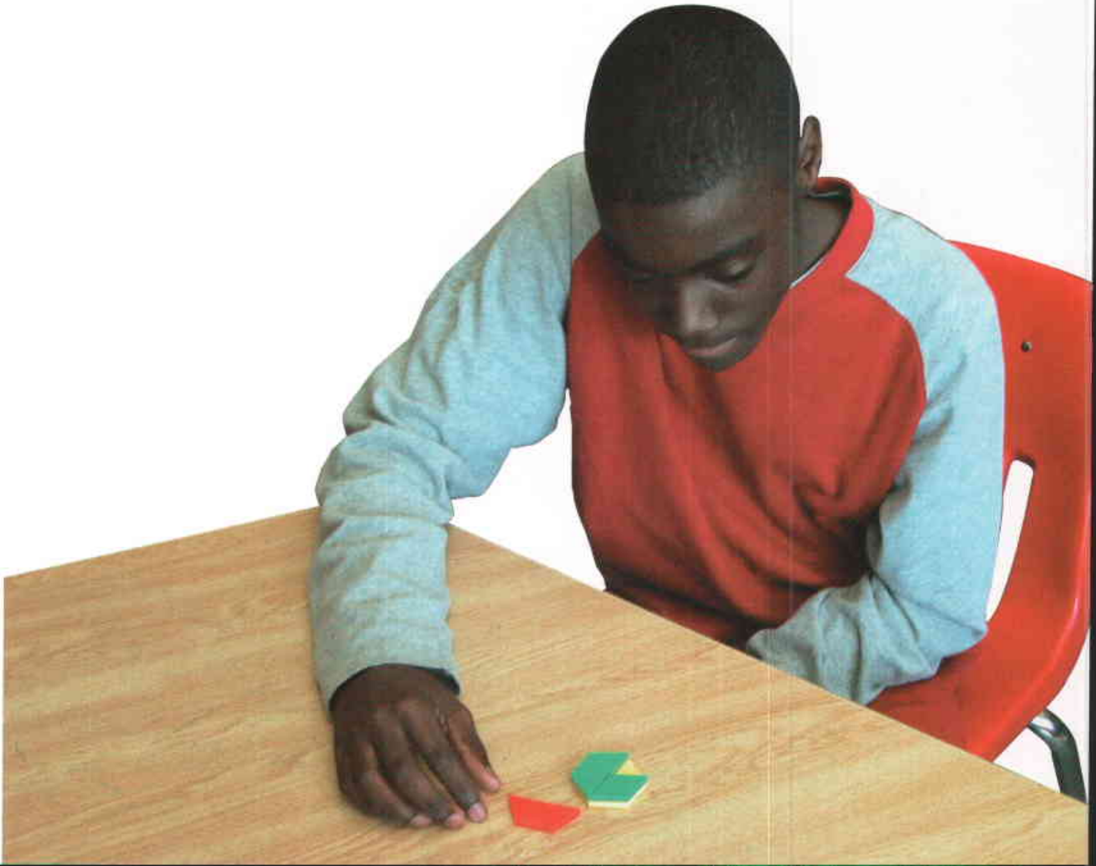
Strategies

What strategies are you using?

1. Use pattern blocks to show subtracting $\frac{1}{2}$. How many different ways can you take away $\frac{1}{2}$? Record your answers.



3. **Reflect** How can pattern blocks help you represent subtracting fractions?

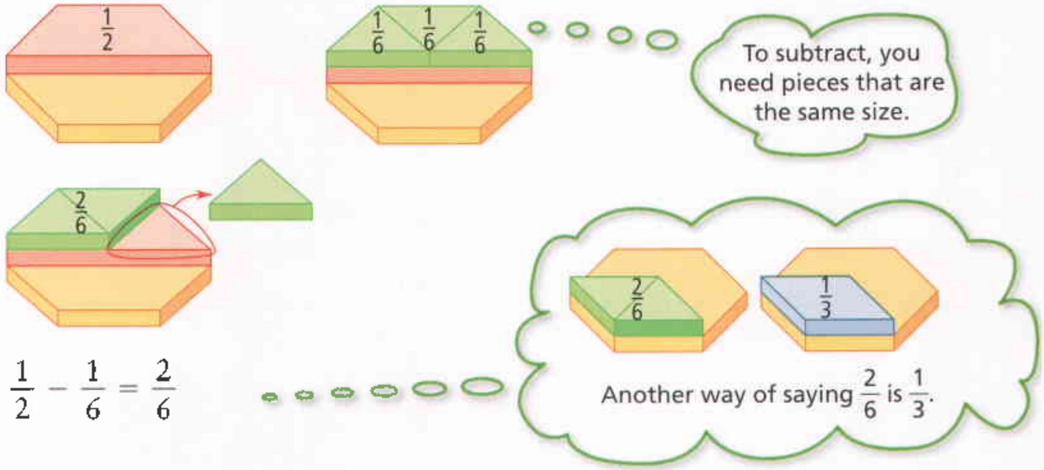


Example: Subtract Fractions

Subtract $\frac{1}{2} - \frac{1}{6}$.

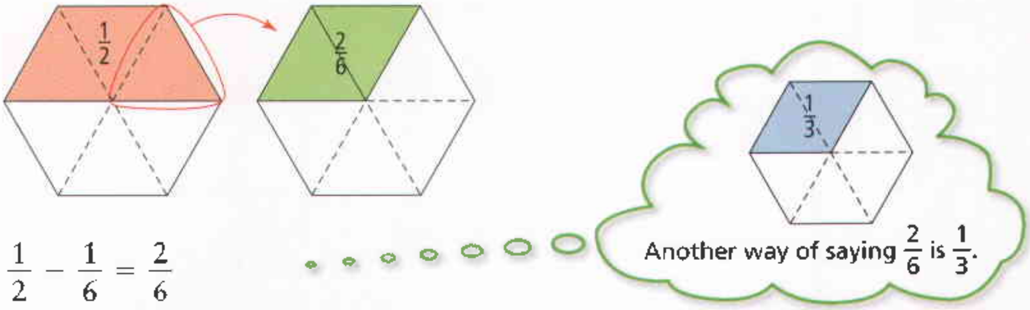
Solution

Method 1: Use Concrete Materials



$\frac{1}{2} - \frac{1}{6} = \frac{2}{6}$

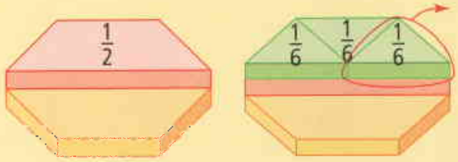
Method 2: Use a Diagram



$\frac{1}{2} - \frac{1}{6} = \frac{2}{6}$

Key Ideas

- When subtracting fractions using concrete materials,
  - represent each fraction using parts of equal size
  - remove the blocks represented by the fraction that is being subtracted
  - identify the fraction that remains



Communicate the Ideas

1. Show $\frac{2}{3}$ in as many ways as you can.

2. Show visually how to subtract $\frac{2}{3} - \frac{1}{6}$.

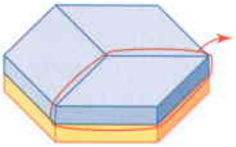
Check Your Understanding

Practise

For help with questions 3 to 10, refer to the Example.

3. Write a subtraction sentence to represent each diagram.

a)



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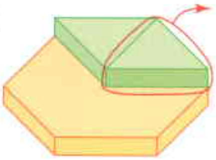

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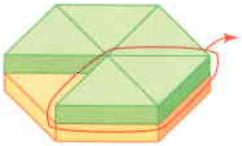
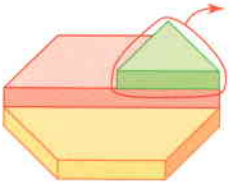


4. Draw a diagram to show the answer to each part in question 3.

5. Write a subtraction sentence to represent each diagram.

a)



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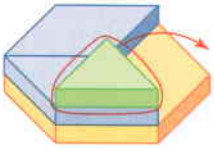
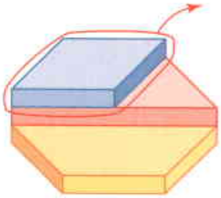


c)



6. Draw a diagram to show the answer to each part in question 5.

7. Use concrete materials or diagrams to show each subtraction.

a) $1 - \frac{1}{2}$

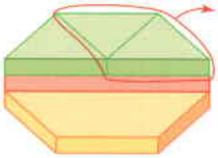
b) $\frac{5}{6} - \frac{1}{6}$

c) $\frac{2}{3} - \frac{1}{3}$

d) $1 - \frac{2}{3}$

8. Write a subtraction sentence to represent each diagram.

a)



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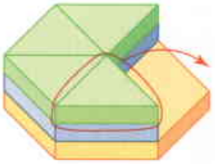


9. Draw a diagram to show the answer to each part in question 8.

10. Use concrete materials or diagrams to show each subtraction.

a) $\frac{1}{2} - \frac{2}{6}$

b) $\frac{1}{2} - \frac{1}{3}$

c) $\frac{1}{3} - \frac{1}{6}$

d) $\frac{5}{6} - \frac{1}{3}$

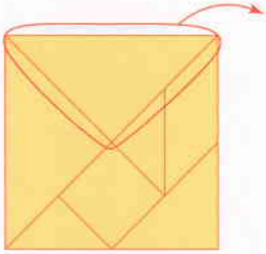
Apply

11. Use concrete materials or diagrams to show each subtraction.

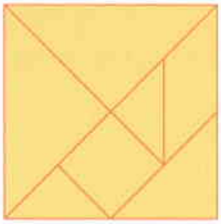
a) $\frac{5}{6} - \frac{1}{2}$

b) $\frac{2}{3} - \frac{1}{2}$

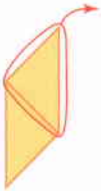
12. One of the tangram pieces is removed from a finished tangram. Write a subtraction sentence to show the fraction of the completed tangram that remains.



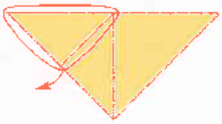
13. Use the tangram diagram to help you answer the following questions.



a) Suppose 1 parallelogram = 1 whole. Write a subtraction sentence to represent this diagram.

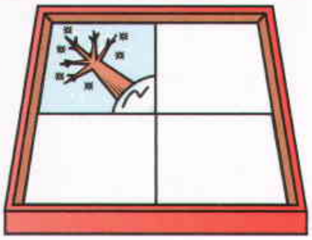


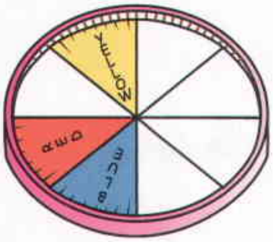
b) Suppose 1 large triangle = 1 whole. Write a subtraction sentence to represent this diagram.



Chapter Problem

14. Write a subtraction sentence to represent each puzzle. Explain what each sentence represents. Solve it.





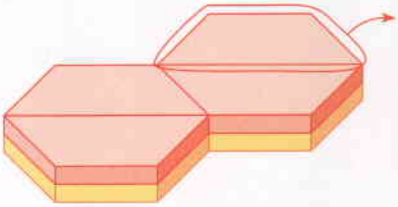
15. Use concrete materials of your own choice to make up two subtraction questions. Draw diagrams to show your questions. Challenge a classmate to solve your questions using the materials you chose.

Extend

16. Suppose 2 hexagons = 1 whole.

a) What fraction of the whole does one red trapezoid represent?

b) Write a subtraction sentence to represent this diagram.



c) Show the answer to the subtraction sentence.

17. Suppose 2 hexagons = 1 whole.

a) Show $\frac{1}{2} - \frac{1}{3}$ visually.

b) Explain how this representation differs from the representation when 1 hexagon = 1 whole.

c) How is the answer to part a) related to the answer when 1 hexagon = 1 whole?

3.3

Find Common Denominators

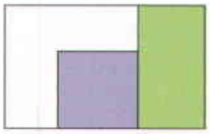
Focus on...

- common denominators

Materials

- paper
- pencil crayons

Estimate how much of the rectangle is covered. Is it more or less than $\frac{1}{2}$? Is it more or less than 1?



Discover the Math

How can you find a common denominator?

1. Fold a square piece of paper one way into quarters. Open it up and colour $\frac{1}{4}$.

2. Now, fold the paper into thirds the other way.

3. Open up the paper.

a) How many parts is the paper divided into now?

b) How many sections are shaded? Name an **equivalent fraction** for $\frac{1}{4}$.

4. Use a fresh piece of paper. Fold the piece of paper into thirds one way. Open it up and colour $\frac{1}{3}$.

5. Now, fold the paper into quarters the other way.

6. Open up the paper. Name an equivalent fraction for $\frac{1}{3}$.

7. **Reflect** How can paper folding help you find a **common denominator**?

equivalent fractions

- two or more fractions that represent the same part of a whole or a group

common denominator

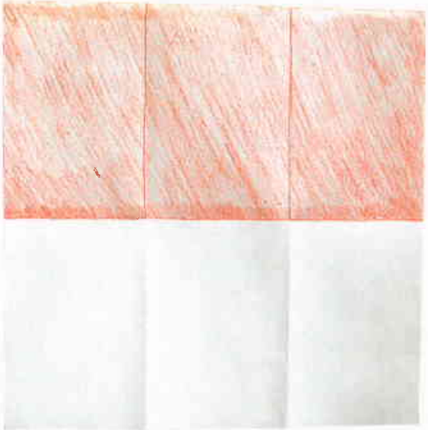
- a number that is a common multiple of the denominators of a set of fractions
- a common denominator for $\frac{1}{2}$ and $\frac{1}{5}$ is 10

Example: Find a Common Denominator

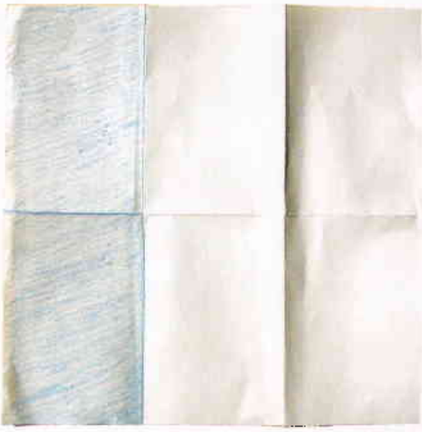
Find a common denominator for $\frac{1}{2}$ and $\frac{1}{3}$.

Method 1: Use Paper Folding

Fold a square piece of paper in half one way and in thirds the other way.



$\frac{1}{2} = \frac{3}{6}$

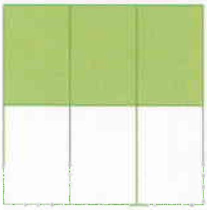


$\frac{1}{3} = \frac{2}{6}$

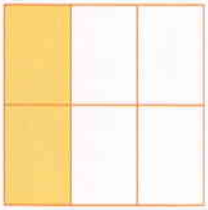
A common denominator for $\frac{1}{2}$ and $\frac{1}{3}$ is 6.

Method 2: Use a Diagram

Divide a square in half one way and in thirds the other way.



$\frac{1}{2} = \frac{3}{6}$



$\frac{1}{3} = \frac{2}{6}$

A common denominator for $\frac{1}{2}$ and $\frac{1}{3}$ is 6.

Method 3: Use Multiples

The denominator of $\frac{1}{2}$ is 2. **Multiples** of 2 are 2, 4, 6, 8, ...

The denominator of $\frac{1}{3}$ is 3. Multiples of 3 are 3, 6, 9, 12, ...

The multiple 6 is in both lists.

A common denominator for $\frac{1}{2}$ and $\frac{1}{3}$ is 6.

Strategies

Look for a pattern

Literacy Connections

Reading the Symbol ...

When you see ..., this means "and so on." The list continues.

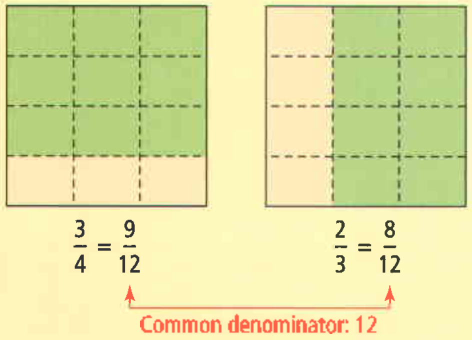
multiple

- the product of a given number and a natural number
- multiples of 2 are 2, 4, 6, 8, ...

