CHAPTER 12

Patterning and Equations

Geometric designs can give walls, floors, and furniture an attractive finish.

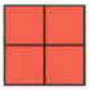
Some designs use circles, squares, or other geometric forms in patterns.

You can use patterning and equation skills to plan similar designs of different sizes. In this chapter, you will work with many patterns, including tile designs. You will describe patterns using variable expressions and determine the number of items in various stages of patterns. You will use equations and patterning to solve problems.

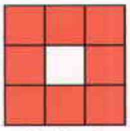
Chapter Problem

Vicki is making coffee tables. She is going to copy the floor pattern here for one of her designs.

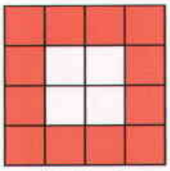
Extend the pattern to show two more tile designs. Explain how the steps in your pattern are related.



Design 1



Design 2



Design 3

Get Ready

Use the Order of Operations

In math, the correct **order of operations** is

Making Connections

You used the order of operations in Chapter 4.

B Brackets, then
O Order:
D, M Division and Multiplication, from left to right
A, S Addition and Subtraction, from left to right

For example,

$12 - 4 \times 2$
$= 12 - 8$ **Multiply first.**
$= 4$ **Then, subtract.**

$2 \times (3 + 4)$
$= 2 \times 7$ **Do brackets first.**
$= 14$

1. Evaluate each expression.

a) $5 \times 2 + 3$
b) $3 \times 12 - 5$
c) $2 \times 6 + 2 \times 7$
d) $9 \times 12 \div 4$

2. Evaluate each expression.

a) $3 \times (5 + 2)$
b) $(15 - 7) \div 4$
c) $(3 \times 1.5 + 0.5) \times 13$

Translate Into Mathematics

To solve a problem, you sometimes need to translate words into operations. For example, "the *sum* of 3 and 8" translates as $3 + 8$.

For example, find the number that is 12 *more than* the *product* of 2 and 7:

$2 \times 7 + 12$
$= 14 + 12$
$= 26$

More than tells me to add. *Product* means to multiply. I multiplied first, because of BODMAS.

Literacy Connections

Which Operation?

$+$ sum, grows by, more than, increased
$-$ less than, difference, subtracted from, decreased
$\times$ product, times, double
$\div$ share equally, the quotient

3. Translate each statement using the $+$, $-$, $\times$, and $\div$ operations. Calculate the result.

a) the sum of 3 and 13
b) four less than 7
c) double 6
d) the difference of 13 and 8
e) the product of 8 and 9

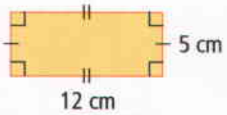
4. For each question, translate, and calculate the result.

a) Share $20 equally among 4 people. How much does each person get?
b) If 77 grows by 13, how big does it get?
c) What is 6 less than double 17?
d) What is 5 more than the product of 8 and 6?

Work With Formulas

When you work with formulas, substitute what you know, and evaluate using the order of operations.

To find the perimeter of this rectangle, apply the formula $P = 2(l + w)$.



$P = 2(l + w)$
$P = 2 \times (12 + 5)$
$P = 2 \times 17$
$P = 34$

2(l + w) means add l and w, then multiply by 2.

The perimeter of the rectangle is 34 cm.

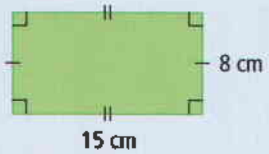
To find the rectangle's area, apply the formula $A = lw$.

$A = lw$
$A = 12 \times 5$
$A = 60$

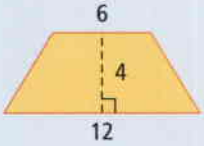
lw means multiply $l \times w$.

The area of the rectangle is 60 cm^2.

5. Find the perimeter and the area of this rectangle.



6. Find the area of this trapezoid. Use the formula $A = (a + b) \times h \div 2$.



Identify and Extend Patterns

When you work with patterns, ask yourself two questions:
- How does the pattern start?
- How does it change from one item to the next?

For example, the pattern 5, 7, 9, … begins with 5 and increases by adding 2. The next two numbers are 11 and 13. You can describe this number pattern as follows:

$5 = 2 + 3$ *or* $5 = 5$
$7 = 2 + 2 + 3$ $7 = 5 + 2$
$9 = 2 + 2 + 2 + 3$ $9 = 5 + 2 + 2$

7. Describe each number pattern in words. Then, find the next two numbers.

a) 4, 8, 12, …
b) 6, 10, 14, …
c) 5, 9, 13, …

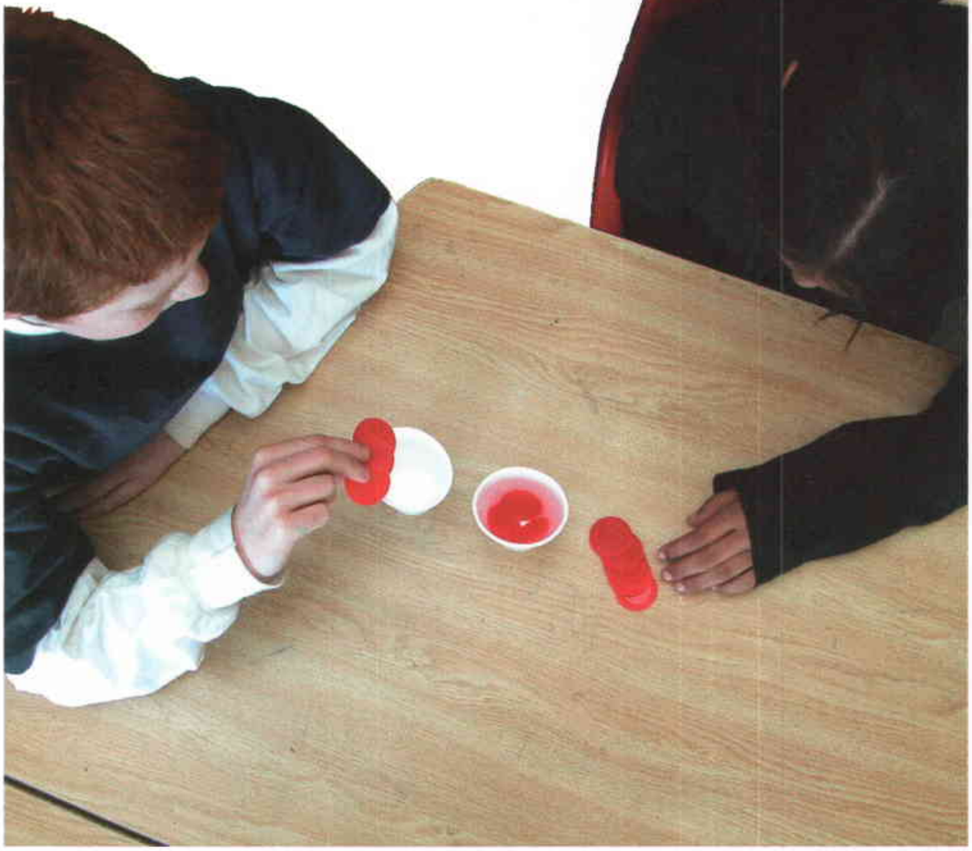
8. a) Extend this pattern to two more diagrams.
b) How many dots are in the 6th diagram? Explain.
c) How many dots are in the 15th diagram? Explain.

12.1

Variables and Expressions

Focus on...

- representing variables
- modelling expressions



You can use cups and counters to model math expressions. What could the cups in the photograph represent? Why do you think they both hold the same number of counters?

Materials

- 2 cups or containers
- supply of counters

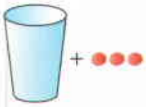
Optional:

- BLM 12.1A Model Expressions Without Manipulatives

Discover the Math

How can you represent variables and model expressions?

1. Use a cup to represent an unknown number of counters. What would this diagram represent?



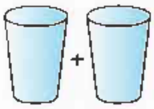
2. Use the **variable** C to represent the counters in the cup. Write an expression to model the total number of counters.

variable

- a letter that represents an unknown number

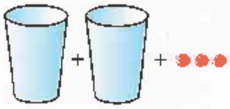
3. **a)** Suppose you put 10 counters in the cup. Instead of an unknown number, you now know the cup has a value. What value?
   b) How many counters will you have in total?
   c) If you let C = 10, what is the value of your expression in step 2?

4. **a)** Each cup in this diagram contains the same number of counters. Use C to show what the diagram represents.



   b) Use cups and counters. Try various numbers for the counters in the cups. Show each solution using the variable C and then using numbers.

5. **Reflect** Describe in words what each set of cups and counters models. Then, use a **variable expression** to model the diagram.

a)


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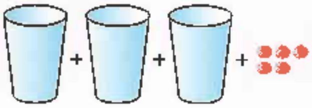


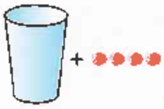
variable expression
- contains variables and operations with numbers
- C + 3 and 2C are variable expressions

Example 1: Model Number Phrases

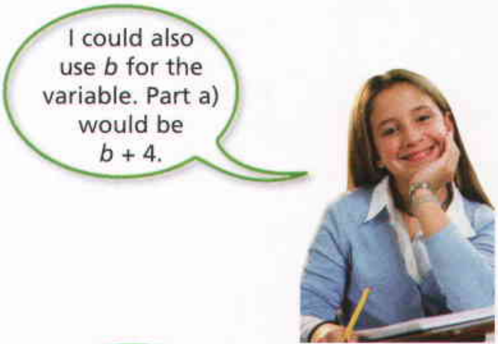
Model each phrase with cups and counters. Then, write each as a variable expression.

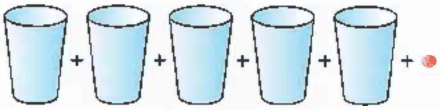
a) 4 more than a number
b) 5 times a number plus 1

Solution

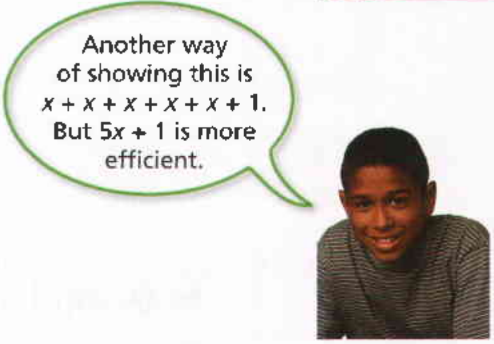
a)


Let p represent the number.
$p + 4$



b)


Let x represent the number.
$5x + 1$



Example 2: Translate Models

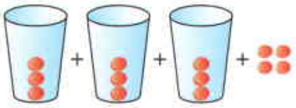
a) Use cups and counters to model the variable expression $3x + 4$.

b) Evaluate the expression for these values of x: $x = 3$, $x = 4$, and $x = 5$.