Key Ideas

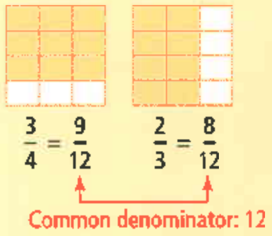
- Paper folding, diagrams, and multiples can be used to find a common denominator.

Paper Folding


$\frac{3}{4} = \frac{9}{12}$ $\frac{2}{3} = \frac{8}{12}$

Common denominator: 12

Diagrams


$\frac{3}{4} = \frac{9}{12}$ $\frac{2}{3} = \frac{8}{12}$

Common denominator: 12

Multiples

The denominator of $\frac{3}{4}$ is 4. Multiples of 4 are 4, 8, 12, 16, 20, ...

The denominator of $\frac{2}{3}$ is 3. Multiples of 3 are 3, 6, 9, 12, 15, 18, ...

Common denominator: 12

Communicate the Ideas

1. Mei is trying to find a common denominator for $\frac{1}{5}$ and $\frac{2}{8}$. She says that a common denominator is 13. Do you agree with her? What might you say to Mei?

2. Brian found both 24 and 42 as common denominators for $\frac{1}{3}$ and $\frac{5}{6}$. Explain how this is possible. What other common denominators are there?
Brian's work:

Multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42
Multiples of 6 are 6, 12, 18, 24, 30, 36, 42
Two common denominators are 24 and 42.

3. A common denominator for $\frac{1}{2}$ and $\frac{1}{3}$ is 6.
What other common denominators could be used? Which denominator would be best to use? Explain why.

Check Your Understanding

Practise

For help with questions 4 and 5, refer to the Example.

4. Use paper folding or a diagram to find a common denominator for each pair of fractions.

a) $\frac{1}{3}$ and $\frac{1}{4}$ **b)** $\frac{1}{2}$ and $\frac{1}{8}$

c) $\frac{1}{6}$ and $\frac{1}{4}$ **d)** $\frac{1}{5}$ and $\frac{1}{2}$

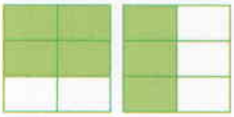
5. Use multiples to find a common denominator for each pair of fractions.

a) $\frac{1}{5}$ and $\frac{1}{3}$ **b)** $\frac{1}{7}$ and $\frac{1}{3}$

c) $\frac{1}{4}$ and $\frac{1}{10}$ **d)** $\frac{1}{6}$ and $\frac{1}{8}$

6. Write two equivalent fractions for the shaded part of each diagram in each pair.

a)



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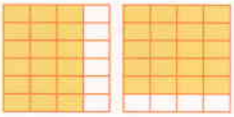
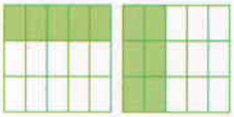
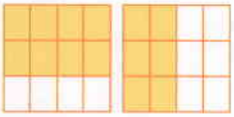


c)



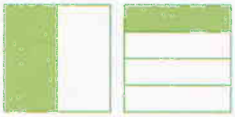
d)



7. State a common denominator for each pair of diagrams in question 6.

8. State a common denominator for each pair of diagrams.

a)



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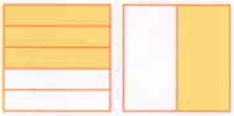


Apply

9. Find two common denominators for each pair of fractions.

a) $\frac{1}{3}$ and $\frac{1}{5}$ **b)** $\frac{1}{4}$ and $\frac{1}{6}$

10. Find three common denominators for each pair of fractions.

a) $\frac{2}{3}$ and $\frac{1}{2}$ **b)** $\frac{1}{2}$ and $\frac{3}{8}$

11. Name all the common denominators between 1 and 40 for the fractions $\frac{1}{4}$ and $\frac{2}{3}$.

12. a) Name two common denominators between 10 and 20 for the fractions $\frac{1}{2}$ and $\frac{1}{6}$.

b) Describe how you found the common denominators.

13. Find a common denominator for each pair of fractions. Use equivalent fractions to rewrite each pair of fractions with the common denominator.

a) $\frac{3}{5}$ and $\frac{1}{2}$

b) $\frac{5}{8}$ and $\frac{1}{4}$

Try This!

14. Which fraction in each pair is greater? Show at least one method.

a) $\frac{5}{6}$ and $\frac{2}{3}$

b) $\frac{2}{5}$ and $\frac{1}{4}$

Extend

15. Find a common denominator for each group of fractions. Describe your method.

a) $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$

b) $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$

3.4

Add and Subtract Fractions Using a Common Denominator

Focus on…

- common denominator
- adding and subtracting fractions

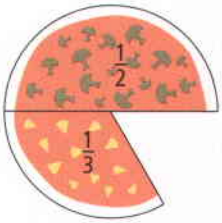
Suppose you invite a friend over for pizza. You order a six-slice pizza. One slice is left after you both eat. What fraction of the pizza has been eaten? How can you show this using numbers?

Discover the Math

How can you add and subtract fractions using a common denominator?

Example 1: Add Fractions

How much pizza is left?

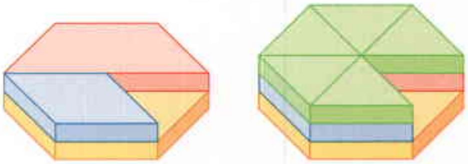


Solution

Strategies
Make a model

Method 1: Use Manipulatives

To add fractions, the pieces have to be the same size.



Each green triangle represents $\frac{1}{6}$.

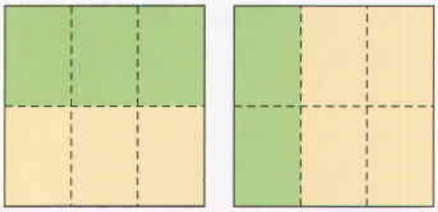
Count the number of pieces.

$\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$

$\frac{5}{6}$ of the pizza is left.

Method 2: Use a Common Denominator

Fold a piece of paper in half one way and then in thirds the other way. Colour $\frac{1}{2}$ of the page. Three sections are coloured. Fold another piece of paper in thirds one way and then in half the other way. Colour $\frac{1}{3}$ of the page. Two sections are coloured.



$\frac{1}{2} = \frac{3}{6}$ $\frac{1}{3} = \frac{2}{6}$

A common denominator is 6.

$$\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6}$$ **Add the numerators**

$$= \frac{5}{6}$$

$\frac{5}{6}$ of the pizza is left.

Example 2: Subtract Fractions

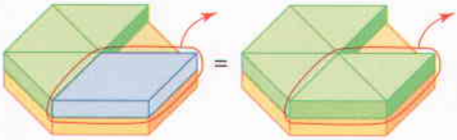
Subtract $\frac{5}{6} - \frac{1}{3}$.

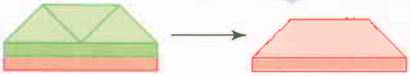
Solution

Method 1: Use Manipulatives

To subtract fractions, the pieces have to be the same size.

$$\frac{5}{6} - \frac{1}{3} = \frac{3}{6}$$





$\frac{3}{6}$ is another way of saying $\frac{1}{2}$.

Method 2: Use Multiples

Multiples of 6 are ⑥, 12, 18, ….

Multiples of 3 are 3, ⑥, 9, 12, ….

The first common denominator in the two lists is 6.

Write equivalent fractions with 6 as a denominator.

$$\frac{5}{6} = \frac{5}{6} \qquad \frac{1}{3} = \frac{1 \times 2}{3 \times 2}$$

$$= \frac{2}{6}$$

To find equivalent fractions, multiply the numerator and denominator by the same number.

$$\frac{5}{6} - \frac{1}{3} = \frac{5}{6} - \frac{2}{6}$$ **Subtract the numerators.**

$$= \frac{3}{6}$$

Literacy Connections

Adding and Subtracting Fractions

3 ← number of shaded pieces

6 ← number of pieces in whole or group

When you add or subtract fractions with the same denominator, add or subtract only the number of shaded pieces (numerator) in each fraction. The number of pieces in the whole or group (denominator) stays the same.

Literacy Connections

Remembering Common Denominators

People who have friends in common share the *same* friends. Fractions that have a common denominator share the *same* denominator.

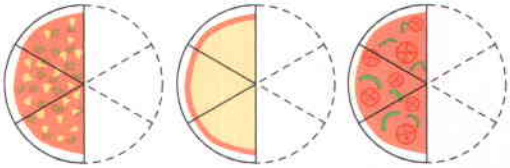
$\frac{1}{2} = \frac{3}{\textcircled{6}}$

$\frac{1}{3} = \frac{2}{\textcircled{6}}$

common denominator

Example 3: Add the Same Fraction

How much pizza is left over?



Solution

Method 1: Use Manipulatives

Strategies
Make a model

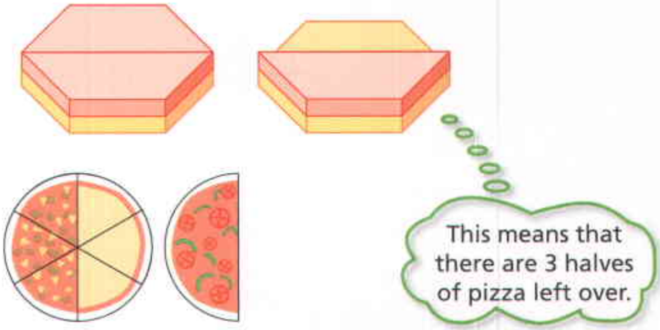
Each red trapezoid represents $\frac{1}{2}$.

There are 3 trapezoids.

$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2}$

$\frac{3}{2}$ can be written as the mixed number $1\frac{1}{2}$.

There are $\frac{3}{2}$ or $1\frac{1}{2}$ pizzas left over.



This means that there are 3 halves of pizza left over.

Method 2: Multiply

Each strip shows $\frac{1}{2}$.

There are 3 half strips.

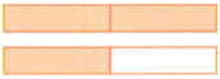
3 halves = $3 \times \frac{1}{2}$

Multiply the whole number 3 by the numerator.

$3 \times \frac{1}{2} = \frac{3}{2}$

$\frac{3}{2}$ can be written as the mixed number $1\frac{1}{2}$.

There are $\frac{3}{2}$ or $1\frac{1}{2}$ pizzas left over.



Key Ideas

- To add or subtract fractions with different denominators, use a common denominator.
- Repeated addition of the same fraction can be written as multiplication.

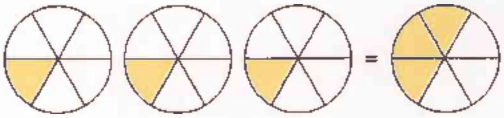
Communicate the Ideas

1. Use diagrams to subtract $\frac{1}{2} - \frac{1}{3}$.

2. Use a common denominator to subtract $\frac{1}{2} - \frac{1}{3}$. Explain why you chose your common denominator.

3. Add $\frac{5}{6} + \frac{1}{2}$ in two different ways. Explain which method you prefer and why.

4. Describe two ways to represent the diagram using numbers.



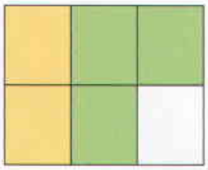
Check Your Understanding

Practise

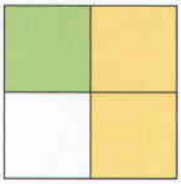
For help with question 5, refer to Example 1.

5. Write an addition sentence to represent the fraction of each figure that is shaded?

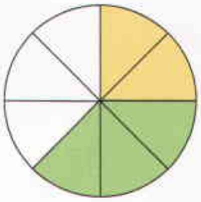
a)



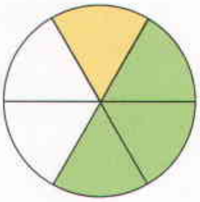
b)



c)



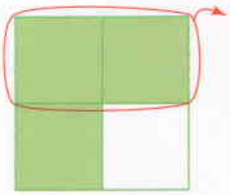
d)



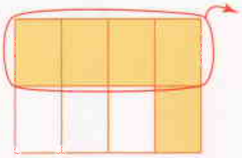
For help with question 6, refer to Example 2.

6. What fraction of each figure remains?

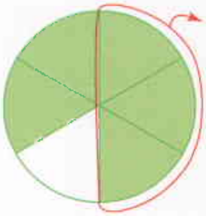
a)



b)



c)



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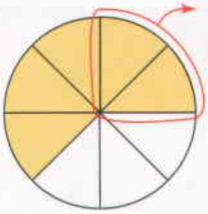
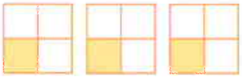


For help with question 7, refer to Example 3.

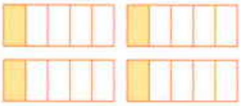
7. In each diagram, what fraction of a whole is shaded?

a)



b)

c)



8. Rewrite each expression with a common denominator. Subtract.

a) $\frac{2}{3} - \frac{1}{6}$ **b)** $\frac{3}{4} - \frac{1}{2}$

c) $\frac{7}{8} - \frac{1}{4}$ **d)** $\frac{3}{5} - \frac{1}{2}$

9. Rewrite each expression with a common denominator. Add.

a) $\frac{1}{5} + \frac{1}{2}$ **b)** $\frac{1}{4} + \frac{5}{6}$

c) $\frac{2}{5} + \frac{2}{3}$ **d)** $\frac{1}{6} + \frac{1}{2}$

10. Write each repeated addition as a multiplication and evaluate. Show your answer as an improper fraction and as a mixed number.

a) $\frac{1}{2} + \frac{1}{2} + \frac{1}{2}$

b) $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$

Apply

11. a) Evaluate each of the following.

$2 \times \frac{1}{2} =$ ■ $3 \times \frac{1}{3} =$ ■

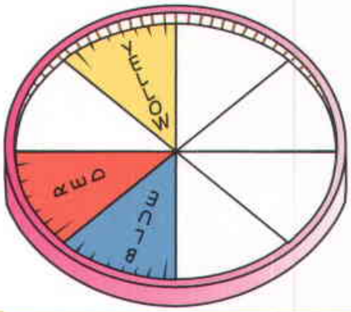
$4 \times \frac{1}{4} =$ ■ $5 \times \frac{1}{5} =$ ■

b) Describe the pattern you see.

c) Predict the value of $20 \times \frac{1}{20}$.

Chapter Problem

12. Use multiplication to describe the number of pieces in this puzzle.



13. Write each repeated addition as a multiplication and evaluate. Show your answer as an improper fraction and as a mixed number.

a) $\frac{3}{2} + \frac{3}{2} + \frac{3}{2} + \frac{3}{2} + \frac{3}{2}$

b) $\frac{4}{3} + \frac{4}{3} + \frac{4}{3}$

14. Cheryl and Monica volunteered to shovel the snow from Monica's driveway.

If the two girls shovelled the whole driveway, were they both correct? Explain.

15. Kayla used the following diagram and explanation to add $\frac{1}{4} + \frac{3}{8}$.

$$\frac{1}{4} + \frac{3}{8} = \frac{1 + 3}{4 + 8}$$
$$= \frac{4}{12}$$

a) Explain the error in Kayla's solution.

b) Redraw the diagram to show the correct answer. Show the correct equation.

16. Jamal ate $\frac{1}{3}$ of a pizza for lunch. His friend Kevin ate $\frac{1}{4}$ of the same pizza.

a) What fraction of the pizza did the two friends eat?

b) What fraction of the pizza was left over? Draw a diagram to show your answer.

17. Which is greater, $\frac{2}{5} + \frac{1}{2}$ or $\frac{2}{3} + \frac{1}{6}$? Show how you know.

18. Which is greater, $1 - \frac{2}{3}$ or $1 - \frac{5}{8}$? Show how you know.

19. Add. Hint: Use multiples to find a common denominator.

a) $\frac{1}{2} + \frac{1}{3} + \frac{1}{4}$

b) $\frac{1}{4} + \frac{2}{5} + \frac{3}{10}$

20. a) Draw a diagram to show $\frac{1}{4} + \frac{1}{2}$.

b) Draw a diagram to show $\frac{2}{3} + \frac{1}{6}$.

c) Which sum is larger? How do you know?

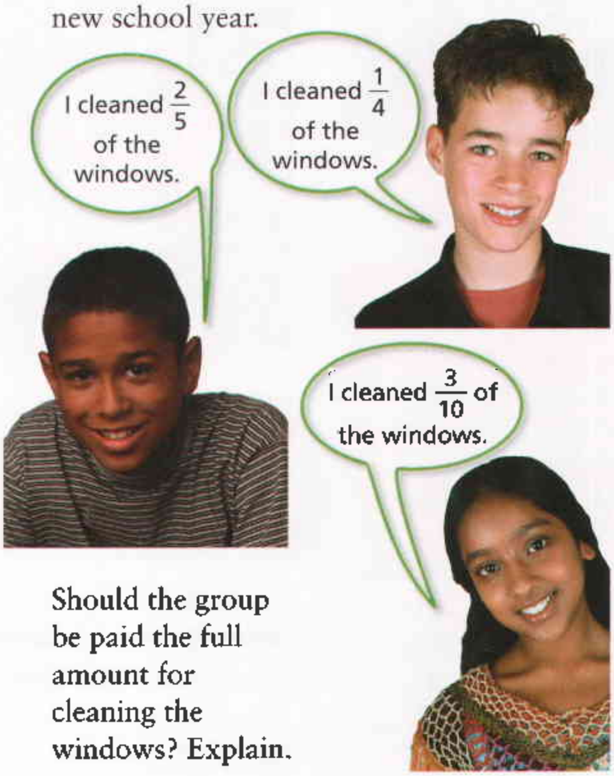
Extend

21. Use diagrams to show how you would add $1 + \frac{1}{2} + \frac{1}{4}$.

22. Evaluate.

a) $1 + \frac{1}{3} + \frac{3}{4}$ **b)** $2 - \frac{2}{5} + \frac{1}{2}$

23. Three students were hired to clean the school windows before the start of the new school year.



Should the group be paid the full amount for cleaning the windows? Explain.

Making Connections

Order of Operations and Fractions

You learned about the order of operations in Chapter 1. The order of operations also applies to fractions. Add or subtract the fractions in brackets first.

Evaluate.

a) $\left(\frac{1}{4} + \frac{1}{2}\right) - \frac{1}{3}$ **b)** $\frac{1}{2} + \left(\frac{2}{3} - \frac{1}{3}\right)$

3.5

More Fraction Problems

Focus on...

- solving problems with fractions
- using appropriate strategies and calculation methods
- explaining the problem solving process

Count the leftover sandwich pieces on the tray.
How many sandwiches are left?

Literacy Connections

Reading Problems

Read the problem. Write it in your own words.

What information are you given?

What information do you need?

What is the problem asking you to do?

Discover the Math

What different strategies can you use to solve problems containing fractions?

Example 1: Leftover Sandwiches

For a party, Janine's mother serves a tray with sandwiches cut in quarters. After the party, there are 9 pieces left. How many sandwiches is this?

Solution

Understand

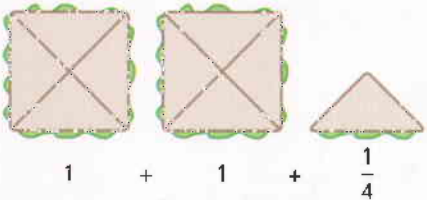
Find how many sandwiches are left.

Plan

Count the sandwich pieces left on the plate. How many whole sandwiches is this?

Do It!

Strategies
Make a picture or diagram



There are $2\frac{1}{4}$ sandwiches left.

Look Back

Multiply.

$9 \times \frac{1}{4} = \frac{9}{4}$

$\frac{9}{4}$ is another way of saying $2\frac{1}{4}$.

Example 2: Leftover Muffins and Oranges

The coach prepared a snack for her team. She cut muffins in half and oranges in eighths. After the game, there were five pieces of muffin and 13 pieces of orange left. How many muffins and how many oranges is this?

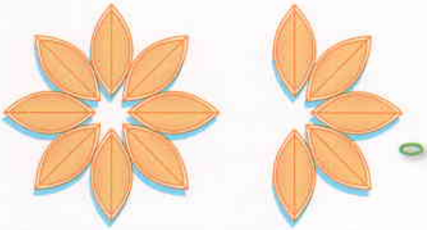
Solution

Strategies
Look for a pattern

Place pieces together to make whole muffins and whole oranges.


1 muffin = 2 pieces 2 muffins = 4 pieces 3 muffins = 6 pieces

There are more than 2 whole muffins. This is $2\frac{1}{2}$.


1 orange = 8 pieces 2 oranges = 16 pieces

There are less than two whole oranges. This is $1\frac{5}{8}$.

There are $2\frac{1}{2}$ muffins and $1\frac{5}{8}$ oranges left.

Key Ideas

- When solving a problem, read what is being asked. Write it in your own words.
- Decide what strategy to use. Use a diagram or manipulatives if you need to.
- Check your answer by using a different strategy to solve the problem.

Communicate the Ideas

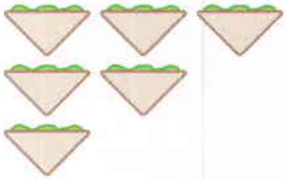
1. Describe one strategy you could use to solve this problem.

The head server in a restaurant cuts pies into 6 equal pieces. At the end of the day, there are 8 pieces left. How much pie is left?

2. Solve the pie problem using a different strategy. Show your solution.

3. In a group or as a class, discuss the various strategies you used for questions 1 and 2. Which strategies are most efficient? Explain.

4. How many whole sandwiches are there? How do you know?

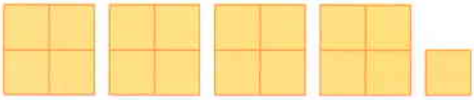


Check Your Understanding

Practise

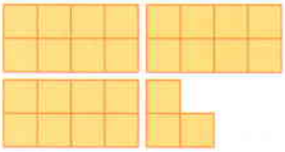
5. What fraction does each set of diagrams represent?

a)



b)

c)



6. Write a fraction for each situation.

a) 5 red pens out of a package of 12 pens

b) 3 baseballs out of a box of 8 balls

c) 2 green T-shirts in a stack of 5 T-shirts

Apply

For help with questions 7 to 9, refer to the Example.

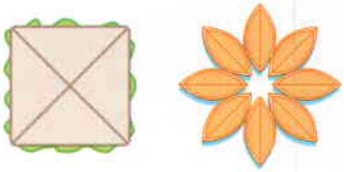
7. A plate contains sandwiches cut into quarters. After lunch, there are 5 pieces left. How many sandwiches is this?

8. On pizza day, several 8-slice pizzas are ordered. At the end of lunch, there are 13 slices left. How many pizzas is this?

9. Several paper plates are cut in half for an arts and crafts class. After the class, there are 7 pieces left. How many paper plates is this?

For help with questions 10 and 11, refer to Example 2.

10. A large snack plate contains sandwiches cut into quarters and oranges cut into eighths. At the end of snack time, there are 14 pieces of sandwich and 12 pieces of orange left. How many sandwiches and how many oranges is this?



11. The school chef cut up muffins and pizzas to give to his helpers. He cut muffins in half and pizzas in sixths. After serving his helpers, the chef had three muffin pieces and eight pizza slices left. How many muffins and how many pizzas is this?

12. A fruit plate contains orange wedges. Each wedge is $\frac{1}{8}$ of a whole orange. There are enough orange wedges left on the plate to make $2\frac{1}{4}$ whole oranges. How many wedges are left on the plate?

13. Sixteen marbles are placed in a bag. Five of the marbles are blue, 3 are purple, 2 are white, and 6 are green.

a) What fraction of the marbles is each colour?

b) What fraction of the marbles is green or blue?

c) What fraction of the marbles is not purple?

14. Aleta offered her 5 friends watermelon wedges one hot summer afternoon. She cut two slices of watermelon into equal-sized wedges. She and her friends each ate one wedge. What fraction of a whole slice was each wedge?

15. There are 25 students in Selena's class. Her teacher divides the class into 5 teams for a game.

a) What fraction of the class is on each team?

b) The next day, four students are absent. What fraction could the teacher divide the class into now to have an equal number of students on each team? Describe your solution.

16. The length of a rectangular carpet is 15 m. The width is $\frac{1}{3}$ of its length.

a) Sketch a diagram of the carpet and label the given information.

b) Find the area of the carpet.

c) Find the perimeter of the carpet.

Try This!

17. Malak's birthday cake is cut into 12 pieces. His family eats $\frac{1}{3}$ of the cake for dessert.

a) What fraction of the cake is left over? Describe your solution.

b) The next day Malak shares the leftover cake with friends. He and his friends each have one piece and the cake is all gone. How many friends did he share with?

c) If Malak cut the leftover pieces of cake in half, how many friends could he share with? Describe your solution.

Extend

18. a) The sum of each row and column must be 1. Find the missing values.

$\frac{1}{3}$	$\frac{1}{4}$	🟥
$\frac{7}{12}$	🟥	$\frac{1}{6}$
🟥	$\frac{1}{2}$	$\frac{5}{12}$

b) Create a similar fraction square in which the sums equal 2.

19. Ms. Getfit is a math teacher who also teaches physical education. To get her students thinking about math in their physical education classes, Ms. Getfit uses a new format for a relay race. There are 8 teams competing and each team has 3 people. The first person on each team runs $\frac{1}{4}$ of a lap of the track. The second person runs $\frac{1}{6}$ of a lap of the track, and the third person runs $\frac{1}{3}$ of a lap of the track. How many laps of the track are run in total by all 8 teams combined? Describe your solution.

CHAPTER
3

Review

Key Words

Match each term with its meaning. In your notebook, write each term with its correct meaning.

1. The ______ of 4 are 4, 8, 12, 16, ….

2. $\frac{6}{4}$ is an example.

3. $1\frac{1}{2}$ is an example.

4. $\frac{1}{2}, \frac{2}{4}, \frac{3}{6},$ and $\frac{4}{8}$ are examples.

5. 2 is this part of the fraction $\frac{2}{3}$.

6. 4 is this part of the fraction $\frac{3}{4}$.

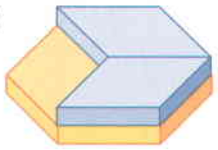
7. $\frac{1}{2}$ and $\frac{2}{3}$ have 6 as a ______ ______.

A numerator
B equivalent fractions
C denominator
D multiples
E divisor
F improper fraction
G common denominator
H mixed number

3.1 Add Fractions Using Manipulatives, pages 86–89

8. Write an addition sentence to represent the fraction of each hexagon that is covered.

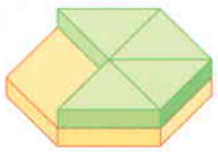
a)



b)

c)

d)



9. Use pattern blocks or diagrams to model each addition.

a) $\frac{1}{2} + \frac{1}{3}$ **b)** $\frac{2}{3} + \frac{1}{6}$

c) $\frac{1}{6} + \frac{1}{2}$

10. Draw a diagram to show the answer to each addition in question 9.

11. Suppose 1 hexagon = 1 whole.

a) Cover a hexagon with 1 trapezoid, 1 rhombus, and 1 triangle. Draw a diagram to represent this situation.

b) Write an addition statement to represent the diagram. Evaluate.

12. Use pattern blocks or diagrams to show each sum.

a) $\frac{1}{3} + \frac{1}{3}$

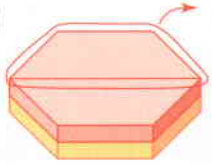
b) $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$

13. What multiplication statement does each part in question 12 represent?

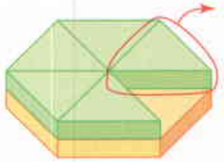
3.2 Subtract Fractions Using Manipulatives, pages 90–93

14. Write a subtraction sentence to represent each diagram.

a)



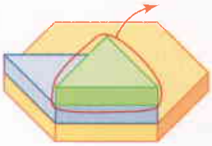
b)



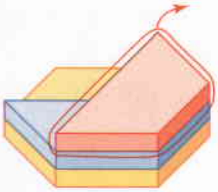
15. Draw a diagram to show the answer to each subtraction in question 14.

16. Write a subtraction sentence to represent each diagram.

a)



b)



17. Draw a diagram to show the answer to each subtraction in question 16.

18. Use pattern blocks or diagrams to model each subtraction.

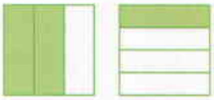
a) $\frac{2}{3} - \frac{1}{3}$ **b)** $\frac{1}{2} - \frac{1}{6}$

c) $\frac{5}{6} - \frac{2}{3}$

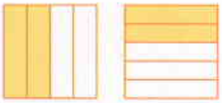
3.3 Find Common Denominators, pages 94–97

19. State a common denominator for each pair of diagrams.

a)



b)



20. Use paper folding or a diagram to find a common denominator for each pair of fractions.

a) $\frac{1}{3}$ and $\frac{5}{6}$ **b)** $\frac{3}{4}$ and $\frac{1}{6}$

21. Use multiples to find a common denominator for each pair of fractions.

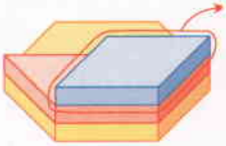
a) $\frac{1}{4}$ and $\frac{2}{3}$ **b)** $\frac{2}{5}$ and $\frac{1}{2}$

22. Find two common denominators between 10 and 20 for $\frac{1}{2}$ and $\frac{1}{3}$.

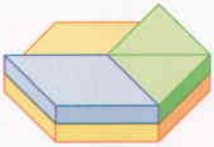
3.4 Add and Subtract Fractions Using a Common Denominator, pages 98–103

23. Write and evaluate the addition or subtraction sentence represented by each diagram.

a)



b)



24. Draw a diagram to represent each addition or subtraction.

a) $\frac{5}{6} - \frac{3}{8}$ **b)** $\frac{3}{4} + \frac{3}{5}$

25. Evaluate.

a) $\frac{9}{10} - \frac{2}{3}$ **b)** $\frac{1}{4} + \frac{5}{6}$

3.5 More Fraction Problems, pages 104–107

26. A plate contains sandwiches cut into thirds. After lunch, there are 7 pieces left over. How many sandwiches is this?

27. Twenty-four cars are parked in a parking lot. Six of the cars are red, 4 are blue, 9 are white, and 5 are grey.

a) What fraction of the cars is each colour?

b) What fraction of the cars is red or blue?

c) What fraction of the cars is not white?

28. A snack plate contains muffins cut half and apples cut into sixths. At the end of snack time, there are 5 muffin pieces and 11 pieces of apple left. How many muffins and how many apples is this?

CHAPTER
3

Practice Test

Strand Questions	NSN 1–14	MEA	GSS	PA	DMP

Multiple Choice

For questions 1 to 4, select the correct answer.

1. What fraction of the hexagon is covered?

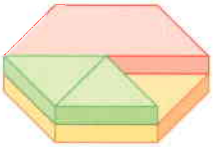
A $\frac{1}{3}$ **B** $\frac{1}{2}$

C $\frac{2}{3}$ **D** $\frac{5}{6}$

2. What fraction of the hexagon is covered?

A $\frac{2}{6}$ **B** $\frac{4}{6}$

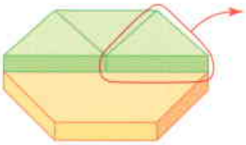
C $\frac{1}{6}$ **D** $\frac{5}{6}$



3. Evaluate the subtraction represented by the diagram.

A $\frac{5}{6}$ **B** $\frac{2}{6}$

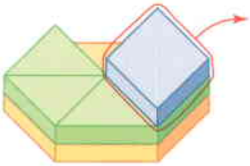
C $\frac{1}{2}$ **D** $\frac{1}{6}$



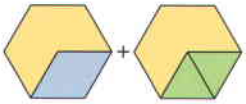
4. Suppose 1 hexagon = 1 whole. The diagram models

A $\frac{5}{6} - \frac{1}{3}$ **B** $\frac{1}{3} + \frac{2}{6}$

C $\frac{5}{6} + \frac{1}{3}$ **D** $\frac{1}{3} - \frac{2}{6}$



5. Suppose 1 hexagon = 1 whole.



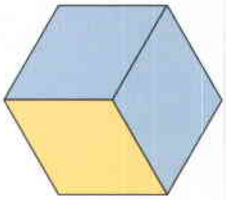
The diagram shows

A $\frac{2}{3} + \frac{1}{6}$ **B** $\frac{1}{2} + \frac{1}{3}$

C $\frac{1}{3} + \frac{2}{6}$ **D** $\frac{3}{6} + \frac{1}{3}$

Short Answer

6. Suppose 1 hexagon = 1 whole. The diagram shows 2 blue rhombi covering 1 yellow hexagon.



You want to subtract $\frac{1}{2}$.

a) Draw a diagram to show this subtraction.

b) Write and evaluate the subtraction statement represented.