Solution

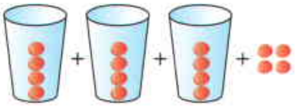
a)

b) *Method 1: Use Cups and Counters*



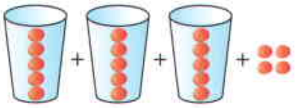
For $x = 3$: $3 + 3 + 3 + 4$
$= 13$

I could also say $3 \times 3 + 4 = 13.$



For $x = 4$: $4 + 4 + 4 + 4$
$= 16$

I could say $3 \times 4 + 4 = 16.$



For $x = 5$: $5 + 5 + 5 + 4$
$= 19$

I could say $3 \times 5 + 4 = 19.$

Method 2: Substitute Into the Expression $3x + 4$

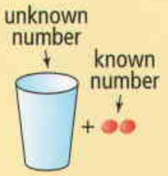
For $x = 3$:
$3 \times 3 + 4$
$= 9 + 4$
$= 13$

For $x = 4$:
$3 \times 4 + 4$
$= 12 + 4$
$= 16$

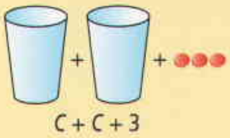
For $x = 5$:
$3 \times 5 + 4$
$= 15 + 4$
$= 19$

Key Ideas

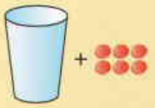
- A container, such as a cup, can be used to model an unknown number.
- Counters, such as integer chips or algebra tiles, can be used to represent known numbers.



- A variable represents an unknown number. For example, you could use C to represent the contents of the cups in the diagram.



- Variable expressions have a combination of variables, numbers, and operations. For example, another way of showing the expression in the diagram is 2C + 3.
- You can substitute a number for a variable, and then evaluate a variable expression. For example, evaluate the expression on the right if there are 7 counters in the cup.

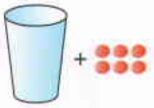


$$\begin{aligned} &C + 6 \\ &= 7 + 6 \\ &= 13 \end{aligned}$$

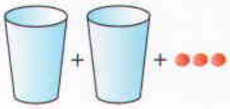
Communicate the Ideas

1. **a)** Identify the number and the variable in the expression C + 6.

b) Describe two advantages of using letters, instead of cups, for variables.

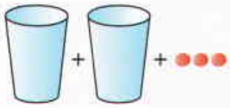


2. Develop a variable expression for the diagram. Describe how you did it.

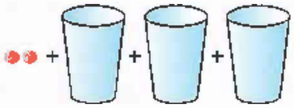


3. The class was asked to model "2 more than 3 times a number." Look at the following solutions. Identify any errors. How would you correct them?

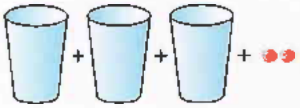
a) Mark's model:



b) Rhiann's model:



c) Kayla's model:



4. **a)** Use cups and counters to model the variable expression $5a + 2$.

b) Create another variable expression that means the same thing but does not use multiplication. Justify your response.

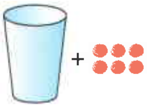
Check Your Understanding

Practise

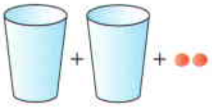
For help with questions 5 to 8, refer to Example 1.

5. Write each model as a variable expression.

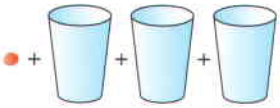
a)



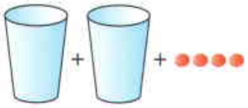
b)



c)

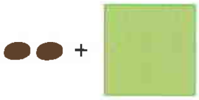


d)

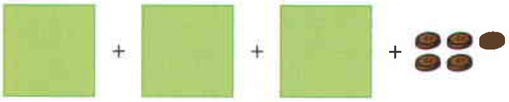


6. John uses a large square to represent an unknown quantity and buttons to represent counters. What variable expression is modelled by each diagram?

a)



b)



c)

7. Model each phrase with cups and counters.

a) a number plus 5

b) double a number

8. Model each phrase with cups and counters.

a) 2 more than 3 times a number

b) 4 less than 5 times a number

For help with questions 9 to 12, refer to Example 2.

9. Use cups and counters to model each variable expression.

a) $2a + 3$ **b)** $4n + 5$

c) $2 + 5x$ **d)** $3w + 10$

10. a) For each expression in question 9, put 2 counters in each cup. Evaluate the expression.

b) Repeat part a) using 3 and then 4 counters in each cup. Evaluate the expression.

11. Substitute $k = 5$ and evaluate each expression.

a) $k + 6$ **b)** $k - 3$

c) $3k$ **d)** $2k + 3$

e) $12 - k$ **f)** $4 + 3k$

12. For each expression in question 11, substitute $k = 8$ and evaluate.

Apply

13. Model each variable expression.

a) $4 + 3t$

b) $7t - 4$

c) $5 + 6t$

d) $10t + 3$

14. For each expression in question 13, substitute $t = 3$ and evaluate.

15. Show each phrase with a variable expression, using addition, subtraction, or multiplication.

a) \$10 *more than* an unknown price

b) the *product* of 8 and Jessica's age

c) the area *increased by* 10 cm^2

d) *double* the length

16. Write a variable expression for each phrase.

- **a)** 10 cm *shorter* than you
- **b)** 15 *less than* your opponent's score in a card game
- **c)** *triple* the elephant's mass
- **d)** the *sum* of 12 and Kenneth's points

17. The expression $5h + 3$ describes Sonja's pay scale for baby-sitting.

- **a)** Model this expression with cups and counters.
- **b)** Explain what you think the expression means.
- **c)** If Sonja baby-sits from 6.30 P.M. to 11.30 P.M., how much does she earn?

18. You can use the expression $\frac{n}{20}$ to estimate your average walking speed, in kilometres per hour. The variable n is the number of steps you take in 1 min. Estimate your walking speed if you take 80 steps in 1 min.

19. **a)** Evaluate the expression $3x + 4$ for each value: $x = 1, 2, 3, 4$, and 5.

- **b)** Describe the pattern you see in your answers. What could produce this pattern?

20. You can use the expression $\frac{m}{13}$ to estimate the total volume of blood in your body, in litres. The variable m is your mass, in kilograms.

- **a)** Estimate how much blood a 39-kg teenager has.
- **b)** Estimate how much blood a 70-kg adult has.
- **c)** Estimate the volume of blood in your body.

Try This!

21. **a)** Model this phrase with a variable expression: double the area, increased by 5 cm^2.

- **b)** Evaluate your expression in part a) for each area: 5 cm^2, 10 cm^2, 15 cm^2, 20 cm^2.
- **c)** Describe a pattern rule for your expression values.
- **d)** Try to extend the pattern. Identify the 10th value.

Extend

22. The average January temperature can be estimated for any city in the world. The expression $33 - 0.75L$ estimates the temperature, in degrees Celsius, for a city with a latitude of L degrees. What is the estimated mean January temperature for

- **a)** Kingston, Jamaica, at 17°N?
- **b)** Yellowknife, Yukon, at 62°N?
- **c)** London, England, at 52°N?
- **d)** Is the estimate for part c) accurate? Explain.

Go to www.mcgrawhill.ca/links/math7 and follow the links to find out.

Internet Connect

Literacy Connections

Latitude

Latitude measures how far north or south you are from the equator. The equator is 0°. The North Pole is 90°N. The South Pole is 90°S.

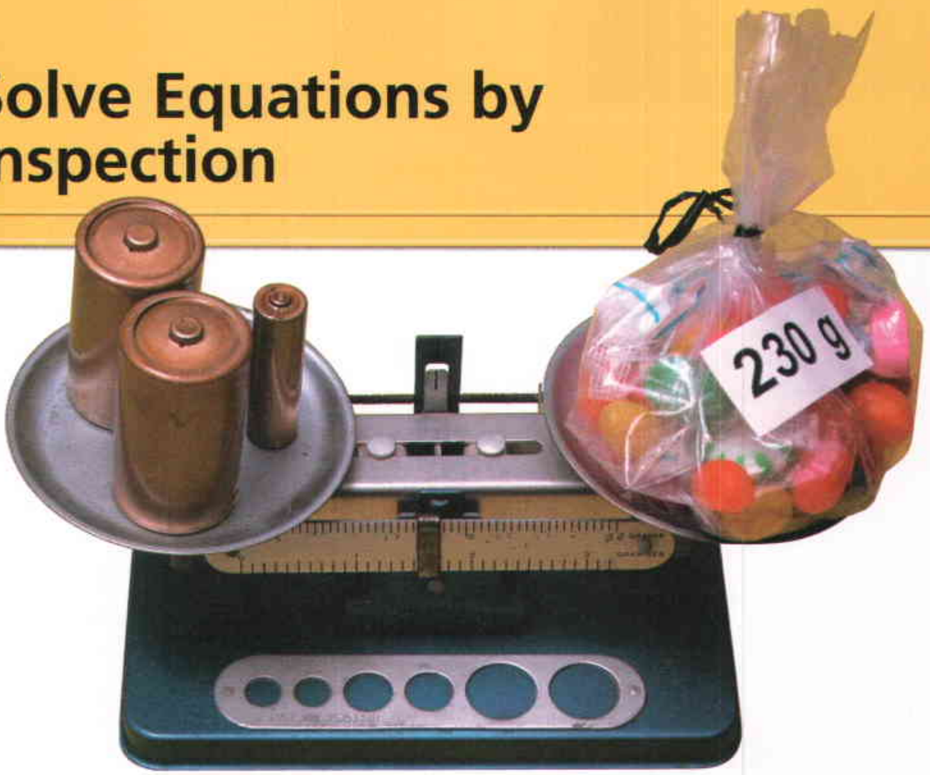
23. **a)** Substitute $x = 5$ into each of the expressions $x + 4$ and $2x$. Evaluate each expression.

- **b)** Which expression gives the greater result? Will this be true for all values of x? Justify your answer.

12.2

Solve Equations by Inspection

Focus on...
- mental math
- substituting into equations



This balance is showing 230 g. The smaller mass is 30 g. What are each of the larger masses?

What does the equation $30 + 2m = 230$ have to do with this situation?