7. Rewrite each repeated addition as a multiplication statement and evaluate. Show your answer as an improper fraction and a mixed number, if necessary.

a) $\frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$

b) $\frac{2}{7} + \frac{2}{7} + \frac{2}{7} + \frac{2}{7} + \frac{2}{7}$

8. Find a common denominator for each pair of fractions.

a) $\frac{1}{2}$ and $\frac{3}{4}$ **b)** $\frac{2}{3}$ and $\frac{2}{5}$

9. Find two common denominators between 10 and 30 for $\frac{1}{3}$ and $\frac{1}{4}$.

10. Evaluate.

a) $\frac{3}{8} + \frac{2}{3}$

b) $\frac{5}{6} - \frac{4}{5}$

11. a) Which is greater, $1 - \frac{3}{4}$ or $1 - \frac{3}{10}$?

b) Explain the strategy you used to solve part a).

12. Eric and Afsha are both trying to add $\frac{3}{5} + \frac{2}{3}$. Eric says the answer is $1\frac{4}{15}$. Afsha says the answer is $\frac{5}{8}$. Which student is correct? Show how you know.

13. Several apples are cut into slices for a class snack. Each slice is $\frac{1}{12}$ of a whole apple. After the snack, there are enough apple slices left over to make $1\frac{1}{3}$ whole apples. How many apple slices are left over?

Extended Response

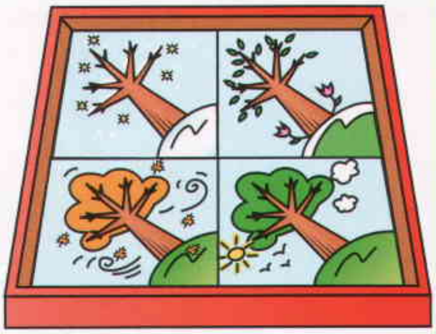
14. Mia and Steven both found a common denominator for $\frac{1}{2} + \frac{2}{3} + \frac{3}{4}$. Mia said, "The common denominator is 12." Steven said, "The common denominator is 24."

a) Who is correct? Explain.

b) Use a common denominator to evaluate $\frac{1}{2} + \frac{2}{3} + \frac{3}{4}$

Chapter Problem Wrap-Up

1. You are asked to create a puzzle for the grade 6 class to help them understand fractions. Your puzzle is to be $\frac{1}{6}$ red, $\frac{1}{6}$ blue, and $\frac{1}{3}$ green. The rest of the puzzle is to be yellow.



a) Design a puzzle that fits this description. Use grid paper, coloured tiles, pattern blocks, centimetre cubes, or another material of your choice.

b) Show how you know your puzzle is $\frac{1}{3}$ green.

c) What fraction of your puzzle is yellow? Show how you know.

2. Create a puzzle that is $\frac{1}{4}$ red, $\frac{1}{4}$ blue, and $\frac{1}{3}$ green. The rest is yellow. Describe as many fraction relations as you can about this puzzle. Use pictures, words, and numbers in your report.

Data Management and Probability

- Collect and organize data on tally charts.
- Develop concepts of probability.
- Identify favourable outcomes, and state probabilities.
- List outcomes using tree diagrams, modelling, and lists.
- Use and apply probability, including in sports and games.

Key Words

tally chart
frequency table
probability
outcome
favourable outcome
random
tree diagram
simulation

CHAPTER 4

Probability and Number Sense

Auto racing is a thrilling sport. Drivers with good winning records are in hot demand. That is because these drivers and their teams have a good chance of winning again.

What makes each race exciting is the fact that each driver has an equal chance of winning. The driver's skill and the speed of the team that checks and fuels the car, however, increase the chances of winning.

By the end of this chapter, you will be able to develop an exciting race-car game. You will be able to check the probability of each player winning. You will also be able to make the game fair for at least two players. Have fun developing and playing the games in this chapter!

Chapter Problem

Some board games are based on auto racing. You can use spinners or number cubes to provide moves or chances.

What makes a game fair? What can you do to give each player an equal probability of winning a game?

Get Ready

Calculate Means

The **mean** or average of a set of numerical data is the sum of the data, divided by the number of data items. For example, look at this set of student heights.

Carrie	165 cm
Arturo	178 cm
Jamilla	156 cm
Robert	158 cm
Wendy	153 cm
Jésus	165 cm

mean = (165 + 178 + 156 + 158 + 153 + 165) ÷ 6
= 162.5

The mean height for this group of students is 162.5 cm.

1. Find the mean of each set of data.
- **a)** 2, 4, 2, 2, 4, 4
- **b)** 150 cm, 170 cm, 158 cm, 166 cm, 184 cm
- **c)** 54 kg, 38 kg, 49 kg, 61 kg, 55 kg, 64 kg
- **d)** 13 mm, 17 mm, 12 mm, 16 mm, 18 mm
- **e)** 12.3 m, 11.9 m, 12.7 m, 13.0 m, 12.5 m, 11.9 m, 12.1 m, 12.8 m
- **f)** 35 jellybeans, 32 jellybeans, 37 jellybeans, 35 jellybeans, 34 jellybeans

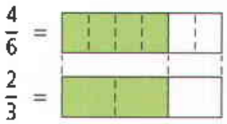
2. Four different groups performed 50 coin tosses each. Their results were as follows:

Group 1	23 heads, 27 tails
Group 2	25 heads, 25 tails
Group 3	21 heads, 29 tails
Group 4	28 heads, 22 tails

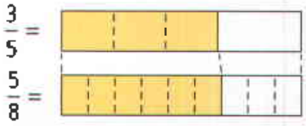
Calculate the mean number of heads.

Equivalent Fractions

These fraction strips show that the fractions $\frac{4}{6}$ and $\frac{2}{3}$ are equivalent.



The fractions $\frac{3}{5}$ and $\frac{5}{8}$ are not equivalent.



3. Draw fraction strips for each pair of fractions. State which pairs are equivalent.
- **a)** $\frac{2}{4}$ and $\frac{1}{2}$
- **b)** $\frac{2}{3}$ and $\frac{1}{2}$
- **c)** $\frac{2}{5}$ and $\frac{3}{8}$
- **d)** $\frac{7}{9}$ and $\frac{42}{54}$

4. a) Maria has 15 T-shirts, 6 of which are white. Express the number of white T-shirts as a fraction of the total number.
b) Find an equivalent fraction for your fraction in part a). Hint: Use 5 as the denominator.

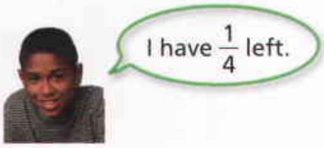
Compare and Order Fractions

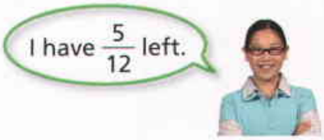
You can compare fractions by showing them with a common denominator.

For example, Devon and Monica are comparing the amount of chocolate bar they have left.

Making Connections

You worked with fractions in Chapter 3.





To find a common denominator, list the multiples of 4 and 12.

The multiples of 4 are 4, 8, $\boxed{12}$, 16, 20, …. $\frac{1 \times 3}{4 \times 3} = \frac{3}{12}$ **Twelve is a common denominator.**

The multiples of 12 are $\boxed{12}$, 24, …. $\frac{5}{12}$

The fraction $\frac{5}{12}$ is greater than $\frac{3}{12}$. Monica has more chocolate left.

5. Write each pair of fractions with a common denominator. Which fraction is greater?

a) $\frac{1}{3}$ and $\frac{3}{8}$ **b)** $\frac{2}{5}$ and $\frac{1}{2}$ **c)** $\frac{4}{6}$ and $\frac{7}{10}$

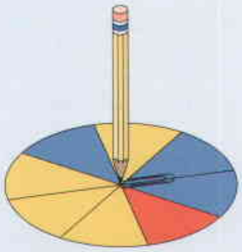
6. Order these fractions from least to greatest.

$\frac{7}{10}$ $\frac{1}{6}$ $\frac{11}{15}$ $\frac{2}{3}$ $\frac{3}{5}$

7. Look at this spinner.

a) Express each colour as a fraction of the whole spinner.

b) Write the colours in the order of their fractions, from least to greatest.



Convert Fractions to Decimals

To change a fraction to a decimal, divide the denominator into the numerator.

$\frac{7}{10}$ = $\boxed{\text{C}}$ 7 $\boxed{+}$ 10 $\boxed{=}$ 0.7 $\frac{1}{4}$ = $\boxed{\text{C}}$ 1 $\boxed{\div}$ 4 $\boxed{=}$ 0.25

8. Show each fraction as a decimal.

a) $\frac{3}{10}$ **b)** $\frac{6}{10}$ **c)** $\frac{23}{100}$

9. Show each fraction as a decimal.

a) $\frac{1}{2}$ **b)** $\frac{3}{8}$ **c)** $\frac{2}{5}$

4.1

Introducing Probability

Focus on...

- probability
- outcomes

At the beginning of a soccer game, a referee tosses a coin. The winning captain chooses which end of the field to take.

How do the team captains know that they each have an equal chance of winning the toss? How can you express this chance as a number?

Discover the Math

Materials

- coin

tally chart

- used to record experimental results or data
- counts the data

frequency table

- used to show the total numbers of occurrences in an experiment or survey

probability

- the chance that something will happen
- often expressed as a proper fraction, or a decimal between 0 and 1

How can you answer questions about chance?

1. Trudy, the Snowcats captain, calls the toss at the start of a soccer match. Last night, Trudy tossed a coin 100 times and got 60 heads, so she decides to call heads. Is Trudy making a winning decision? Explain.

2. Test Trudy's decision by tossing a coin for about 5 min.

a) Before you start, estimate the number of heads and the number of tails you will record.

b) Record your results in a **tally chart** and **frequency table**, like this one.

	Tally	Frequency
Heads	卌	
Tails	|||	
Total Trials		

3. a) Estimate the **probability** of getting heads, based on your results. State your estimate as a fraction, then convert it to a decimal.

b) Based on your results, comment on Trudy's decision.

4. Combine results from the whole class. Use the decimal estimates to calculate the mean of the probabilities. What do you notice?

5. **Reflect** Can you think of another way to determine the probability? Use your ideas to help Trudy see the coin toss in a different way.

Example: Calculate Probability by Outcomes

A jar of 30 jellybeans has 7 red, 6 black, 4 yellow, 5 orange, and 8 green jellybeans.

a) How many possible **outcomes** are there when you pick a jellybean from the jar?

b) If you are hoping for a red jellybean, how many **favourable outcomes** are there?

c) What is the probability of picking a red jellybean, if you are not looking as you pick? Write the probability as a fraction and a decimal, rounded to the nearest hundredth.

d) What is the probability of picking a black jellybean at **random**, as a fraction and a decimal?

outcome

- one possible result of a probability experiment

favourable outcome

- an outcome that counts for the probability being calculated

random

- a choice or pick in which each outcome is equally likely
- if you do not look as you pick, you are choosing at random

Solution

a) There are 30 jellybeans in the jar. So, there are 30 possible outcomes.

b) There are 7 red jellybeans, so there are 7 favourable outcomes.

c) $$\text{Probability(red jellybean)} = \frac{\text{favourable outcomes}}{\text{all outcomes}}$$

=

$$= \frac{7}{30}$$

$$\doteq 0.23$$

The probability of a red jellybean is $\frac{7}{30}$, or approximately 0.23.

d) There are 6 black jellybeans.

$$\text{Probability(black jellybean)} = \frac{\text{favourable outcomes}}{\text{all outcomes}}$$

$$= \frac{\text{number of black jellybeans}}{\text{total number of jellybeans}}$$

$$= \frac{6}{30}$$

$$= 0.2$$

The probability of a black jellybean is $\frac{1}{5}$ or 0.2.

The probability $\frac{6}{30}$ is equivalent to $\frac{1}{5}$. That's a 1 in 5 chance.

Strategies

How else might you show the fraction $\frac{6}{30}$?

Key Ideas

- Probabilities can be estimated from repeated trials of an experiment.
- Probabilities can also be calculated.
- Probabilities can be shown as a fraction or as a decimal.

$$\frac{7}{30} = 7 \div 30$$
$$\doteq 0.23$$

$\boxed{C}$ 7 $\boxed{\div}$ 30 $\boxed{=}$ 0.233333333

$$\text{Probability(red jellybean)} = \frac{\text{favourable outcomes}}{\text{all outcomes}} = \frac{\text{red jellybeans}}{\text{all jellybeans}} = \frac{7}{30}$$

- The outcomes of an experiment are the possible results. For example, in a coin toss, the two outcomes are heads and tails. The probability of getting heads is $\frac{1}{2}$ or 0.5.

Communicate the Ideas

1. Kyle says, "Let's use a coin toss. I call heads. I have one chance in two of winning." Devon says, "I call tails. My chance of winning is 0.5." Both Kyle and Devon are correct. Explain.

2. In some sports, a coin toss is not used. Discuss the different ways a sporting event can be started, using probability to make choices.

3. You are asked to pick a letter at random from the bag.

a) What chance do you have of picking the letter A?

b) What chance do you have of picking the letter S?

c) Justify your answers.

d) Try it. Use a tally chart to show your first 50 picks. Which letter did you pick the most often?

Check Your Understanding

Practise

4. Copy and complete this tally chart and frequency table.

	Tally	Frequency
Heads	卌 卌 卌 卌 |	
Tails	卌 卌 卌 卌 卌 ||	
	Total Trials	

5. Copy and complete this tally chart and frequency table.

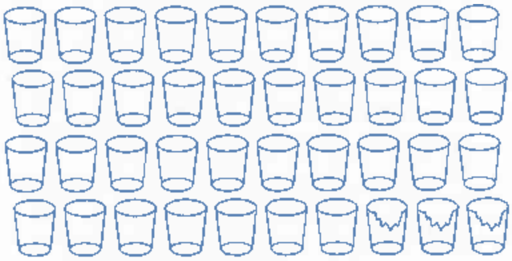
	Tally	Frequency
Red	卌 卌 |||	
Blue	卌 卌 卌 |	
Yellow	卌 卌 |	
	Total Trials	

6. In each situation, state the total number of outcomes, and the number of favourable outcomes.

a) Helen likes roll-neck sweaters. She picks a sweater at random.

b) Four winning tickets will be drawn.

c) Christina wants an unchipped glass. She picks a glass at random.



7. For each part of question 6, state the probability as a fraction.

For help with questions 8 to 10, refer to the Example.

8. In a jar of 25 jellybeans, there are

- 13 zippy zingers
- 3 stomach stirrers
- 2 tongue twisters
- 7 face freezers

a) How many possible outcomes are there?

b) How many favourable outcomes are there for each flavour?

c) What is the probability of picking a zippy zinger at random? Write the probability as a fraction and as a decimal.

d) Find the probabilities of picking each of the other flavours at random. Show the probabilities as fractions and as decimals.

Apply

9. Suppose the numbers of each jellybean flavour in question 8 were doubled.

a) Predict what will happen to the probability of each flavour.

b) Determine the probability of picking each flavour at random. Do your findings confirm your prediction? Explain.

10. a) For the jar of jellybeans in question 8, what is the probability of picking a jellybean of *any* flavour at random? Explain.

b) What is the probability of picking a knee knocker jellybean at random? Explain.

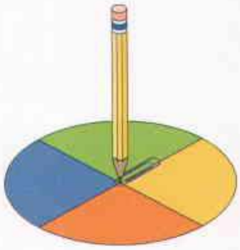
11. Karina's sock drawer has 3 pairs of grey socks, 4 pairs of white socks, and 6 pairs of tan socks. Each pair is rolled together. Every morning Karina picks a pair of socks at random.

a) What is the probability that Karina will pick out a pair of white socks?

b) Which colour of socks is Karina most likely to pick? Which colour of socks is she least likely to pick? Justify your answer.

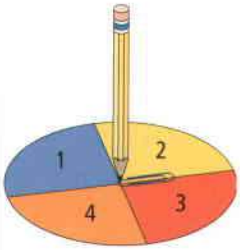
c) Explain why the probabilities change, depending on the colour.