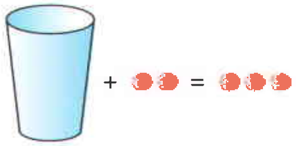
Discover the Math

Materials
- 5 cups or containers
- supply of counters

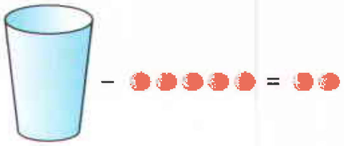
How can you find the solution to an equation using mental math?

1. Model each diagram with cups and counters. Determine the number of counters in each cup.

a)



b)



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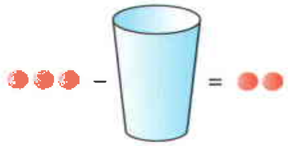
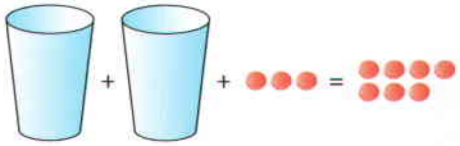


d)



2. **a)** Describe how you came up with the answer in each part of step 1.
b) How could you use mental math to answer step 1?
c) How can you model step 1 using a balance, like the one in the picture?

3. **a)** Use one of your strategies from step 2 to find the value of the variable n in the equation $n + 4 = 5$.
b) Is your answer to part a) the only **solution**? Explain.

solution (of an equation)
- a number that makes an equation true
- $y = 3$ is the solution to $y + 1 = 4$

4. How could you use mental math to find the value of the variable in each equation?
a) $p - 5 = 2$
b) $w + 5 = 10$
c) $5 \times c = 10$
d) $3 \times x = 6$

5. **Reflect** Explain the similarities between your method in step 1 and your method in step 4. Describe how you can use mental math to solve simple equations.

Literacy Connections

The pictures of balances are designed to help you visualize the idea of balancing an equation.

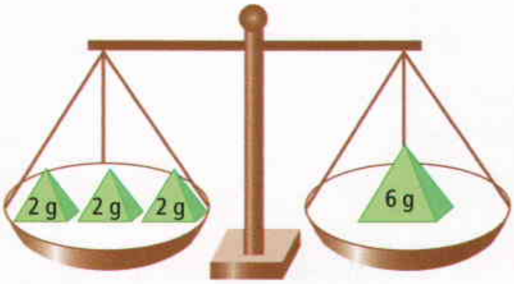
Example 1: Solve by Inspection

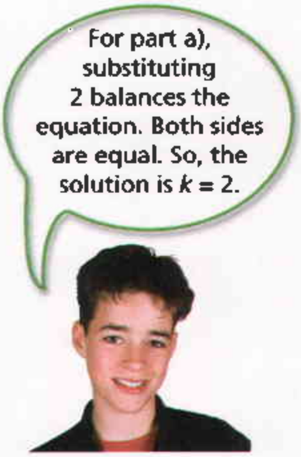
For each equation, use mental math to try a number as a solution.

a) $3k = 6$
b) $m + 3 = 12$
c) $7 - x = 4$
d) $\frac{a}{3} = 4$

Solution

a) $3 \times 2 = 6$
So, the solution is $k = 2$.





b) $9 + 3 = 12$
So, the solution is $m = 9$.

c) $7 - 3 = 4$
The solution is $x = 3$.

d) $\frac{12}{3} = 4$
The solution is $a = 12$.

Example 2: Test for Solutions

Which of the numbers 5, 6, and 7 is a solution to the equation $4x + 3 = 27$?

Solution

I know $x = 6$ is the solution. So, I think $x = 7$ won't work. I'll check.

Try each number.

$4 \times 5 + 3$
$= 20 + 3$
$= 23$
$x = 5$ does not work.

$4 \times 6 + 3$
$= 24 + 3$
$= 27$
With $x = 6$, the equation is true.

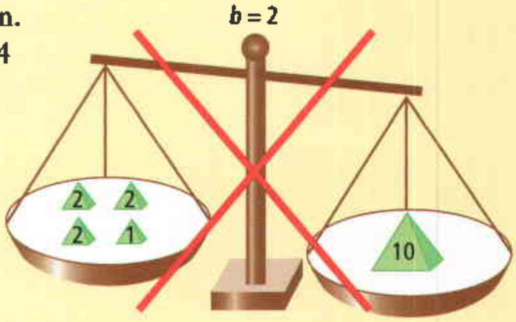
$4 \times 7 + 3$
$= 28 + 3$
$= 31$
$x = 7$ does not work.

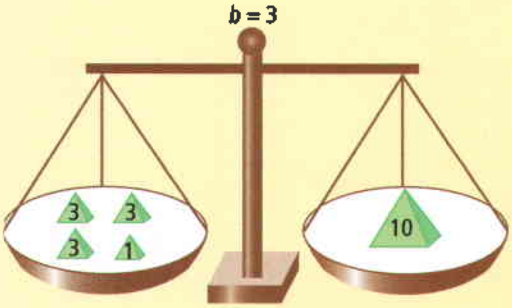
The solution is $x = 6$.

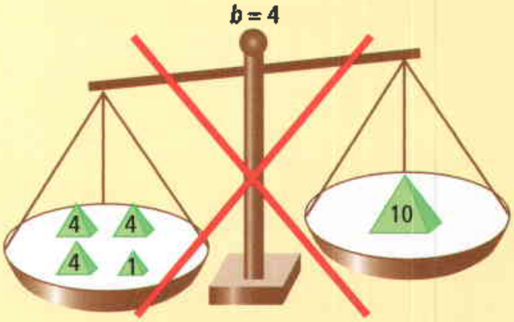
Key Ideas

- A number is the solution to an equation if the two sides of the equation are equal after substituting.
- Simple equations can be solved by inspection, using mental math. For example, what mass would balance the scales in the top diagram?

- You can test possible solutions to an equation. For example, which of $b = 2$, $b = 3$, and $b = 4$ is the solution to $3b + 1 = 10$? Testing shows that $b = 3$ is the solution.
  $3 \times 3 + 1$
  $= 9 + 1$
  $= 10$

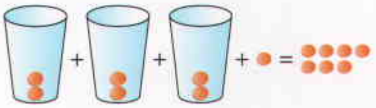






Communicate the Ideas

1. The diagram shows how Tom used modelling to solve an equation.
   a) What is being tested here?
   b) Do the cups show a possible solution? Explain.



2. Describe how to use inspection to solve the equation $9k = 72$.

3. To solve the equation $7 + 6y = 31$, you are given these values to test: $y = 3$, $y = 4$, $y = 5$, and $y = 6$. Will you have to test all four values? Explain why or why not.

Check Your Understanding

Practise

For help with questions 4 to 9, refer to Example 1.

4. What mass will keep the balance?

a)

b)

c)

5. What number on each weight will keep the balance?

a)

b)

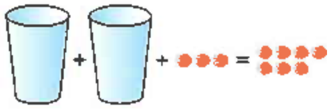
c)

6. Find the value of each cup to make the sentence true.

a)

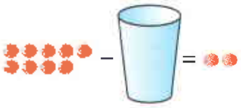
b)

c)

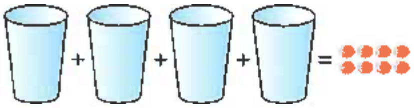


7. Find the value of each cup to make the sentence true.

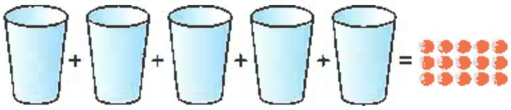
a)



b)



c)



8. Solve by inspection or using manipulatives.

a) $7b = 21$ **b)** $x + 15 = 25$

c) $8v = 56$ **d)** $13 = 3 + n$

e) $r - 10 = 0$ **f)** $19 - g = 13$

9. Solve by inspection.

a) $n + 7 = 26$ **b)** $35 = y - 15$

c) $63 = 7t$ **d)** $2a + 2 = 10$

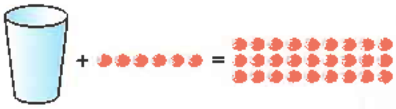
e) $25 - x = 16$ **f)** $30 = 5q + 5$

For help with questions 10 to 15, refer to Example 2.

10. How many counters should go in the cup? Test the possible solutions. Which number balances the equation?

Test $b = 11$, 12, and 13.

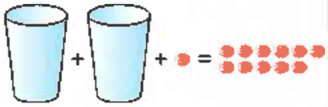
11. How many counters should go in the cup? Justify your answer.



Test $y = 21$, 22, and 23.

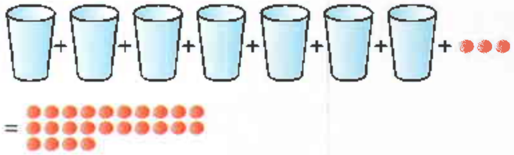
12. For each diagram, how many counters should go in each cup? Test the possible solutions.

a)



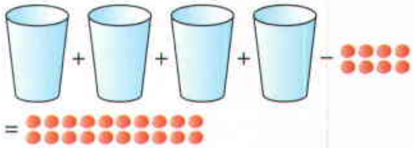
Test $c = 3$, 4, and 5.

b)



Test $k = 2$, 3, and 4.

c)



Test $t = 6$, 7, and 8.

13. Show whether or not $x = 4$ is the solution to each equation.

a) $6x = 24$ **b)** $17 - x = 10$

c) $36 \div x = 8$ **d)** $x + 27 = 33$

e) $3x + 1 = 15$ **f)** $20 = x + 10$

14. Determine whether $k = 9$ or 10 is the solution to each equation.

a) $k + 5 = 14$ **b)** $k - 8 = 1$

c) $k + 6 = 16$ **d)** $2k = 18$

e) $k \div 2 = 5$ **f)** $16 = 2x - 4$

15. Solve each equation.

a) $h + 7 = 10$ **b)** $6m = 36$

c) $y - 17 = 21$ **d)** $f \div 7 = 11$

e) $50c = 250$ **f)** $3 + 20x = 83$

Apply

16. Draw a balance to show the equation $230 = 30 + 2m$.

- **a)** What total mass does the balance show?
- **b)** What does the equation tell you about the smaller mass? the two larger masses?
- **c)** Solve the equation to determine each larger mass.

17. **a)** Write two different equations that do *not* have 3 as a solution. Each equation should use a different operation (addition, subtraction, multiplication, division).

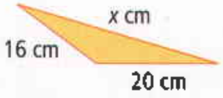
- **b)** Describe how you came up with your equations.

18. If Annie had $5 more in her pocket, she could afford a $14 CD. An equation modelling this is $x + 5 = 14$.

- **a)** Explain what the variable x represents. How do you know?
- **b)** Solve the equation to find how much money Annie has.

19. Kathy walks at a speed of 100 m per minute on her way to school. She lives 1500 m from school. Kathy's walk can be modelled by the equation $100t = 1500$. In this equation, t is the time, in minutes, it takes Kathy to get to school. Solve the equation to determine how long it takes Kathy to get to school.

20. The perimeter of this triangle is 70 cm. An equation modelling the perimeter is $x + 20 + 16 = 70$. Determine the length of side x.



Try This!

21. **a)** Write two different equations that have 5 as a solution. Each equation should use a different operation (addition, subtraction, multiplication, division).

- **b)** Describe how you came up with your equations.

Extend

22. A competitor's total score in a diving competition can be calculated using the formula $T = Mn$.

- M is the mean score.
- T is the total score.
- n is the number of judges.

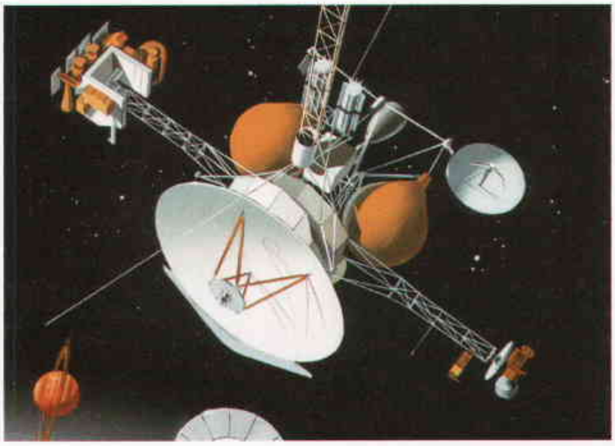
If Alex's mean score was 8 and his total score was 48, how many judges were there?

23. The sum of 3 and a number is –10.

- **a)** Write an equation to model this situation.
- **b)** Solve the equation to find the unknown number.

24. When you know your speed and travel time, the distance can be calculated using the formula $d = s \times t$.

- **a)** How fast are you driving if it takes 5 h to go 350 km?
- **b)** The Cassini space probe arrived at Saturn in 2004. How fast did the probe travel if it took 5 h to go 2400 km?



Did You Know?

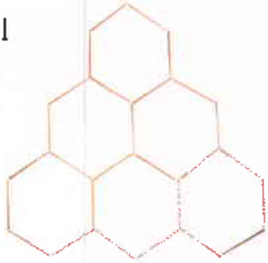
Saturn's largest moon, Titan, has a thick methane atmosphere. Cassini will teach scientists a lot about this strange moon.

12.3

Model Patterns With Equations

Focus on…
- geometric and other patterns
- modelling with equations

In this honeycomb pattern, each row has one more cell than the row above it. What equation models the row of cells that uses 21 toothpicks?



Discover the Math

Materials
- supply of counters
- 4 cups or containers

Optional:
- BLM 12.3A Develop an Equation Step by Step

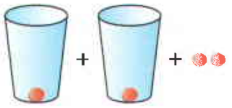
How can you use patterning to write equations?

1. Use counters to make this pattern of rectangles. Extend the pattern as far as Rectangle 5.

2. a) Rectangle 1 can be modelled as in the photo

Model Rectangles 2 and 3 using cups and counters. Hint: In your models, continue the pattern started in the photograph. Think about what part of the pattern each cup will represent.

b) Use a table like the one below to record what you are doing in each model.

Rectangle number	Cups and Counters	Equation
1	 2 cups with 1 counter each + 2 counters	2 × 1 + 2 = 4
2	2 cups with ■ counters each + 2 counters	2 × ■ + 2 = ■
3		

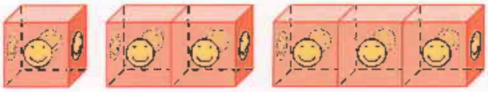
3. a) Extend your table to predict what will happen in Rectangles 4 and 5.
b) Check by modelling. Revise your equation, if necessary.

4. Reflect How did patterning help you develop your equations?

Example 1: Write an Equation for a Patterning Problem

Cosmic Toys makes coloured rods.

- Cubes are joined in rows to make the rods.
- The rods are then dipped in paint.
- Finally, a smiley face sticker is fixed to every exposed vertical face.



For example, the two-cube rod has 6 smiley faces.

How many cubes are needed to have 30 smiley faces?
Model this problem with an equation.

Solution

Understand

The equation has to model the number of smiley faces on a rod. The expression for the number of smiley faces should equal 30.

Plan

1. Use a table to organize the pattern and create a set of equations.
2. Use the equations in the table to help you write an equation that models the rod with 30 smiley faces.

Do It!

Strategies
Make a table or chart

1.

Number of Cubes	Number of Smiley Faces	Equation
1	1 at the front + 1 at the back + 2 at the ends = 4	$2 \times 1 + 2 = 4$
2	2 at the front + 2 at the back + 2 at the ends = 6	$2 \times 2 + 2 = 6$
3	3 at the front + 3 at the back + 2 at the ends = 8	$2 \times 3 + 2 = 8$
4	4 at the front + 4 at the back + 2 at the ends = 10	$2 \times 4 + 2 = 10$

2. Let n represent the number of cubes. The equation for the block pattern with 30 smiley faces is $2n + 2 = 30$.

If you double the number of cubes and add 2, you get the number of smiley faces.

Look Back

Substitute the values 1, 2, 3, and 4 for n and check against the equations in the table. For example:

$$\begin{aligned} 2n + 2 &= 2 \times 1 + 2 \\ &= 4 \end{aligned}$$

One cube has 4 smiley faces.

Example 2: Model Patterns With Equations

Study this pattern of marble diagrams.


Diagram 1


Diagram 2


Diagram 3

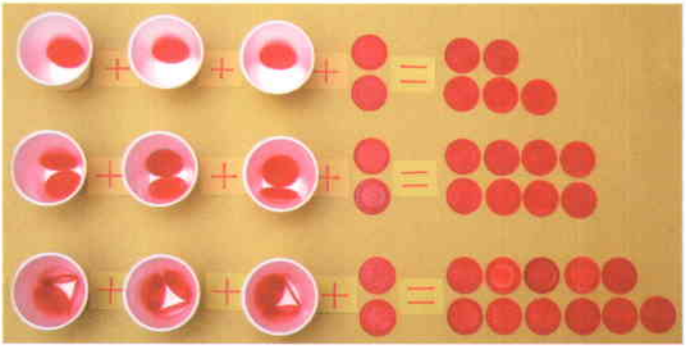
a) Use equations to model the first three diagrams.

b) Use an equation to model the diagram with 17 marbles. Explain what your equation means.

Solution

Strategies
Act it out

a)



Let d stand for the number of the diagram.

Diagram 1: $3d + 2 = 5$
Diagram 2: $3d + 2 = 8$
Diagram 3: $3d + 2 = 11$

The letter d also stands for the number of counters in each cup. In the model for diagram 2, there are 3 cups with 2 counters in each cup. Plus there are 2 more counters. I can use the equation $3d + 2 = 8$.

b) The equation is

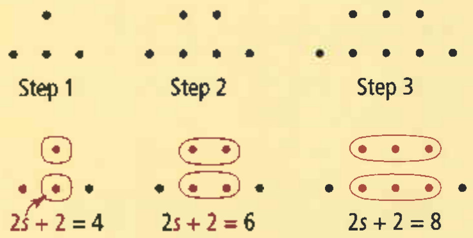
$$\overset{\text{number of counters in each cup}}{3}\underset{\text{number of the diagram}}{d} + \overset{\text{number of extra counters}}{2} = \underset{\text{total number of counters}}{17}$$

$3 \times 5 + 2 = 17$
This must be the 5th diagram.

I used the same variable expression for Diagrams 1, 2, and 3. I must use the same expression for the diagram with 17 marbles. The equation is $3d + 2 = 17$.

Key Ideas

- The items in a pattern can be translated into equations, using numbers or variables.
- An equation for a pattern can be explained in terms of the pattern. For example, look at the pattern to the right.

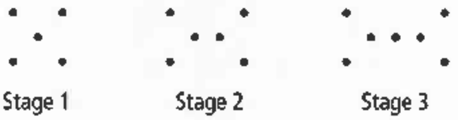


- You can write an equation in different ways. For example, the equations $2x + 3 = 12$, $12 = 2x + 3$, and $3 + 2x = 12$ all mean the same thing.

Communicate the Ideas

1. Helen wrote an equation as $2x + 3 = 7$. Kenneth wrote it as $7 = 2a + 3$. Jeeva wrote it as $7 = 3 + 2m$. Are all of these the same or different? Explain.

2. Which stage in this pattern has 19 dots?



Sameh started by writing
2 + 1 + 2 = 5
2 + 2 + 2 = 6
2 + 3 + 2 = 7

Rosie started by writing
An expression for the pattern is 4 + n.

a) Complete each solution. What are the advantages of each method?
b) Can you think of any other methods?

3. a) Kim wrote the expression $n - 3$ for a dot pattern. Draw three stages of a dot pattern for this expression.
b) Make an equation to show the number of dots in the third stage of your dot pattern.
c) Will your equation be exactly the same as those developed by other students? Explain.

Literacy Connections

Choosing Variables

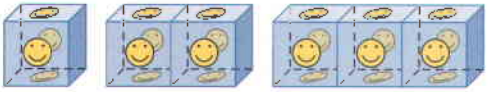
Any letter can be used for a variable. Try to use a letter that helps you remember what the variable represents. In the Key Ideas, *s* refers to the step number.

Check Your Understanding

Practise

For help with questions 4 and 5, refer to Example 1.

4. These rods have smiley stickers on all their square faces except the two end faces.



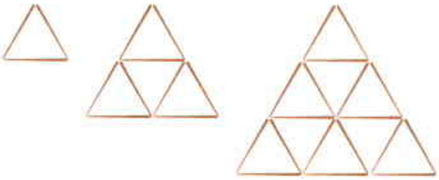
a) Copy and complete this table.

Number of Cubes	Number of Smiley Faces	Equation
1	1 (front) + 1 (back) + 1 (top) + 1 (base) = 4	$4 \times 1 = 4$
2		
3		

b) How long is the rod with 28 smiley faces? Write an equation for this rod.

5. a) Copy and complete this table for the toothpick pattern shown.