12. Danae and Tom are about to start a board game. The game comes with a four-section spinner. Explain how to use the spinner to decide who goes first.



13. What's wrong? Cheryl repeatedly spun a spinner with four equal sections and got these results:

Colour	Total
orange	3
blue	4
yellow	0
red	2



Is something wrong with Cheryl's spinner? How could she check?

14. Jonah's sock drawer is filled with unmatched socks. Each morning he pulls out two socks at random. On Monday morning, there are 2 green socks, 3 purple socks, and 1 orange sock in Jonah's drawer.

a) Use slips of paper in a paper bag to model Jonah's sock drawer. Record colours.

b) Pick two slips of paper at random. Note whether they match or not. Hint: Do not put the first slip of paper back before picking the second. Then, put both back.

c) Copy this tally chart and frequency table.

	Tally	Frequency
Matched		
Unmatched		
	Total Trials	

Perform step b) repeatedly for 5 min. Record your results in the tally chart.

d) Use your results to estimate the probability that Jonah will get a pair of matched socks.

15. a) Veronica ate 2 red, 2 yellow, 1 orange, and 5 green jellybeans from a jar. She left behind 5 red, 6 black, 2 yellow, 4 orange, and 3 green jellybeans. Find the probability of picking each jellybean colour

- before Veronica started eating
- after Veronica had eaten her 10 jellybeans

b) Do you think Veronica picked her jellybeans at random? Explain.

Try This!

16. A number cube was rolled 100 times. These results were recorded.

Number	Total
1	16
2	20
3	13
4	7
5	27
6	17

a) Use these results to estimate the probability of rolling a 5.

b) Do you think the cube is fair? Explain. How could you check?

c) If the same cube were rolled 6000 times, would you be surprised if the number 5 turned up 980 times? 320 times? Explain.

Extend

17. Create a probability experiment that involves picking an item out of a container. Use at least 50 items.

a) In repeated trials of the experiment, decide whether you should replace, or not replace, the item after each pick. Explain your decision.

b) Carry out repeated trials for 2 min. Then, estimate the probabilities of picking each type of item.

c) Count possible outcomes to determine the actual probabilities.

d) Compare your results for parts b) and c). Think about the items you used. Are all outcomes equally likely? Explain.

4.2

Organize Outcomes

Focus on...
- probability in games
- modelling outcomes
- favourable outcomes

Board games have been around for thousands of years. In many board games, you must roll a number cube or spin a spinner to move.

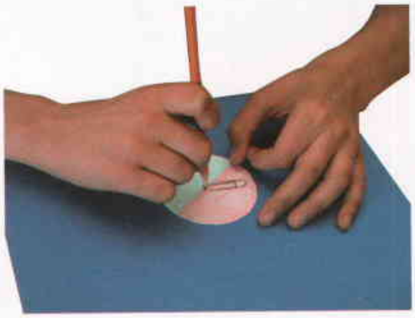
Suppose you are three board spaces away from "home." You need to roll a 3 or more. What is the probability that you will get home on this move?

Discover the Math

Materials
- white cardboard
- compasses or circular object to trace around
- ruler
- pencil crayons
- 2 paper clips
- 2 pencils

Optional:
- BLM 4.2A Spinner Templates

How can you organize outcomes to help with calculations?



1. Create a spinner with two equal-sized sections, as in the photograph. Colour the sections red and blue.

2. Create a spinner with four equal sections, coloured red, blue, yellow, and purple.

3. Create a way of organizing the combined outcomes when you spin both spinners.

4. Predict the probability of spinning red and yellow in the same spin.

5. Spin the pair of spinners repeatedly for about 5 min. Record your results in an organized list. Use your results to estimate the probability of spinning red and yellow.

Strategies
Make an organized list

6. **Reflect** How well does your organizer present your results? How could you improve it?

Example 1: Use a List or Modelling

Create an organizer, with probabilities, for

a) a spinner with equal red and blue sections

b) a spinner with equal red, blue, yellow, and purple sections

Solution

a) *Method 1: Make a List*

spinning red $\quad \text{probability(red)} = \dfrac{\text{red}}{\text{red or blue}} = \dfrac{1}{2}$

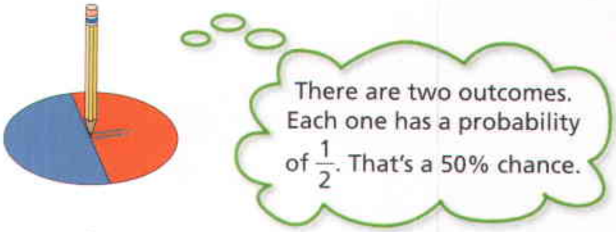
spinning blue $\quad \text{probability(blue)} = \dfrac{\text{blue}}{\text{red or blue}} = \dfrac{1}{2}$

Method 2: Test the Spinner

Make and test the spinner. You should find that the two outcomes are equally likely.

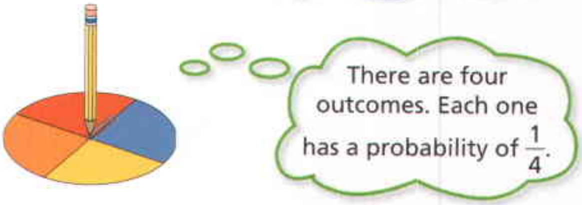
$\text{Probability(red)} = \dfrac{1}{2}$

$\text{Probability(blue)} = \dfrac{1}{2}$



b) spinning red $\frac{1}{4}$ $\quad$ spinning blue $\frac{1}{4}$

spinning yellow $\frac{1}{4}$ $\quad$ spinning purple $\frac{1}{4}$



Strategies

Make a model

Literacy Connections

Reading Percents

The expression "50-50 chance" means a probability of 50%. You can read 50% as $\frac{50}{100}$. What simple fraction is equivalent to this?

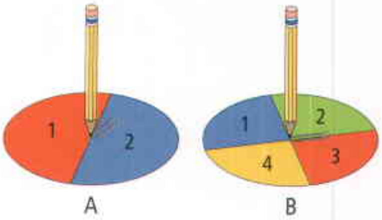
Example 2: Use a Tree Diagram

a) Create a **tree diagram** to show the possible outcomes from spinners A and B.

b) What is the probability of spinning a 1 and a 4?

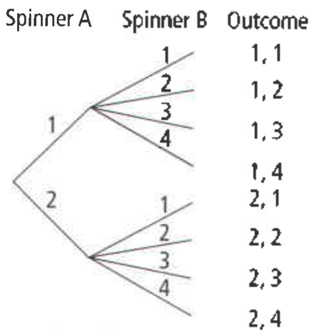
c) What is the probability of spinning a 1 and a 2?

d) What is the probability of spinning a total of 4?



Solution

a)



tree diagram

- diagram that shows outcomes as sets of branches
- useful for organizing combined outcomes

b) There is only one favourable outcome, (1, 4).

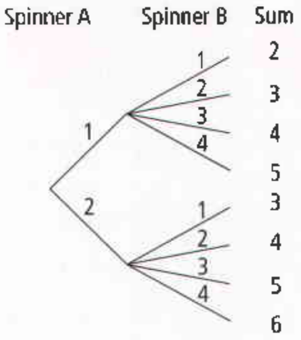
$$\text{Probability}(1 \text{ and } 4) = \frac{\text{favourable outcomes}}{\text{all outcomes}} = \frac{1}{8}$$

c) (1, 2) and (2, 1) are the favourable outcomes.

$$\text{Probability}(1 \text{ and } 2) = \frac{\text{favourable outcomes}}{\text{all outcomes}} = \frac{2}{8}$$

Another way of saying this is $\frac{1}{4}$.

d)



I changed the combined outcomes from pairs to totals. Two outcomes show a sum of 4.

$$\text{Probability(total of 4)} = \frac{\text{favourable outcomes}}{\text{all outcomes}} = \frac{2}{8}$$

Literacy Connections

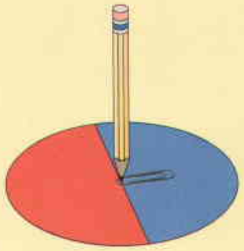
Reading Tree Diagrams

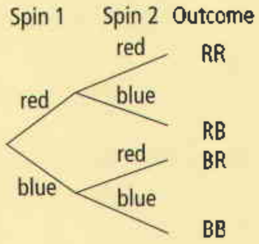
Read tree diagrams from left to right.

- The branches on the left of the tree show the outcomes for one spinner.
- The branches on the right show the outcomes for the other spinner.
- At the far right of the diagram, read off the combined outcomes.

Key Ideas

- Tree diagrams and lists help to organize the outcomes of a probability experiment.
- Tree diagrams and lists can be used to determine probabilities. With this spinner, the probability of spinning red and then red in two spins is $\frac{1}{4}$.
- Modelling with physical objects can help with creating an organized list or tree diagram.





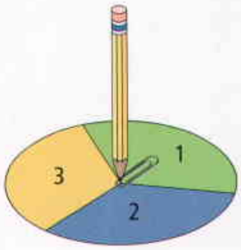
Communicate the Ideas

1. How can a tree diagram help you organize the possible outcomes from this spinner and cube?

2. Draw a spinner for each set of probabilities.

a) spinning red $\frac{1}{3}$ spinning blue $\frac{1}{3}$ spinning green $\frac{1}{3}$

b) spinning yellow $\frac{1}{5}$ spinning purple $\frac{4}{5}$



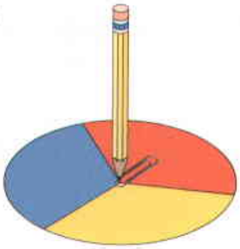
Check Your Understanding

Practise

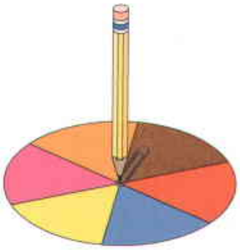
For help with questions 3 and 4, refer to Example 1.

3. Create an organized list of outcomes, with probabilities, for each spinner.

a)

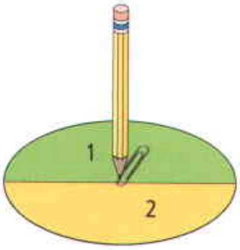


b)

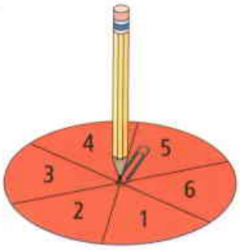


4. Create an organized list, with probabilities, for each spinner.

a)

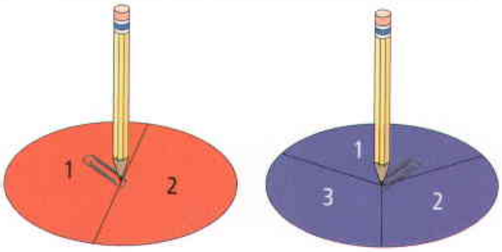


b)



For help with questions 5 to 10, refer to Example 2.

5. Two spinners are numbered as shown.

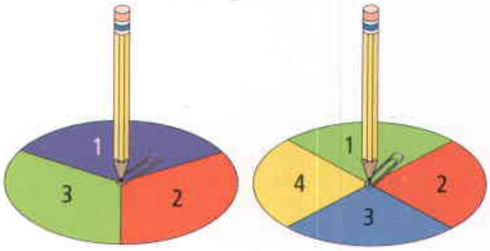


a) Create a tree diagram for spinning both spinners.

b) What is the probability of spinning a 1 and a 3?

c) What is the probability of spinning a 1 and a 2?

6. a) What is the probability of spinning a 1 and a 3 on these spinners?



b) What is the probability of spinning a 2 and a 4?

c) What is the probability of spinning a total of 4?

Apply

7. a) Choose numbers for your own pair of spinners.

b) Draw a tree diagram, showing the total scores.

c) For each total score, determine the probability.

8. A number cube is labelled 2, 3, 3, 4, 5, 6.

a) What is the probability of rolling a 4?

b) What is the probability of rolling a 3?

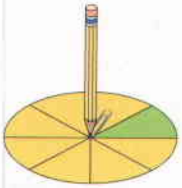
c) What is the probability of rolling a number less than 3?

9. Chantal needs to roll 3 or more on a number cube to win the game. Determine the probability that Chantal wins. Justify your answer.

10. a) What is the probability of spinning green?

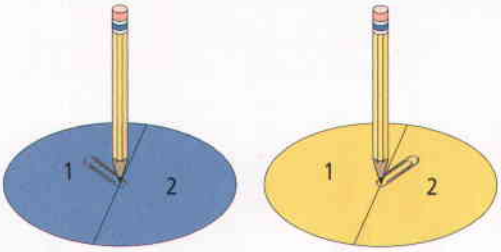
b) What is the probability of spinning yellow?

c) Draw a diagram of a red-blue spinner with probability(red) $= \frac{3}{5}$.



Chapter Problem

11. A car rally game uses two spinners.

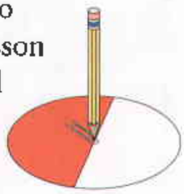


a) What are the possible sums when you add the results from these two spinners?

b) What is the probability of getting each sum?

Try This!

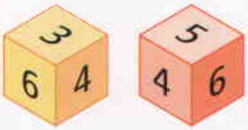
12. Frances asked Davin to explain the probability lesson she missed. Davin decided to use a spinner to explain predicted probability of outcomes and tree diagrams.



a) Draw a tree diagram to show the outcomes and predicted probability of spinning the spinner twice.

b) Frances wanted to test Davin's theory so she spun the spinner twice and got two reds. She argued with Davin that the results on her spin were different than what he explained. Write Davin's response.

Extend



13. Roll two number cubes. If the sum of the numbers showing is an even number, then Player 1 wins. If the sum is an odd number, Player 2 wins.

a) Play the game 50 times. Record who wins in a tally chart.

b) Estimate the probability of each player winning.

c) Explain whether one player is more or less likely to win. Justify your reasoning.

14. a) Create a probability game. Explain what materials are needed. List the rules clearly.

b) Explain whether some events in the game are more likely than others. If not, explain why not.

c) Create an organizer for your game. You can use a tree diagram, an organized list, or your own type of organizer.

d) Report on the probabilities involved in your game. If some probabilities cannot be determined from your organizer, try experimenting to get estimates.

Making Connections

What does math have to do with pop flavours?

Do you still drink the same beverages as you did in kindergarten? Do you like the same flavours?

The Ontario Beverage Company makes seven different flavours of pop. Over the past two years, they have been keeping track of how much of each flavour has been sold.

Drink Flavour	Sales (1000s)
Koala Cola	13
Lizard Lime	5
Lemur Lemon	8
Gorilla Grape	3
Roary Root Beer	9
Oliphaunt Orange	10
Jumping Ginger	2

1. Based on this information, estimate the probability that a customer will buy Oliphaunt Orange.

2. Which flavours should the company discontinue? Explain, in terms of probability.

4.3

Use Outcomes to Predict Probabilities

Focus on...
- listing outcomes
- predicting probabilities

Examine this set of numbered cards.

What is the probability of drawing a card numbered 3?

Discover the Math

How can you work with outcomes to predict probabilities?

Making Connections

You will find the rules to *Crazy Eights* on page 130.

Example 1: Probability in Number Cards

Crazy Eights can be played with a set of 40 cards, as pictured on this page. Determine the probability of each draw.

a) the 8 of ♠

b) any one of the Crazy Eights

c) any red card

Solution

a) There is only one 8 of ♠, so the probability of picking that card is $\frac{1}{40}$.

b) There are four 8s altogether, so the probability of picking an 8 is $\frac{4}{40}$ or $\frac{1}{10}$.

c) *Method 1: Count Outcomes*
There are 20 red cards, so the probability of picking a red card is $\frac{20}{40}$ or $\frac{1}{2}$.

Method 2: Use Proportions
Exactly half of the set is red, so the probability of picking a red card is $\frac{1}{2}$.

Strategies
Solve a simpler problem

Example 2: Outcomes of Multiple Coin Tosses

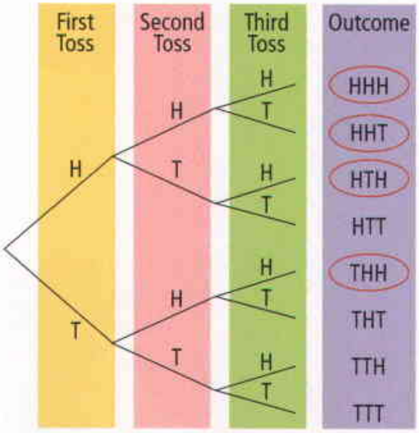
Madison and Robyn want to share a bag of potato chips. They cannot agree on the flavour. They decide to toss a coin and play "the best of three wins."