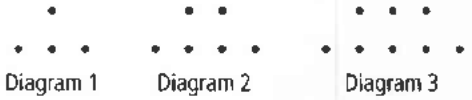
Number of Rows	Perimeter	Equation
1	1 + 1 + 1	$3 \times 1 = 3$
2	2 + 2 + 2	
3		



b) Write an equation for the number of rows of triangles needed to have a perimeter of 27 toothpicks.

For help with questions 6 and 7, refer to Example 2.

6. a) Use equations to model the first three diagrams in this pattern.



b) One diagram has the equation $2 + 2d = 14$. How many dots does the diagram have? Explain.

7. Study this pattern of marbles.

a) Use equations to model the first 3 diagrams.

b) Use an equation to model the diagram with 17 marbles. Explain what your equation means.

8. Write each equation in two other ways.

a) $k + 7 = 13$ **b)** $6n + 3 = 21$

9. Write each equation in two other ways.

a) $37 = 4w + 5$ **b)** $6 + 2a = 26$

Apply

10. a) Use equations to model the first three diagrams.

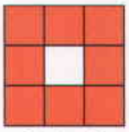
b) Use an equation to model the diagram with 75 model cars.

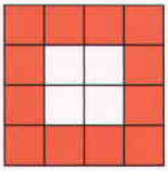
11. A pattern of blocks is shown. A certain number of blocks are needed to have 20 unexposed vertical faces. Write an equation for this problem.

Chapter Problem

12. Vicki makes coffee tables for sale. The visuals below show her first three designs.


Design 1


Design 2


Design 3

a) Develop a variable expression for the number of red tiles in each design. Hint: You can use tiles to model this.

b) Predict how many red tiles will be in Design 5. Explain why you are making this prediction.

c) Make a model of Design 5. If necessary, revise your ideas.

13. If c represents an unknown number of interlocking cubes, write a situation that could be modelled by each equation.

a) $c - 2 = 4$

b) $2c + 1 = 23$

c) $4c - 2 = 14$

Try This!

14. a) Create a pattern that could be modelled by this expression.

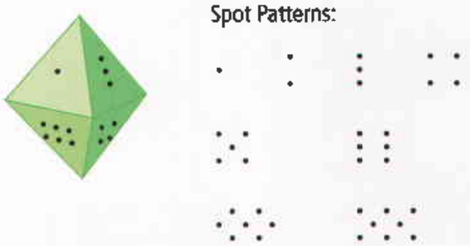
$$8 + 3h$$

b) Write equations for the first 3 diagrams in the pattern.

c) Write an equation for the 7th diagram in the pattern. What does this equation model? Explain.

Extend

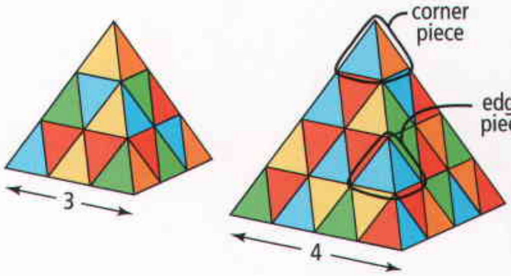
15. These four-sided and eight-sided dice have spot patterns on their faces. The patterns have 1, 2, 3, and 4 spots for the four-sided die, and 1, 2, 3, 4, 5, 6, 7, and 8 spots for the eight-sided die. Let m represent the number of four-sided dice. Let n represent the number of eight-sided dice.



a) Write an equation, using m and n, for a combination of dice with a total of 102 spots.

b) Investigate the possible total numbers of spots, using different combinations of dice.

16. Cosmic Toys makes a line of pyramid puzzles. The building block is a pyramid with equilateral triangles for its base and side faces. Pyramid puzzles with base lengths 3 and 4 look like this.



How many edge and corner pieces would a puzzle with base length n have?

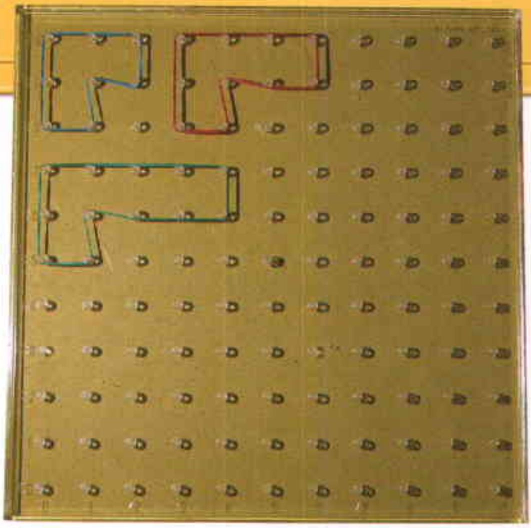
12.4

Solve Equations by Systematic Trial

Focus on…

- modelling with equations
- systematic trial

This geoboard shows a geometric pattern. If the pattern were extended, which item would have a perimeter of 32?

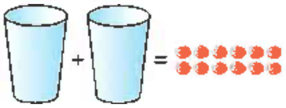


Discover the Math

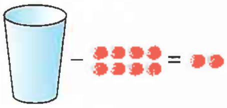
How can you use systematic trial to solve an equation?

1. Each cup contains an unknown number of counters. When more than one cup is used, each contains an equal number of counters. What number of counters in the cup makes each picture true? For each picture, record each guess until you get the correct answer.

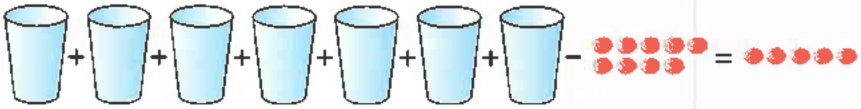
a)



b)



c)



2. Describe your method of finding the numbers of counters in step 1.

What visual can I use to show this?

3. Use your method to answer this question: If you triple the number of counters and add 15 more counters, you have 63 counters. How many counters did you start with?

4. Make up two examples of your own. Share them with a partner. Ask your partner to describe his or her solving strategy.

5. **Reflect** Can you improve your method? If so, describe the improved method. If not, explain why your method is effective as it is.

Example 1: Model and Solve Pattern Problems

a) Write an equation for the diagram with 26 marbles.

b) Solve your equation. State what the solution means.

Diagram 1

Diagram 2

Diagram 3

Solution

a)

Diagram Number	Number of Marbles	Pattern
1	5	$3 \times 1 + 2$
2	8	$3 \times 2 + 2$
3	11	$3 \times 3 + 2$
d	26	$3 \times d + 2$

The equation is $3d + 2 = 26$.

Strategies
Make a table or chart

Use d to stand for the number of the diagram.

b) Use systematic trial.

Try $d = 5$.

$3 \times 5 + 2$
$= 15 + 2$
$= 17$

Too small.

Try $d = 10$.

$3 \times 10 + 2$
$= 30 + 2$
$= 32$

Too large.

Try $d = 8$.

$3 \times 8 + 2$
$= 24 + 2$
$= 26$

Correct.

In Diagram 8, there will be 26 marbles.

Example 2: Practise Systematic Trial

Solve each equation.

a) $2x - 7 = 25$

b) $7a + 12 = 54$

Solution

a) Try $x = 20$:

$2 \times 20 - 7$
$= 40 - 7$
$= 33$
Too large.

I need a number that is more than 25 when I double it. I'll try 20.

Try $x = 15$:

$2 \times 15 - 7$
$= 30 - 7$
$= 23$
Slightly too small.

Try $x = 16$:

$2 \times 16 - 7$
$= 32 - 7$
$= 25$
Correct.

The solution is $x = 16$.

b) Try $a = 5$:

$7 \times 5 + 12$
$= 35 + 12$
$= 47$
Too small.

Try $a = 7$:

$7 \times 7 + 12$
$= 49 + 12$
$= 61$
Too large.

Try $a = 6$:

$7 \times 6 + 12$
$= 42 + 12$
$= 54$
Correct.

5 is too small and 7 is too large. The solution should be $a = 6$.

The solution is $a = 6$.

Example 3: Model and Solve Number Pattern Problems

a) Develop an expression to model the number pattern 2, 7, 12, 17, ...

b) Which step is the number 52?

Solution

a) $1 \times 5 - 3 = 2$
$2 \times 5 - 3 = 7$
$3 \times 5 - 3 = 12$
$4 \times 5 - 3 = 17$
The expression $n \times 5 - 3$ or $5n - 3$ describes the number pattern.

The steps in the pattern repeatedly jump by 5. Show repeatedly adding 5 as multiplying the step number by 5.

b) The equation $5n - 3 = 52$ models the step in the pattern that equals 52.

Try $n = 10$.

$5 \times 10 - 3$
$= 50 - 3$
$= 47$
Slightly too small.

Try $n = 11$.

$5 \times 11 - 3$
$= 55 - 3$
$= 52$
Correct.

Strategies
Use systematic trial

The 11th step in the pattern is 52.

Key Ideas

- To solve equations by systematic trial, substitute values for the variable until you get the correct answer.
- Use a reasonable guess for the first value you substitute. Think about the relationship between the numbers in the equation.
- For each value that you substitute, think about the previous values. Were they too large? too small? For example, solve $3n - 8 = 31$.

$$\begin{aligned}&3(12) - 8\\&= 36 - 8\\&= 28\end{aligned}$$

$$\begin{aligned}&3(13) - 8\\&= 39 - 8\\&= 31\end{aligned}$$

Communicate the Ideas

1. A question asks you to solve the equation $4w - 1 = 15$. What does the word "solve" mean?

2. When solving $5k + 15 = 60$, Mario chose 10 for his first value of k. Jenna chose 20. Which number is the better choice, and why?

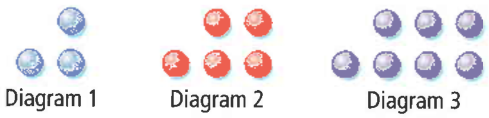
3. Kajan has one more than twice as many candies as Lena.
  a) Model this situation visually or with cups and counters.
  b) Explain whether each equation could model the situation.
    - $2L + 1 = 15$, where L is the number of candies Lena has
    - $2K + 1 = 15$, where K is the number of candies Kajan has
  c) In each equation in part b), what does the number 15 mean?

Check Your Understanding

Practise

For help with questions 4 to 7, refer to Example 1.

4. Study the pattern of marbles.



a) Copy and complete this table.

Diagram Number	Number of Dots	Pattern
1	1 on the left, 2 on the right = 3	$1 + 1 \times 2$
2	1 on the left, 4 on the right = 5	$1 + 2 \times 2$
3		
d		

b) Write an equation for the diagram with 15 dots.

c) Solve your equation. State what the solution means.