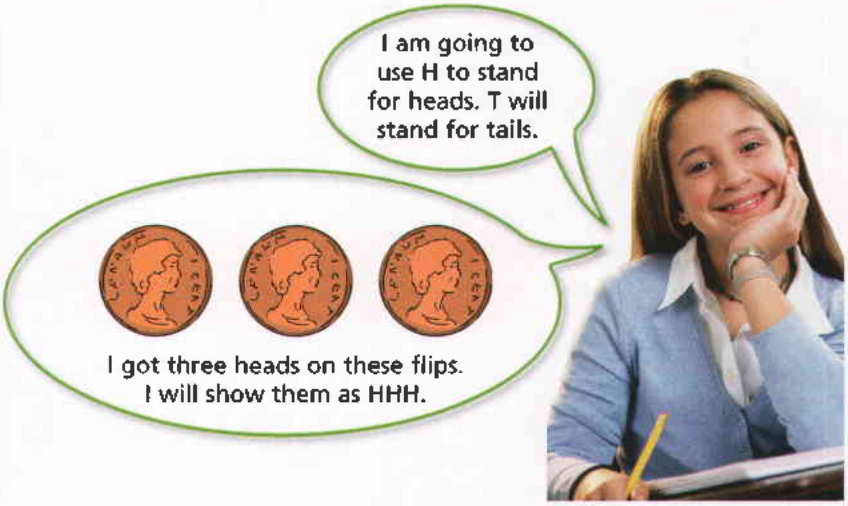
- If there are at least two heads out of three tosses, Madison decides on the flavour.
- Otherwise, Robyn decides.

What is the probability that Madison gets to decide?

Solution

Method 1: Use a Tree Diagram





Four favourable outcomes show two heads. So, the probability that Madison gets to decide is $\frac{4}{8}$ or $\frac{1}{2}$.

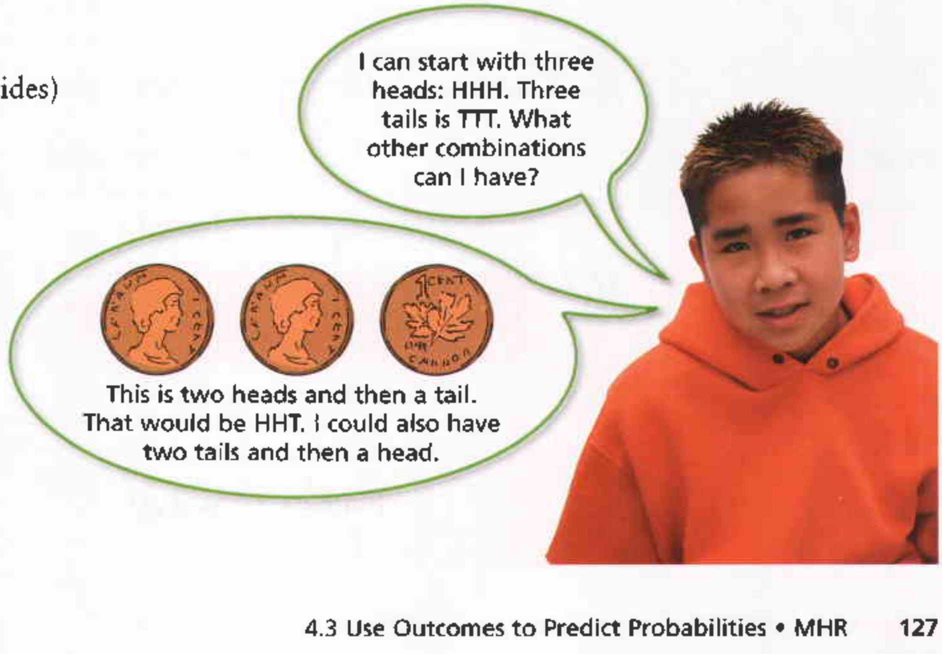
Method 2: Create a List

HHH TTT HHT TTH HTH THT HTT THH

The favourable outcomes for Madison are HHH, HHT, HTH, and THH.

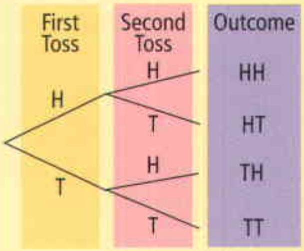
So,

$$
\begin{aligned}
&\text{Probability(Madison decides)}\\
&= \frac{\text{favourable outcomes}}{\text{all outcomes}}\\
&= \frac{4}{8}\\
&= \frac{1}{2}
\end{aligned}
$$



Key Ideas

- Outcome organizers such as tree diagrams and lists help to predict probabilities.
- When choosing a method to solve a probability problem, think about what might work best for the problem.



Communicate the Ideas

1. John e-mailed a friend about the probability lesson he missed. John explained that the teacher used a tree diagram to determine all the possible outcomes of tossing a coin three times. What could John tell his friend about the probability of winning a best of three?

2. For each situation, describe an appropriate method to illustrate the possible outcomes.

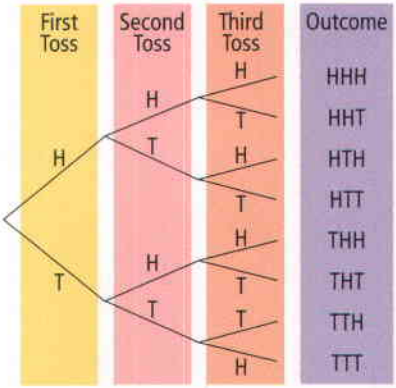
a) rolling odds or evens with a number cube

b) drawing a Crazy Eight from a 40-card set

3. Look at this tree diagram. What method would you use to find each probability, and why?

a) exactly two heads

b) at least one head and at least one tail



Check Your Understanding

Practise

For help with questions 4 to 6, refer to Example 1.

4. Determine the probability of each draw from a 40-card set. (See the picture on page 126.)

a) the 3 of ♦

b) the 7 of ♣

c) any black card

d) any 9

e) any card except a Crazy Eight

5. Determine the probability of each draw from a 40-card set. (See the picture on page 126.)

a) the 4 of ♠

b) the 7 of ♥

c) any red Crazy Eight

d) a 7 or 8

e) a 3 or 4

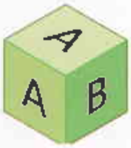
6. Look at this 20-card set. Which probabilities in question 4 change? Which stay the same? Explain.

For help with questions 7 to 10, refer to Example 2.

7. For each situation, list all the possible outcomes.

a) You roll a standard number cube.

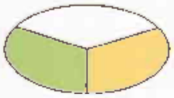
b) You roll a cube labelled A, A, B, B, B, and C.



c) You draw a jellybean from a bag with the colour selection shown.

8. For each part of question 7, choose one outcome. Calculate the probability.

9. You flip a coin and spin a three-section spinner. List the possible outcomes.



Apply

10. What is the probability of drawing the 8, the 9, or the 10 of ♠ from this set? Explain.

11. For each situation,

- state the probability
- justify your response

a) rolling an odd number on a number cube

b) rolling a 2 or a 5 on a number cube

12. What is the probability of picking a vowel from a bag containing the letters of the word MILLION? Explain.

13. A letter is drawn repeatedly and at random from the word MATHEMATICAL. You can choose one letter. Every time your choice is drawn, you get a point. Letters are replaced after each draw. Which letter should you choose? Explain.

14. Madeleine and Adeena disagree about probabilities with a standard number cube. Madeleine knows that rolling an odd number is more likely than rolling any individual number. Explain how Madeleine should convince Adeena.

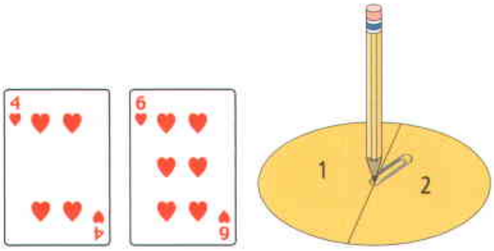
15. Mya decides to use an 8-sided die when playing her favourite board game.

a) How do the probabilities differ from using a standard 6-sided number cube?

b) Another 8-sided die is numbered 1, 1, 2, 3, 4, 4, 4 and 5. Which number is most likely to be rolled? Explain.

Chapter Problem

16. A car rally game uses two number cards and a spinner. Pick a card at random. Then, spin the spinner. Subtract.



a) What are the possible differences (card number – spinner number)?

b) Try it for 3 min. List your results.

c) What is the probability of getting a 3? Explain.

Try This!

17. Create your own probability game that uses simple probability. You can use number cubes, spinners, coloured chips, or other materials. Find and explain the probabilities for your contest.

Extend

18. Rafina is choosing from a menu. She can select one item from each category.

noodle: Shanghai or udon
protein: chicken, beef, or tofu
sauce: spicy Thai or peanut
fruit drink: mango or watermelon

a) Draw the tree diagram displaying Rafina's possible choices.

b) Is the probability of each protein choice really the same? Justify your reasoning.

c) How is the number of choices reduced if Rafina decides on chicken? Explain, in terms of your tree diagram.

19. Two friends are using a coin to play "best of three wins." What is the probability that the winner is chosen after the second toss?

Making Connections

What does math have to do with card games?

Here is one version of the rules for *Crazy Eights*.

- Shuffle the cards. Deal all the cards out evenly to the players.
- The player to the dealer's right begins by laying down a card.
- Players take turns to play cards. Play always moves to the right.
- Usually, you must follow by suit or number. For example, if the 3 of hearts has just been played, you must play another 3 or another heart.
- You can play a Crazy Eight on any turn. You don't have to follow the same suit or number. If you do so, you immediately play a second card. The next player must now follow your second card.
- If you cannot play any card, you miss a turn.
- The winner is the first person to play his or her final card.

1. Play *Crazy Eights* with two or three friends.

2. Discuss what you learned about winning strategies.

3. How does understanding probability help you in *Crazy Eights*?

4.4

Extension: Simulations

Focus on...
- simulations
- probability experiments
- simulations to make best probability estimates

Companies often use "instant-win" promotions to entice customers. How good a deal is this sort of promotion?

Discover the Math

Materials

Any one of
- number cube
- six-section spinner
- slips of paper and a container

Optional:
- BLM 4.4A Run a Simulation

simulation
- a probability experiment used to model a real situation

How can you simulate probability problems?

1. The bottle cap for a new brand of pop has a letter inside it. If you collect all the letters that spell Y-O-U-W-I-N, you win a game console. How many bottles of pop do you think Jasmine needs to buy to win the game console?

2. Select a tool to simulate buying the pop. Describe a method for running your **simulation** and recording your results.

3. Use your simulation method to test how many bottles Jasmine buys. Compare your simulation results with your estimate from step 1. Were they close? Explain why or why not.

4. a) Compare your results with some other classmates, or as a class.
b) Use the combined results to calculate the mean number of bottles Jasmine would have to buy.

5. Reflect Consider the simulations you compared in step 4.
a) Explain what the simulations tell you about the contest.
b) Why was it useful to compare results?

Example: Simulate With a Spinner

Jasmine used a spinner to simulate the promotion. She spun the spinner and recorded her results in a tally chart. She continued to spin until she had landed on all six letters.

a) How many bottles of pop did Jasmine have to "buy"?

b) Which letter appears to have been last? Explain your reasoning.

Letter	Tally
Y	𝍸 |
O	𝍸
U	|
W	||
I	||||
N	||||

Solution

a) Add up the tallies. Jasmine bought 22 bottles of pop.

b) The letter U only has 1 tally. It must have been the last letter.

Key Ideas

- A simulation is an experiment that can be used to model a real situation involving probabilities.
- There are many different ways to simulate a situation.

Communicate the Ideas

1. Will simulation outcomes be the same for each student? Explain.

2. Jasmine used a spinner to simulate buying bottles of pop with the letters for Y-O-U-W-I-N. What other methods might she have used?

Check Your Understanding

Practise

For help with questions 3 and 4, refer to the Example.

3. This tally chart shows the results of a simulation.

a) How many rolls were needed to get all six letters?

b) Which letter was last? How do you know?

Letter	Tally
S	||
C	||||
O	|
R	|||
E	||

4. This tally chart shows the results of a number cube simulation.

a) How many rolls were needed to get all six numbers?

b) Which numbers could have been last? Explain your reasoning.

Number	Tally
1	|||
2	||
3	|
4	𝍸
5	|
6	||||

5. Describe an item that could be used to simulate each situation. Explain why each item is appropriate.

a) A student does not know the answer to a True or False question.

b) You are picking 1 pizza topping at random from 8 choices.

c) You can win a phone by collecting the letters on bottle caps to spell P-H-O-N-E.

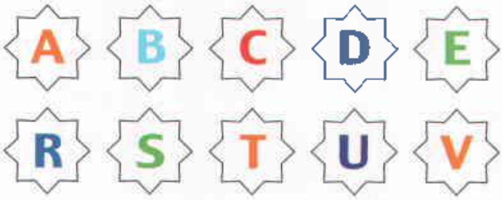
Apply

6. a) Create a simulation item for question 5c). Using your item, simulate the contest and record your results.

b) Explain the results of your simulation.

c) Explain how the results of your simulation compare to those of your classmates.

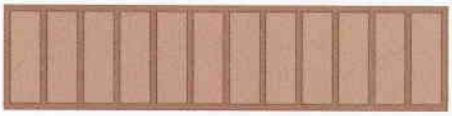
7. Your favourite chocolate bar has a letter on the inside wrapper. You can win if you collect B, A, and R. There are 10 different letters on the inside wrappers.



a) Develop a simulation item. Simulate this contest.

b) Run the simulation 10 times. For each simulation, how many bars were needed to get B, A, and R?

c) Did you need to buy the same number of bars in each simulation? Explain.



8. At a street intersection, 100 vehicles, on average, pass through in an hour. 50% are passenger cars, 25% are vans, and 25% are trucks.

a) Design a simulation item to simulate the situation, and conduct the simulation.

b) Predict what the next 10 vehicles will be.

9. Describe a situation that can be simulated using each item.

a) five marbles in a bag, each of a different colour

b) a spinner divided in half, and one of the halves is divided into two quarters

c) a cube labelled A, A, B, B, C, C

10. Palo has been practising his aim at darts. Out of 10 throws, he hits the bulls-eye 3 times.

a) Estimate the probability of Palo hitting a bulls-eye.

b) Estimate the probability of Palo not hitting a bulls-eye.

c) Describe an item that can be used to simulate the situation. Hint: How could you use a spinner?

d) Predict whether Palo's next five throws will include any bulls-eyes.

e) Conduct the simulation and compare with your prediction.

Extend

11. The pop company decides to change the contest. To win a game console, customers must collect the letters to spell W-I-N-N-E-R.

a) Describe two different ways the contest could be run. What is the probability of drawing an N, with each way?

b) Explain how a number cube can be used to simulate the contest. Which way in part a) does the simulation fit? Explain.

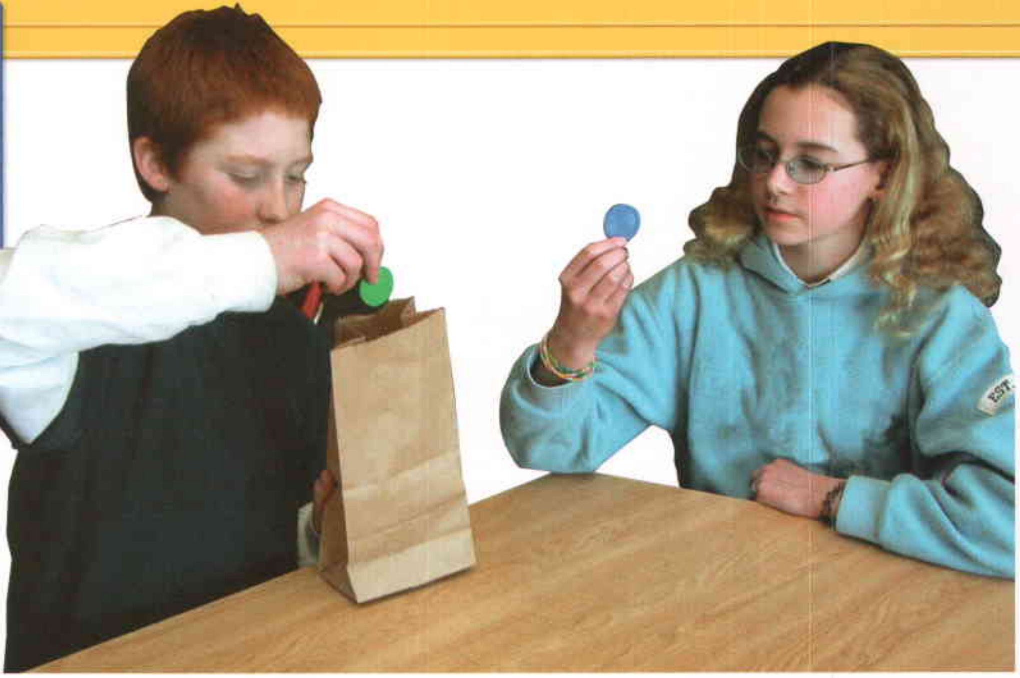
c) For each way in part a), create an imaginary tally chart that shows the result of a simulation. Explain your method.