5. Study the pattern of dots.

a) Copy and complete this table.

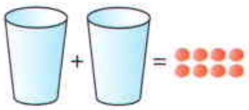
Diagram Number	Number of Dots	Pattern
1	2 on the left, 1 in the middle, 2 on the right = 5	$2 + 1 \times 1 + 2$
2	2 on the left, 2 in the middle, 2 on the right = 6	$2 + 2 \times 1 + 2$
3		
d		

b) Write an equation for the diagram with 11 dots.

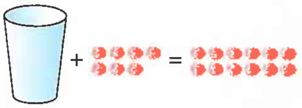
c) Solve your equation. State what the solution means.

6. What number makes each situation true? Solve by inspection or by systematic trial.

a)

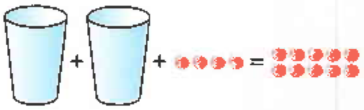


b)

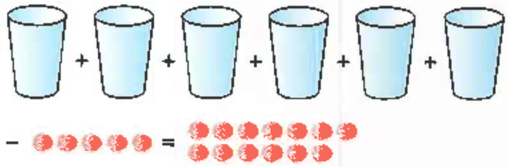


7. Solve by inspection or by systematic trial.

a)



b)



For help with questions 8 and 9, refer to Example 2.

8. Solve each equation by systematic trial.

a) $2x + 3 = 17$ **b)** $3q + 2 = 20$

c) $5 + 7z = 19$ **d)** $20 = 6 + 2a$

9. Solve each equation by systematic trial.

a) $2m + 12 = 56$ **b)** $8w + 5 = 37$

c) $34 = 9 + 5p$ **d)** $5n - 2 = 33$

For help with questions 10 to 12, refer to Example 3.

10. a) Develop an expression to model the number pattern 7, 10, 13, 16, ….

b) Which step is the number 34?

11. a) Develop an expression to model the number pattern 10, 17, 24, 31, ….

b) Which step is the number 45?

12. a) Develop an expression to model the number pattern 19, 18, 17, 16, ….

b) Which step is the number 7?

Apply

13. Solve by systematic trial.

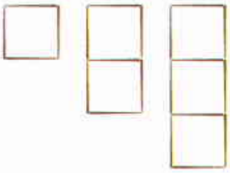
a) $7 + 3c = 91$ **b)** $123 - a = 97$
c) $10x - 6 = 84$ **d)** $48 = 9c - 6$

14. Solve each equation.

a) $30 - 2m = 18$ **b)** $64 - 3k = 34$
c) $100 - 5c = 35$ **d)** $89 = 45 + 2b$

15. a) Five times a number plus 13 gives 48. Model this with an equation.
b) What is the number?
c) Explain your solution method.

16. a) Develop an expression to model the shrinking number pattern 49, 45, 41, 37, ... Hint: What is the jump in this pattern? What number, minus the jump, gives the first step of 49?
b) Which step is the number 13?



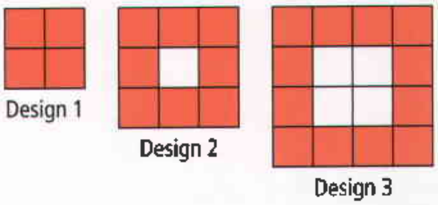
17. Study the toothpick pattern. Use equations to determine the number of squares that give
a) a total of 52 toothpicks
b) a perimeter of 48 toothpicks

18. a) It cost Jenna \$50 to set up her business selling smiley pins. She charges \$2 a pin. In her first week, Jenna made a profit of \$80. The equation modelling her profit is $2n - 50 = 80$, where n is the number of pins sold. How many pins did Jenna sell?
b) Explain the strategy you used. How else might you have solved the problem?

19. a) Write an equation, containing multiplication and addition, that has 8 as a solution.
b) Write an equation, containing division and subtraction, that has 8 as a solution.

Chapter Problem

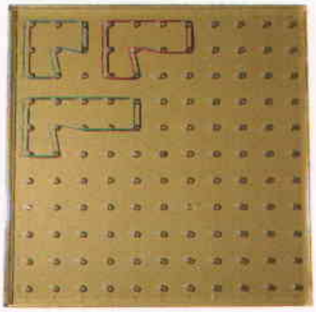
20. The diagrams show the first three of Vicki's coffee table designs.



a) Develop a variable expression for the number of red tiles in each design.
b) Which design will need 24 red tiles? Show and explain your solution.

Try This!

21. Suppose this geoboard pattern were extended. Write and solve an equation for the item in the pattern with a perimeter of 32.



Extend

22. Solve each equation.

a) $1.4y + 15 = 29$
b) $150 = 2.5w + 50$

23. For a new style of rod, Cosmic Toys decides to put a smiley face sticker on every exposed square face.

The number of exposed smiley stickers in a rod of length n can be calculated using an equation. Is there a rod with 49 exposed smiley faces? 106 exposed smiley faces? Justify your answers.

12.5

Model With Equations

Focus on...

- modelling real-world problems
- applying equation skills

Banks and stores need quick ways of counting their coins. Rolls of $2 coins contain 25 coins, or $50. What would be a quick way of counting these coins? How could you model their value using an equation?

Discover the Math

How can you model and solve a problem using an equation?

1. Barbara is counting the money in her piggy bank. She has a $10 bill and a number of $2 coins.

a) Barbara modelled the number of $2 coins with the expression $2T$. What does T represent?

b) Explain why the value of Barbara's $2 coins can be modelled with the expression $2T$.

2. Explain why the total value of Barbara's coins can be modelled with the expression $2T + 10$.

3. Barbara has a total of $18.

a) Write an equation modelling this total.

b) Describe the equation in words.

c) Solve the equation. Explain what the solution means.

4. **Reflect** How would you change the equation if Barbara had a total of $30? Explain.

Example 1: Systematic Trial in Formulas

The perimeter of a rectangle is 32 cm. The length is 9 cm. What is the width of the rectangle?

Solution

Method 1: Work With the Formula

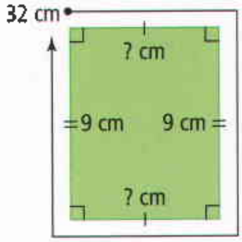
$P = 2l + 2w$

$32 = 2(9) + 2w$

$32 = 18 + 2w$

$2w$ is unknown

So, $32 = 18 + 2w$. Solve for the width.



Strategies
Make a picture or diagram

Try $w = 8$:

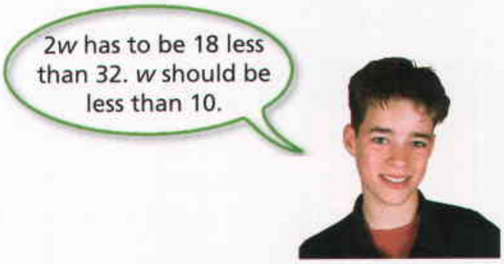
$18 + 2 \times 8$
$= 18 + 16$
$= 34$

The right side is 34. 8 is slightly too high.

Try $w = 7$:

$18 + 2 \times 7$
$= 18 + 14$
$= 32$

7 is correct.



The width of the rectangle is 7 cm.

Method 2: Work Backward

Two of the sides are 9 cm. So, the leftover perimeter is

$32 - 2 \times 9$
$= 32 - 18$
$= 14$

Strategies
Work backward

The two widths total 14 cm. So,

$2w = 14$
$w = 7$

The width of the rectangle is 7 cm.

Strategies
What other strategy might you use?

Example 2: Model With an Equation

At a garage sale, paperback books are priced at \$2 each.
One customer buys \$12 worth of books.

a) Solve this problem using a method of your choice.
b) Use an equation to solve this problem.
c) Compare your solutions from a) and b). What conclusion can you make?

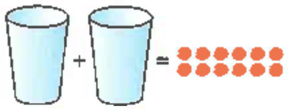
Solution

a) $2 \times 6 = 12$

The solution is $b = 6$. This means that the customer bought 6 books for \$12.

b) Let b represent the number of books.

The equation is $\underbrace{2}_{\text{price of books}}\underbrace{b}_{\text{number of books}} = \underbrace{12}_{\text{total cost}}$.

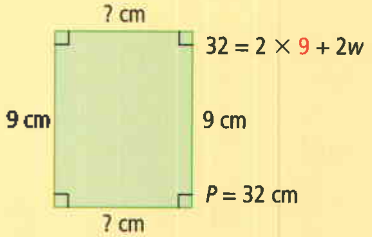


I used cups and counters to model the situation.
2 cups = 12 red counters

c) There are different ways to solve this problem. The three solutions here all work.

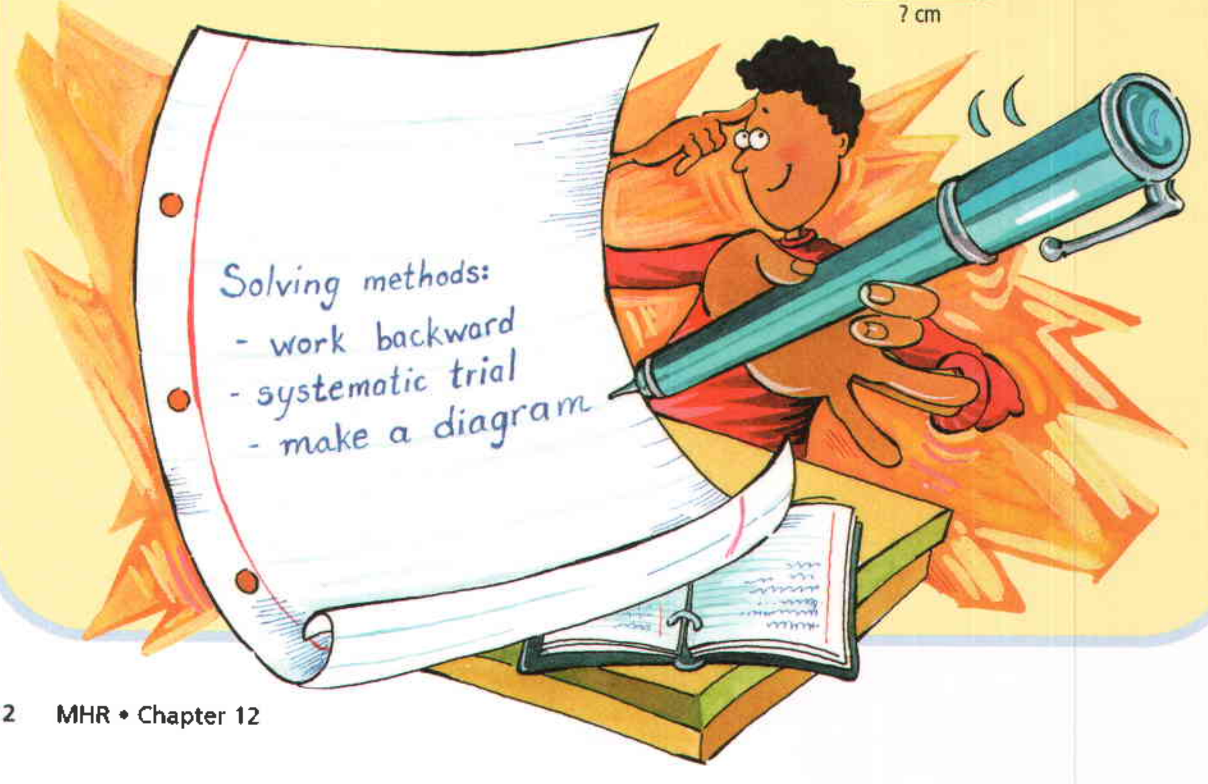
Key Ideas

- Situations can be modelled with an equation by translating the situation into numbers and variables. Use operations to show what happens to the numbers.
- Solve the equation using an appropriate method.