4.5

Apply Probability in Sports and Games

Focus on...

- applying probability
- probability in sports and games



Games are a fun way to practise your knowledge and skills.

What can you tell about the game being played in this photograph? What do you need to know to win?

Discover the Math

Materials

- coloured tiles, counters, or markers
- paper bag

Strategies

Act it out

How can you use probability to help you win?

The game of *Match or No Match* is played in pairs. Here is how you play.

- Place two green tiles and one blue tile in a bag.
- Before starting, decide who will be *Match* and who will be *No Match*.
- Player 1 draws a tile from the bag. Then, Player 2 draws a tile from the bag.
- The *Match* player gets a point if the tiles match. The *No Match* player gets a point if the tiles do not match.
- Replace the tiles. Repeat.

1. Create a tally chart to act as a score sheet.

2. Before playing the game, write two explanations:

a) why you think you might win

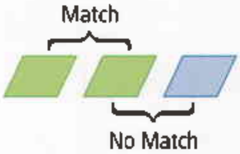
b) why you think you might lose

3. Play for about 5 min. The winner is the person with the most points. Who wins?

4. **Reflect** How could you improve your chances of winning?

Example 1: Use Tree Diagrams to List Game Outcomes

Kaia and Elana play *Match or No Match*. Kaia is *Match* and she thinks that she is less likely to win because of the possible outcomes. She says the game is not fair. Is Kaia correct?

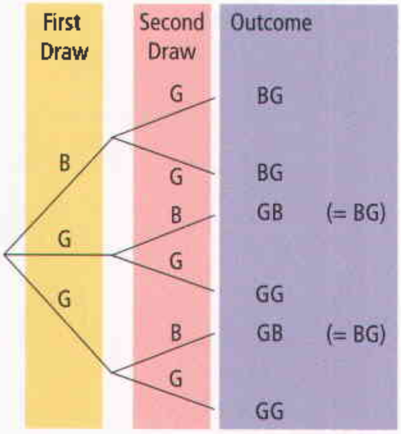


Solution

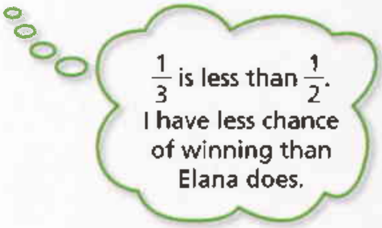
Understand Find the probability of *Match*. That will tell whether Kaia is correct or incorrect.

Plan Use a tree diagram to represent the game and determine the probability of *Match*.

Do It!



There are two favourable outcomes, GG and GG. So, the probability of *Match* is $\frac{2}{6}$ or $\frac{1}{3}$. Kaia is right. The game is not fair.



Look Back

- There are more combinations for *No Match* than for *Match*.
- It does not matter that there are two matching tiles and only one unmatched tile. *Match* is less likely.

Literacy Connections

"Fair" games provide an equal chance for each player to win. If there are two players, there is an equal probability that each player will win. We say that each has a 50% chance of winning. This probability can be shown as the fraction $\frac{1}{2}$.

In a fair game, three players should each have a one in three chance of winning. This can be shown as $\frac{1}{3}$.

Example 2: Probability in Baseball

Baseball star Jumpin' Jack Flazio's batting average is .300.

a) Estimate the probability of Flazio getting a hit.

b) Estimate the probability of Flazio not getting a hit.

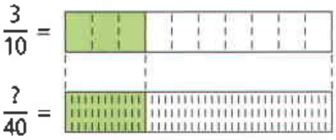
c) How many hits, on average, will Flazio get in 40 at-bats?

Solution

a) A batting average of 0.300 means that Flazio has had 300 hits in 1000 at-bats. So, the estimated probability of scoring a hit is $\frac{300}{1000}$, or $\frac{3}{10}$.

b) Out of every 1000 at-bats, Flazio is expected *not* to get a hit 700 times. The estimated probability of not getting a hit is $\frac{700}{1000}$ or $\frac{7}{10}$.

c) *Method 1: Use Fraction Strips*



$$\frac{3}{10} = \frac{12}{40}$$

Flazio will get 12 hits on average.

Method 2: Use Equivalent Fractions

$$\frac{3}{10} = \frac{3 \times 4}{10 \times 4}$$
$$= \frac{12}{40}$$

Flazio will average 12 hits in 40 at-bats.

Literacy Connections

Reading Sports Data

Batting averages are really fractions out of a thousand. For example, ".342" means $\frac{342}{1000}$, or 0.342. Out of 1000 times at bat, the batter has scored a hit 342 times.

Key Ideas

- Knowledge of probability can help in sports and games. Strategies that give more favourable outcomes improve your chances of winning.
- Batting averages in baseball and goals scored in hockey or soccer are an indication of an athlete's performance.

Batting average .350

Another way of showing this is $\frac{350}{1000}$.

Out of every 1000 times at bat, this player had 350 hits. He did not get a hit 750 times.

Communicate the Ideas

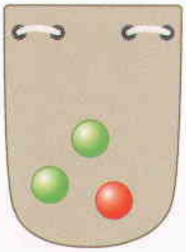
1. Why is it sometimes helpful to predict probabilities in a game?

2. **a)** Elana thinks the game of *Match or No Match* is fair. Do you agree? Explain.
b) Suggest ways of changing the game to make it fair for two players.

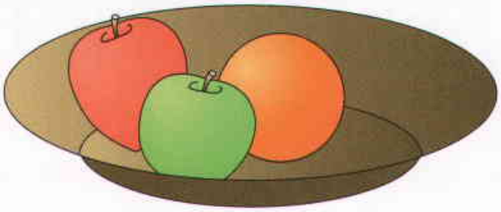
Check Your Understanding

Practise

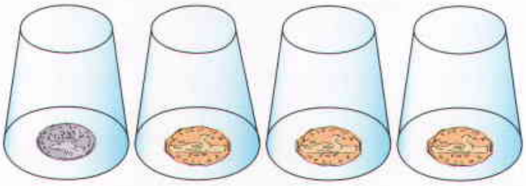
3. Use a tree diagram to show the chances of randomly picking matching marbles.



4. Use a tree diagram to show the probability of randomly grabbing two different types of fruit.



5. Use a tree diagram to show the probability of randomly getting two different coins.



6. **a)** Christian scores 2 goals for every 10 shots on net. What is his chance of *missing* a goal?
b) Ramona wins 6 races of every 15 she enters. What is her chance of *losing* a race?

Apply

For help with questions 7 and 8, refer to Example 1.

7. In *Heads and Tails*, two players each toss a coin. If either or both coins come up heads, Player 1 gets 1 point. If the result is two tails, Player 2 gets 2 points.

a) Draw a tree diagram for the score in one round of the game.
b) Is *Heads and Tails* a fair game? Explain.

8. In *Match or No Match*, imagine adding another blue tile to the bag. You now have two tiles of each colour. How does this affect the probability of matching?

For help with questions 9 and 10, refer to Example 2.

9. Imant's batting average at baseball is .250 or 0.250.

a) Estimate the probability of Imant getting a hit.
b) Estimate the probability of Imant *not* getting a hit, as a fraction and as a decimal.
c) How many hits, on average, will Imant get in 24 at-bats?

10. Raquel averages 17 hits in every 50 at-bats.

a) Estimate Raquel's probability of getting a hit.

b) Write Raquel's batting average with three decimal places. How many hits would Raquel average out of 1000?

11. Lydia entered an egg-and-spoon contest. In 10 runs, she dropped the egg 4 times.

a) Estimate the probability of Lydia dropping the egg.

b) What is the estimated probability that Lydia will *not* drop the egg, the next time she races?

c) Add your answers for parts a) and b) together. Explain your result.

12. A hockey goalie has made 9 saves in 10 shots.

a) The coach has to guess at the probability of the goalie not letting a shot in. What is the best estimate?

b) Out of 100 shots on net, how many goals is the goalie likely to let in?

c) If there were 20 shots on net, how many goals would be expected?

13. Design your own coloured-tiles game. Determine the winning probabilities for each player. Is your game fair? Does it matter who draws first?

14. At Mount St. Louis Ski Resort, 3 ski lifts are open. Five trails are open heading down the mountain.

a) How many different ways can a skier go up and down the mountain? Use an organizer to justify your answer.

b) What is the probability that a skier, choosing at random, will choose any trail in a)? Justify your answer.

15. In MONOPOLY®, a player must remain in jail unless she or he rolls a double in three rolls. Players use two number cubes.

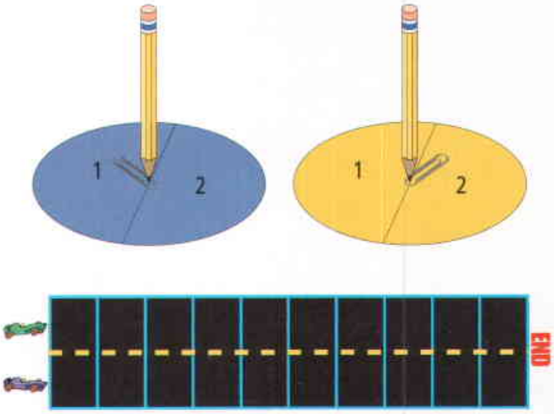
a) What is the predicted probability of rolling doubles in one roll?

b) What is the predicted probability of not rolling doubles in one roll?

c) In terms of the probability, explain why a "get out of jail" card would be useful to have.

Chapter Problem

16. A car rally game uses two spinners. Cars move the sum of each spin. They move along a track 10 spaces long.



Is this game fair? Explain.

Try This!

17. a) In a board game, you can move any number of spaces up to and including the value you roll on a number cube. What is the probability that you will win on your next roll, if you are

- four spaces away from the finish line?
- one space away from the finish line?
- eight spaces away from the finish line?

b) Is this case, explain what a probability of 0 means. What does a probability of 1 mean?

Extend

18. From 35 yards, Didier successfully kicks one field goal in two. What is the probability that Didier scores

a) five field goals in a row?

b) four out of five field goals?

19. James and Karina are playing "best of five" with a coin. James calls tails. After two tosses, Karina has scored two heads.

a) What is the probability that the third toss will be tails?

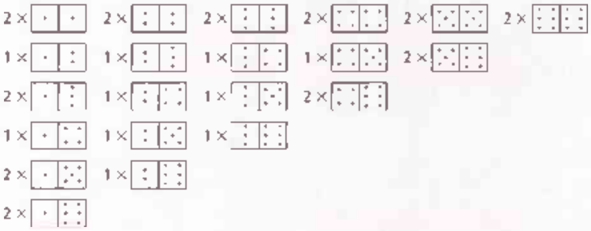
b) Is this probability affected by the results of the first two tosses?

c) What is the probability that Karina will win the best of five, given her two heads so far?

20. Tien Gow, meaning "Heaven Nine," is a Cantonese domino game for four players. It is played with a set of 32 Chinese dominoes. Each player starts with eight randomly chosen dominoes.

- At each move, you must place a new domino touching one that has already been played.
- For each pair of touching dominoes, the halves that touch must have equal numbers of spots.

Which are the best dominoes to hold, and why?



Making Connections

Sharing Birthdays

When is your birthday? Does someone in your class share your birthday? Do two classmates have the same birthday?

Amazingly, the probability of two people in a class of 24 sharing a birthday is more than 50%. This is true even if you don't count twins.

1. Find out whether your class has two people (not twins) with a shared birthday. You can count your teacher.

2. Survey some other classes to find out if class members also have shared birthdays. What did you find?

CHAPTER 4

Review

Key Words

For questions 1 to 3, copy the statement and fill in the blanks. Use some of these words:

tally chart	*frequency table*
probability	*outcomes*
favourable outcomes	*random*
tree diagram	*simulation*

1. The ■■■■■■◉■■■■ of getting heads in a coin toss can be estimated. You would use a ■■◉■■ ■■■■■ and ■■■■■■■◉■ ■■■■■ to record your trials.

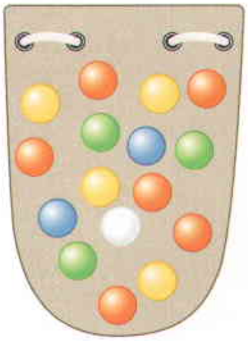
2. To determine a probability, count the total ■■■■■■■◉ and the ■◉■■■◉■■■■■ ◉■■■■■■■.

3. A ◉■■■ ■◉■■■■◉ can be used to organize outcomes.

4. Rearrange the circled letters in questions 1 to 3 to make a key word. Define this word.

4.1 Introducing Probability, pages 116–120

Use this visual for questions 5 to 7.



5. Josephine randomly draws a marble from the bag.

a) Is she as likely to draw a blue marble as a white marble?

b) What is the probability of drawing a yellow marble?

6. What is the probability of drawing

a) an orange or a white marble?

b) any marble other than blue?

7. a) Which colour marble is Josephine most likely to pick? least likely to pick? Why?

b) Explain why the probabilities change, depending on the colour.

8. Work with a classmate. Write the numbers 1 to 10 on separate cards, shuffle them, and place them face down in a row. Take turns turning over a card. If the number on the card is even, Player 1 wins. If it is odd, Player 2 wins. Turn the card back over and shuffle again.

a) Play the game for 5 min. Record who wins in a tally chart and frequency table.

b) Estimate the probability of each player winning.

c) Explain whether one player was more or less likely to win. Justify your reasoning.

4.2 Organize Outcomes, pages 121–125

9. Connie is having trouble deciding which sweatshirt to buy. She has a choice of

- 2 styles: buttoned or pullover
- 3 colours: blue, black, or red

a) Draw a tree diagram showing all her possible choices.

b) How many different choices are there?

c) Decide and explain whether the probability of selecting each sweatshirt combination is the same.

10. For each situation, choose a method for organizing the outcomes. Justify your choice.

a) spinning a two-section spinner marked 1 and 2, rolling a number cube, and adding the values

b) tossing a coin three times

4.3 Use Outcomes to Predict Probability, pages 126–130

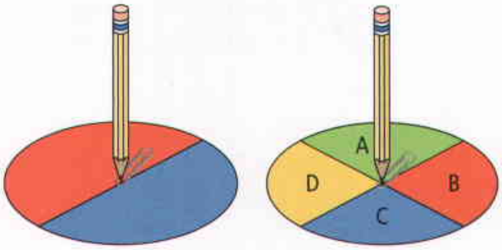
11. For each question,

- state the probability
- explain your reasoning

a) rolling a number less than 4 on a number cube

b) choosing the letter Z in a random draw from letters in the phrase PIZZA BIZARRO

12. Two spinners are spun at the same time.



a) Draw a tree diagram for the possible outcomes.

b) List all the possible outcomes.

c) Which combinations are most likely? Justify your reasoning.

13. A 10-sided die with letters A through J is rolled 60 times. How many times would you expect the outcome E? Why?

4.4 Extension: Simulations, pages 131–133

14. In each box of Yona's favourite cereal, there is a toy figure promoting the movie *Chasm of Doom*. Yona wants to collect all 5 figures.

a) Predict how many boxes Yona will have to buy. Justify your answer.

b) Explain what item you will use to simulate buying the boxes.

c) Perform the simulation. Record your results in a tally chart.

d) Justify why you repeated the simulation the number of times you did.

e) Compare your prediction and your simulation results.

4.5 Apply Probability in Sports and Games, pages 134–139

15. Janet made up a game from a 10 by 10 game sheet with 100 empty squares. She placed the letters A, E, I, O, and U randomly on the board. The letters included 11 As, 7 Es, 12 Is, 4 Os, and 8 Us. She left 9 squares blank. She coloured the remaining squares.

a) If Janet closes her eyes and touches a square, what is the probability that she will touch

- a blank square?
- a vowel?
- neither a blank nor a vowel?

b) Which type of square is she most likely to choose? Justify your response.

CHAPTER

4 Practice Test

Strand	NSN	MEA	GSS	PA	DMP
Questions	5, 6, 11				1–13

Multiple Choice

For questions 1 to 6, select the best answer.

1. What is the probability of guessing a wrong answer on a multiple choice test, when there are 4 options?

A $\frac{3}{4}$ **B** $\frac{1}{4}$ **C** 0 **D** 1

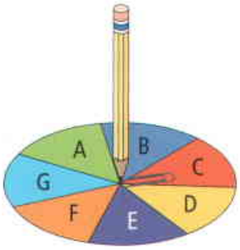
2. A cube numbered 1, 1, 2, 3, 4, 5 is rolled. The probability of rolling less than 3 is

A $\frac{1}{3}$ **B** $\frac{2}{3}$ **C** $\frac{4}{6}$ **D** $\frac{1}{2}$

3. When you roll a number cube, which outcome is most likely?

A rolling a 3
B rolling a number greater than 2
C rolling an even number
D rolling a sum of 7

4. This spinner is spun 50 times. Approximately how many times would you expect the outcome E?



A 4 **B** 7
C 10 **D** 13

5. If your baseball batting average is 0.400, how many hits would you expect to get in 20 at-bats?

A 4 **B** 12
C 8 **D** 6

Short Answer

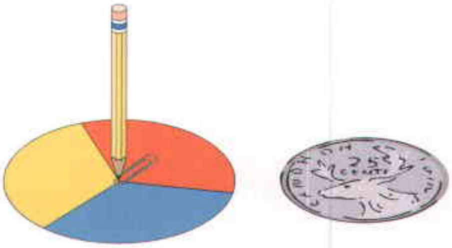
6. Each letter of the word PROBABILITY is written on a separate card and placed face down. You choose a card.

a) What is the probability of choosing the letter I?

b) What is the probability of choosing a consonant?

c) What is the probability of choosing one of the first 5 letters of the alphabet?

7. Su Yeon spins the following spinner and flips the coin.



a) Draw a tree diagram showing the possible outcomes.

b) List all the possible outcomes.

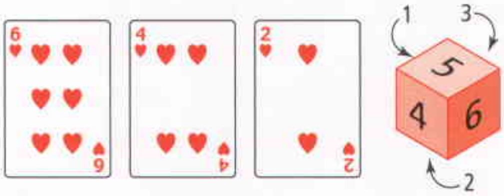
c) What is the probability that Su Yeon spins yellow and the coin lands heads up?

d) What is the probability that she spins red or blue and the coin lands tails up?

8. a) A coin is tossed. State the probability of heads, as a fraction and as a decimal.

b) Compare this value for the probability to an estimate based on repeatedly tossing a coin. How are the values related? How are they different?

9. Rebecca picks a card at random and rolls the number cube.



a) Draw a tree diagram.

b) List all the possible outcomes.

Extended Response

10. Jason orders a submarine sandwich for lunch every Friday. He has a choice of

- bread: white or whole wheat
- filler: turkey, salami, or cheese

a) How many different ways can Jason order his submarine? Use a tree diagram to organize your answer.

b) Is the probability of each submarine choice the same? Justify your reasoning.

11. Vladimir walked into class and was surprised to find a quiz at his desk. Having missed the last week of classes, Vladimir had no knowledge of the quiz content. There were four true-or-false questions that he had to guess at.

a) Based on Vladimir's lack of content knowledge, predict how many true-or-false answers he will get correct. Justify your answer.

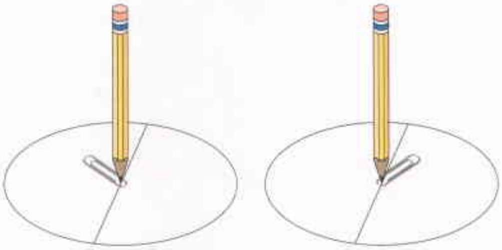
b) Using a spinner, simulate Vladimir's performance on the true-or-false questions. Record your results in a tally chart.

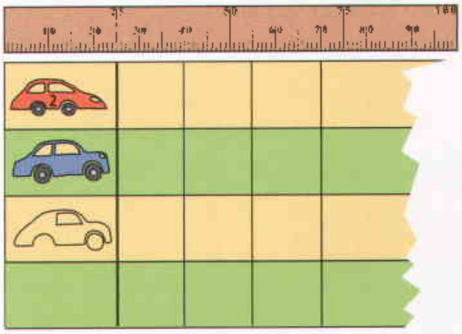
c) Estimate the probability of each score, based on your results.

d) Why might your simulation give different results than your prediction in part a)?

Chapter Problem Wrap-Up

Dana started making a car rally game. Can you help her finish it?





- The game should be fair for at least two people.
- Include a set of rules.

1. Show why your game is fair.

2. Test the game. Modify it if you need to.

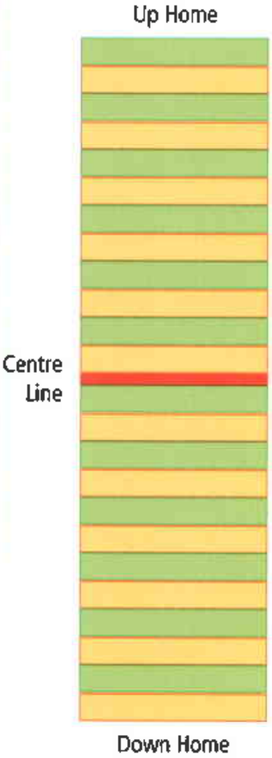
Making Connections

What does math have to do with nutrition?

Dietitians help people eat better. They study what a person eats every day and suggest diet changes. For example, they may suggest ways for the person to lower cholesterol. One way is to reduce the daily fat intake to a fraction of the amount eaten.

1. The average person consumes 2000 calories a day. Marta consumes 3000 calories a day. Marta's nutritionist suggests that she reduce her daily caloric intake to the average. By what fraction does Marla need to reduce her caloric intake?
2. Rick consumes 1800 calories a day. One-third of these are in the form of fat. How many calories does Rick eat as fat?

Making Connections



What are the chances?

In the game *Up and Down*, one player moves counters up the board and the other moves down. The object is to move all your counters off the opposite end of the board.

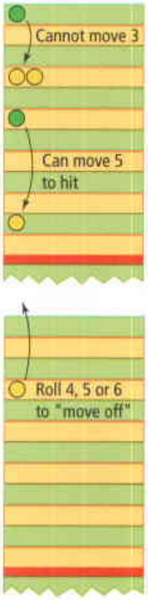
- Begin with all 15 counters stacked on your first space.
- Take turns to roll two number cubes.
- If you roll 3 and 5, for example, you can move one counter 3 spaces and another 5 spaces, or you can move a single counter 8 spaces.
- If you roll doubles, for example, 3 and 3, you get to make *four* moves of 3.
- Spaces with two or more counters of the *same* colour are "protected."
- If you have a single counter on any space, and your opponent has a roll that will reach that space, he or she can "hit" your counter and take it off the board.
- You must bring a counter that has been hit back on before making any other move.
- You can move a counter off if you roll high enough to move it beyond your last space.
- The winner is the first player to move all 15 counters off.

Materials

- 30 counters, half one colour, half another
- 2 sheets of plain paper
- ruler and pencil

Optional

- BLM 3/4 Task A Up and Down



Develop a Fair Game

1. Copy and cut out the spinner and the shapes shown. Use them to make a game that is fair for two players. How will you use the shapes in the spinner? Will you use the outside shapes? If so, how?

2. Use the materials to make a game that is fair for two players.

3. Develop rules and instructions for your game.

4. Use your knowledge of probability to justify your game plan. Explain how each player is equally likely to win.

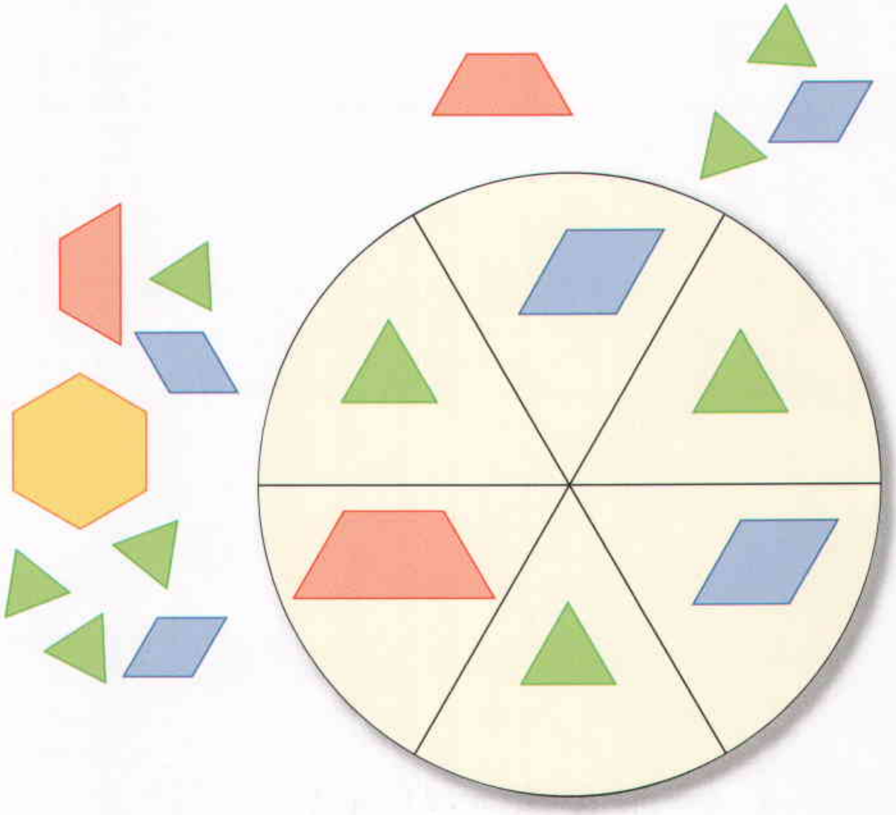
5. Can you develop more than one fair game? Show your results.

Materials

- plain paper
- ruler
- pencil crayons
- scissors

Optional:

- BLM 3/4 Task B Develop a Fair Game

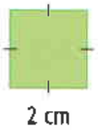


Chapters 1–4 Review

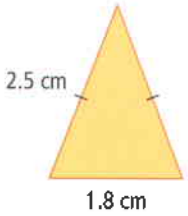
Chapter 1 Measurement and Number Sense

1. Find the perimeter of each shape.

a)

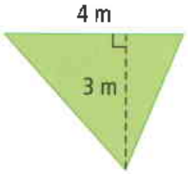


b)

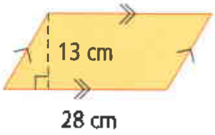


2. Find the area of each shape.

a)



b)



3. Evaluate each expression.

a) $3 + 4 \times 5$

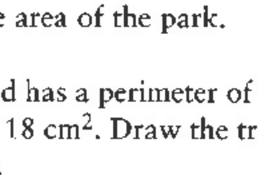
b) $2 \times 6 - 8 \div 2$

c) $5 + 6 \div 3 \times 2$

d) $(7 + 4 - 2) \times (5 - 2)$

e) $3 + 12 \times (16 \div 4 \div 4)$

4. A park is in the shape of a trapezoid, as shown.



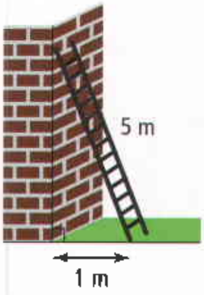
a) Find the length of fence needed to surround the park.

b) Find the area of the park.

5. A trapezoid has a perimeter of 18 cm and an area of 18 cm^2. Draw the trapezoid on grid paper.

Chapter 2 Two-Dimensional Geometry

6. A 5-m ladder leans with its lower end 1 m from the wall. What type of triangle is formed?



7. Sketch and name the geometric shape(s) with the following properties.

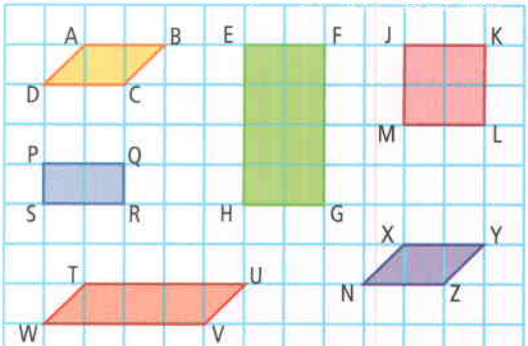
a) three sides; each with a different length

b) three sides; two with the same length

c) four sides of equal length

d) four sides, two opposite pairs have the same length

8. Ms. Jung picked up six erasers left behind by her students. She noticed that they were different shapes and sizes. She sketched the shapes she saw.



a) What geometric shapes can be seen?

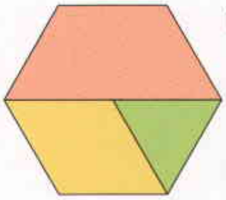
b) Which of the shapes are congruent? Explain.

c) Which of the shapes are similar? Explain.

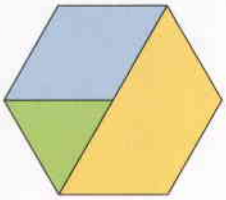
Chapter 3 Fraction Operations

9. Write an addition sentence to represent the fraction of each yellow hexagon that is covered.

a)



b)



10. Use pattern blocks or diagrams to model each subtraction.

a) $\frac{1}{2} - \frac{1}{3}$ **b)** $\frac{2}{3} - \frac{1}{6}$

11. Find a common denominator for each pair of fractions.

a) $\frac{3}{5}$ and $\frac{1}{2}$ **b)** $\frac{1}{3}$ and $\frac{3}{4}$

12. Evaluate.

a) $\frac{1}{2} + \frac{2}{5}$ **b)** $\frac{3}{4} - \frac{2}{3}$

13. Paula spends her Mondays as follows.

8 h sleeping
6 h at school
3 h at gymnastics
2 h doing homework

The rest of the time she is eating or relaxing.

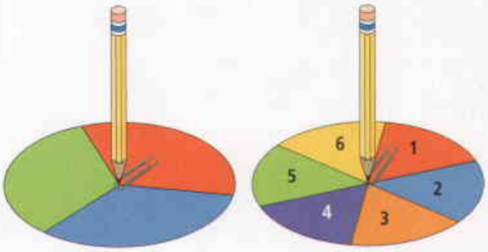
a) What fraction of the day does Paula spend sleeping?
b) What fraction of the day is she at school?
c) How many hours does she spend eating or relaxing? What fraction of the day is this?

Chapter 4 Probability and Number Sense

14. A number cube and a coin are tossed simultaneously.

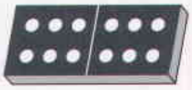
a) Draw a tree diagram showing all the possible outcomes.
b) What is the probability of a 1 and tails?
c) What is the probability of a 2 or a 3 and heads?
d) What is the probability of an odd number and tails?
e) What is the probability of a number less than 4 and heads?

15. Two spinners are spun. Find the probability of spinning each of the following.



a) red and the number 4
b) red or blue and the number 4
c) green and an even number
d) blue or green and a number less than 4

16. A domino set has 28 tiles. Seven of the tiles are doubles. Doubles have the same number of dots on each half. All the dominoes are placed face down on the table. Find the probability of each outcome.



a) a domino that is a double
b) a domino that is not a double

Number Sense and Numeration

- Compare and order fractions, decimals, and percents.
- Solve problems by converting between fractions, decimals, and percents.
- Use estimation to justify or assess the reasonableness of calculations.
- Explain the process of problem solving.

Measurement

- Estimate and calculate areas of trapezoids and irregular two-dimensional shapes.

Data Management and Probability

- Display data on bar graphs, pictographs, and circle graphs.
- Use and apply a knowledge of probability in sports and games.

Key Words

statistic
repeating decimal
percent