$32 = 2 \times 9 + 2w$

$P = 32$ cm



Communicate the Ideas

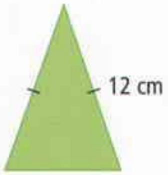
1. When Jane's age is doubled and 3 is added, the result is 17. Which of the following equations are appropriate models? For each equation, explain your answer.

a) $2a + 3 = 17$

b) $17 = 3 + 2a$

c) $3a + 2 = 17$

2. The perimeter of an isosceles triangle is 30 cm. The two equal sides are each 12 cm long. What clues in this statement help you develop an equation?



3. As a math puzzle, Ted wrote the expressions $3f - 6 = 21$ and $2f = 18$ to describe the number of people in his family. Is it possible to have two equations modelling the same situation? Explain.

Check Your Understanding

Practise

4. Write each sentence as an equation. Then, solve.

a) Four less than a number is 3.

b) The sum of a number and 5 is 12.

c) Ten more than the product of a number and 3 is 31.

d) Double a number, decreased by 10, is 15.

5. Write each sentence as an equation. Then, solve.

a) A cost shared by 4 people amounts to $10 each.

b) There are 42 oranges. This is 14 more than double the number of apples.

c) The number of students increased by 15 to 32.

d) 70 cm is 10 cm less than half of Bill's height.

For help with questions 6 to 9, refer to Example 1.

6. The perimeter of a rectangle is 39 cm. Its width is 5 cm.

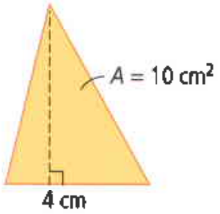
a) Model this situation with an equation.

b) Solve the equation to find the length of the rectangle.

7. The perimeter of a rectangle is 42 cm. The width is 4 cm. What is the length of the rectangle?

8. The perimeter of an isosceles triangle is 20 cm. The two equal sides each measure 6 cm. How long is the third side?

9. The formula for the area of a triangle is $A = b \times h \div 2$. Find h for a triangle with base 4 cm and area 10 cm^2.



For help with questions 10 and 11, refer to Example 2.

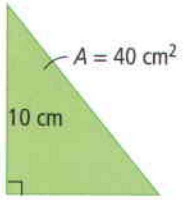
10. The variable d represents an unknown number of DVDs. Write a situation that could be modelled by the equation $20d = 60$.

11. The variable b represents an unknown number of books. Write a situation that could be modelled by each equation.

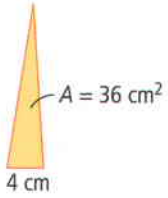
a) $b + 5 = 7$
b) $7b = 28$
c) $b + 14 = 19$

Apply

12. a) Find b for a triangle with area 40 cm^2 and height 10 cm.



b) Find h for a triangle with area 36 cm^2 and base 4 cm.



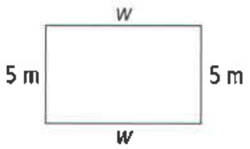
13. Khalid is 1 year older than 3 times his son's age. Khalid is 31 years old. Explain why both $31 = 3s + 1$ and $1 + 3s = 31$ can be used to model this situation.

14. Describe a situation involving money that can be modelled by the equation $20T + 12 = 72$.

15. The Student Council is planning a year-end party. It costs $300 for the D.J. for the evening. They plan to spend $5 per student on food.

a) What is the total cost for s students?
b) The school budgeted $1000 for the party. How many students can attend?
c) How did you solve this problem? What other strategy might you use?

16.

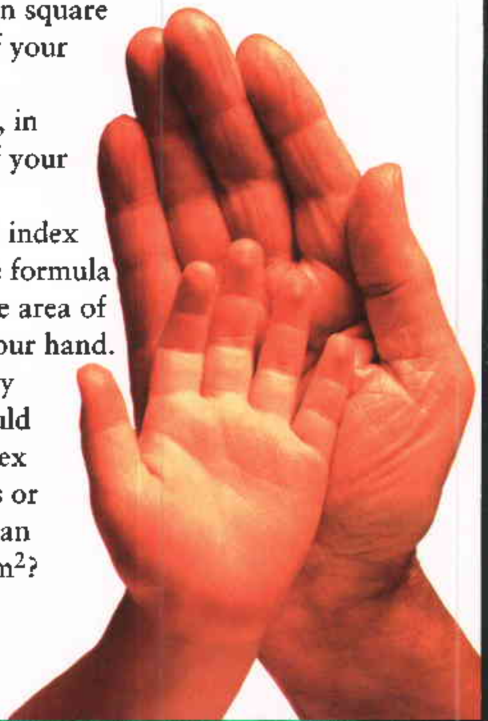


a) The rectangular room shown has a perimeter of 26 m. Write an equation to model the perimeter.
b) The room has area 40 m^2. Write an equation to model the area.

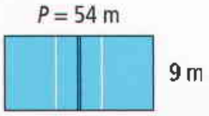
17. An experiment found that you can predict the approximate area of the palm of your hand using the formula $A = 20L - 67$. In this equation,

- A is the area, in square centimetres, of your palm
- L is the length, in centimetres, of your index finger

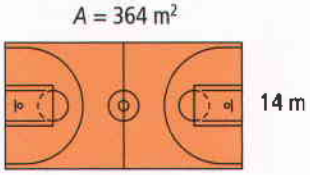
a) Measure your index finger. Use the formula to estimate the area of the palm of your hand.
b) Approximately how long would a person's index finger be if his or her hand had an area of 153 cm^2?



18. A volleyball court is 9 m wide and has a perimeter of 54 m. What is its length?



19. a) A basketball court is 14 m wide with an area of 364 m^2. What is its perimeter?



b) How did you solve this problem? What other strategy might you have used? Which strategy do you prefer, and why?

20. In a science experiment, various masses were suspended from a spring. The stretched length of the spring is recorded in the table.

Mass (g)	5	10	15	20
Stretch Distance (cm)	12	22	32	42

What mass would stretch the spring to a length of 122 cm?

Try This!

21. Sandi is delivering flyers. She earns \$5 plus \$3 per bundle of 100 flyers.

a) How much is Sandi paid to deliver 1 bundle of flyers? 2 bundles of flyers? x bundles of flyers?

b) Sandi earned \$29. How many bundles of flyers did she deliver?

Extend

22. The perimeter of a rectangular playing field is 300 m. The length is double the width.

a) Model this situation with an equation. Use one variable only.

b) How could you simplify the equation?

23. The formula $T = 33 - 0.75L$ estimates the mean January temperature, in degrees Celsius, for a city with a latitude of L degrees. At what latitude would Moscow, Russia, be with a mean January temperature of -9°C?

Making Connections

What do variable expressions have to do with the volume formula?

The formula for the volume of any prism is based on patterns.

The patterning equation for these prisms can be written as
Total number of cubes = number of cubes in the base × the diagram number

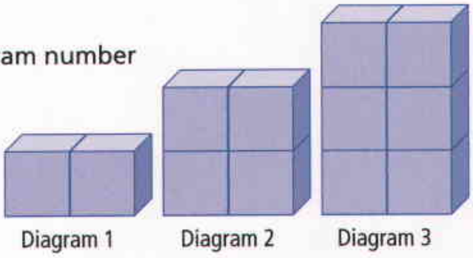
The volume formula is

Volume = area of the base × height

This is the total number of cubes.

This is the number of cubes in the base.

This is the diagram number.



The two equations are the same.

CHAPTER 12

Review

Key Words

Use these words to copy and complete the statements in questions 1 and 2.

variable *variable expression*
solution *equation*

1. In the ■■◉■◉■◉■ $2n + 3 = 5$, the letter n is the ■■■◉■■■■.

2. A ■■■■■■■■ ■■■■■◉■■◉◉ uses numbers, ■■■■■■◉■s, and operations.

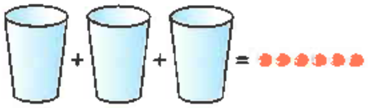
3. Use the circled letters from questions 1 and 2 to finish this sentence: The value that makes an equation true is the ■■■■■■■■.

12.1 Variables and Expressions, pages 386–391

4. Model each phrase with cups and counters.

a) a number, less 3
b) 3 times a number

5. a) Describe, in words, the variable expression that is modelled by this diagram.



b) Write the variable expression.

6. A car is travelling at an average speed of 60 km/h. The expression $60t$ tells you the distance driven, in kilometres. The variable t stands for the time, in hours.

a) How far would this car travel in 3 h?
b) How far would this car travel in 7 h?

12.2 Solve Equations by Inspection, pages 392–397

7. What number on each mass will keep the balance?

a)

b)

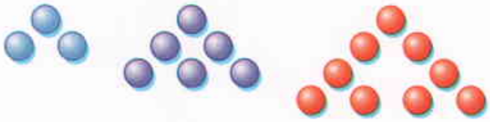
8. Solve each equation by inspection.

a) $c - 12 = 25$
b) $9x = 81$
c) $n - 4 = 10$

9. Ken's parents asked him his average mark on his report card. Ken said that the sum of his 8 marks was 560. What was Ken's average mark?

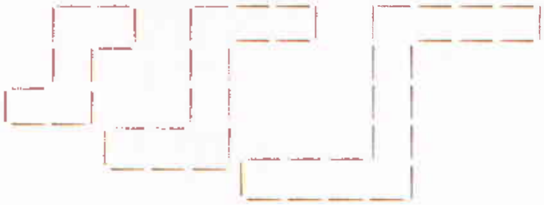
12.3 Model Patterns With Equations, pages 398–403

10. a) Write equations for the first three shapes in this pattern of marbles.



b) Write an equation for the shape that uses 27 marbles.

11. Study this toothpick pattern. Write an equation for the diagram that uses 46 toothpicks.



12.4 Solve Equations by Systematic Trial, pages 404–409

12. Model each situation using an equation.

a) A number increased by 9 is 15.

b) A number, when doubled, gives 24.

c) The product of 4 and a number, minus 5, is 27.

d) Two more than three times a number is 8.

13. Solve each equation in question 12.

14. Solve each equation by systematic trial.

a) $7y - 8 = 55$

b) $70 - 4n = 22$

c) $75 = 40 + 5w$

d) $4y - 6 = 58$

e) $11x + 15 = 202$

f) $25 = 0.5q + 18$

15. Study the pattern of toothpick triangles.

a) Develop an expression for the number of toothpicks needed for a diagram that has n triangles.

b) Write an equation modelling this pattern when 51 toothpicks are used.

c) Solve the equation. How many triangles will be formed with 51 toothpicks?

12.5 Model With Equations, pages 410–415

16. Model each situation using an equation. Solve each equation.

a) Double a length, decreased by 4, is 13.

b) The cost of a computer is 6 times the cost of a printer. The computer costs $1200.

c) Chan's mass is 30 kg less than double Juan's mass. Juan's mass is 45 kg.

d) The regular price of a hardcover novel has been lowered by $6 to $35.

e) Double a number, less 7, is 23.

f) The difference between 180 cm and a person's height is 30 cm.

17. Saturn has 31 moons. Jupiter has 1 fewer than double the number of moons that Saturn has.

a) Model this situation with an equation.

b) How many moons does Jupiter have?

Did You Know?

In 1979, the Voyager 1 space probe discovered rings around Jupiter. Unlike Saturn's rings, they are too faint to be seen by a telescope.

CHAPTER

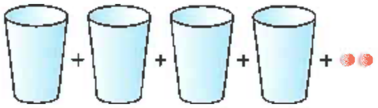
12 Practice Test

Strand	NSN	MEA	GSS	PA	DMP
Questions	2, 9, 11–13			1–13	

Multiple Choice

For questions 1 to 5, select the correct answer.

1. Which expression does the illustration model?



A $4x + 2$ **B** $2x + 4$
C $4x - 2$ **D** $4 + 2x$

2. The solution to the equation $k - 12 = 15$ is

A 3 **B** 12
C 27 **D** none of the above

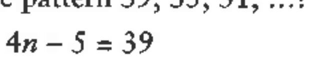
3. "Double a number, increased by 5, is 17" can be modelled as

A $n + 2 \times 5 = 17$ **B** $2n + 5 = 17$
C $17 = 2n - 5$ **D** $5n + 17 = 2$

4. Which dot pattern matches the equation $3n + 1 = 22$?

A


B


C


D


5. Which equation could model a number in the pattern 39, 35, 31, …?

A $4n - 5 = 39$
B $39 - 4n = 17$
C $43 - 4n = 11$
D $42 - 3n = 12$

Short Answer

6. Model each expression using diagrams of cups and counters.

a) $2C + 6$
b) $3C - 2$
c) $7 - C$

7. Model each situation using an equation.

a) Your mother's job takes the same number of hours each day for 5 days. She works a total of 35 h.
b) Your father's job takes the same number of hours each day for 3 days. He works a total of 42 h.
c) \$2 more than 4 times your allowance is \$38.
d) Triple your allowance increased by \$10 gives \$55.

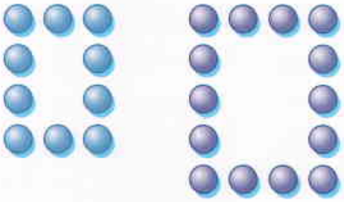
8. Evaluate.

a) $n + 5$ for $n = 7$
b) $3x$ for $x = 10$
c) $8k - 3$ for $k = 2$
d) $12 - b \div 3$ for $b = 30$

9. Solve each equation. Use a method of your choice.

a) $m + 5 = 17$
b) $6k = 18$
c) $4w = 92$
d) $2r - 5 = 19$
e) $15x + 12 = 177$

10. Study this pattern of marbles. Write an equation for the shape that uses 27 marbles.



11. A Mats Sundin collector card sells for \$10 more than a Vince Carter card. A Mats Sundin card sells for \$75.

a) Write an equation modelling this situation. Explain how you developed the equation.

b) Solve the equation to find the selling price of a Vince Carter collector card.

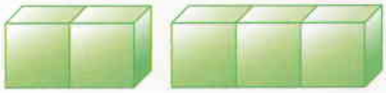
12. A roll of \$2 coins contains 25 coins, or \$50. Write and solve an equation to model each situation below. Explain what your variable represents for each situation.

a) The total number of toonies is 175.

b) The total value of the toonies is \$350.

Extended Response

13. Acme Toy Company makes rods from cubes. The cubes are joined end to end as shown and then dipped in paint.



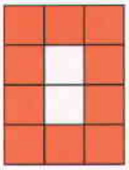
Develop and solve an equation to find the total number of cubes used when 86 faces are painted.

Chapter Problem Wrap-Up

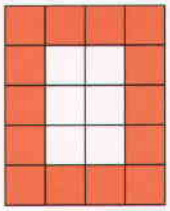
Vicki is planning a set of rectangular tables. Here are her first two designs.

1. Write a variable expression to model the number of red tiles in this pattern. Explain your method.

2. Design your own table pattern using red and white tiles. Use pictures, words, numbers, and equations to show and discuss your pattern.



Design 1



Design 2

Making Connections

Equation Puzzle

Copy this three-by-three grid onto a piece of paper or thin cardboard. Make sure each section is square. Then, cut out the nine squares.

Rearrange the squares so that each equation is next to its solution. There will be extra equations and numbers around the outside edge that do not match up.

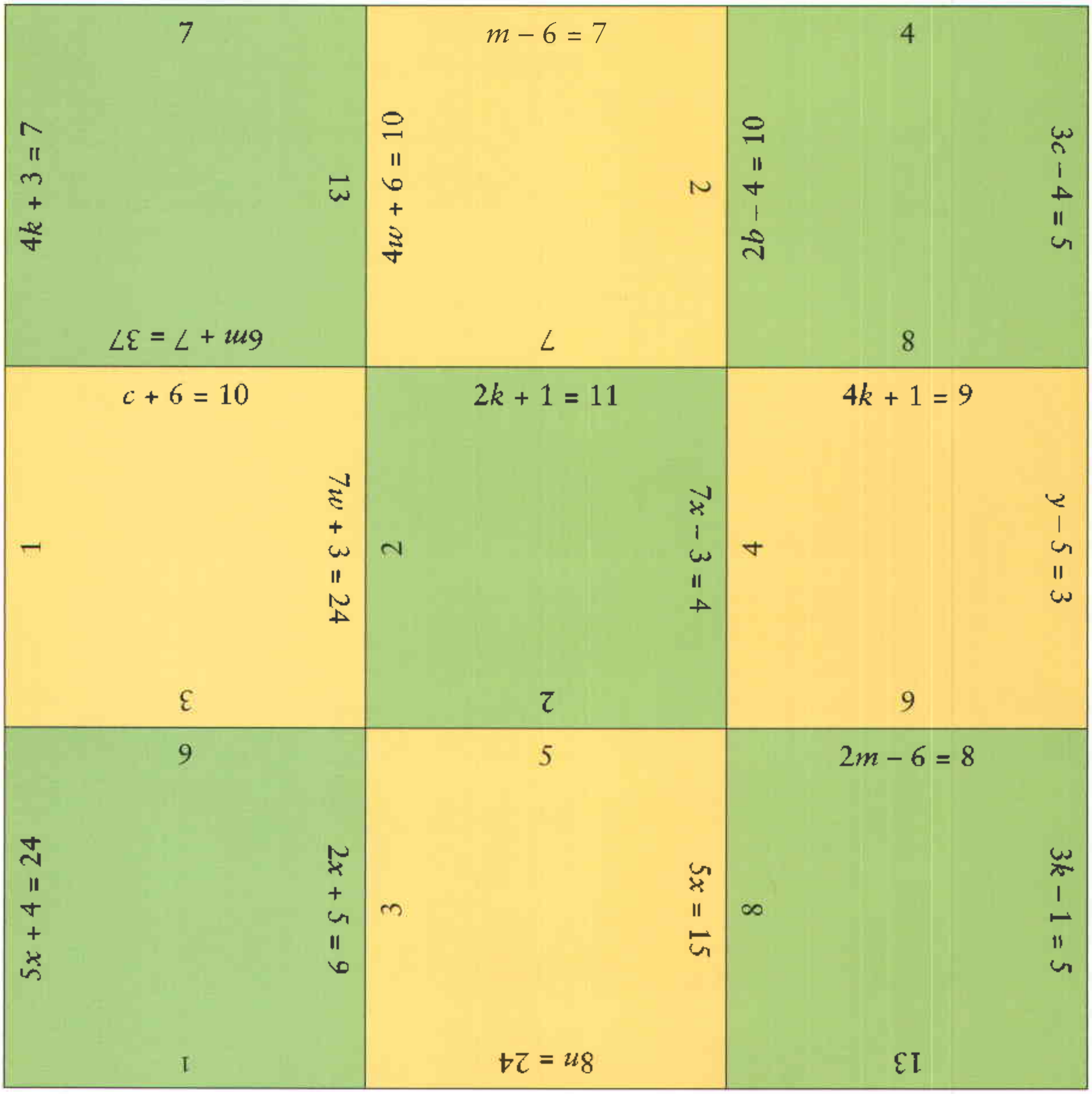
Hint: The piece from the top left corner slides, with no turn, to become the centre piece.

Materials

- scissors

Optional:

- BLM 11/12 Task A Equation Puzzle



Magic Squares

1. Neeta and Sam found this card in the class math centre. They were not sure how to solve it. Help them find the missing numbers.

Magic Square

2		
	5	1
		8

What numbers belong in the empty squares? Remember that the rows, columns, and diagonals of a magic square all add to the same total.

2. Neeta made up this magic square. Sam says that it is not a magic square. Who is correct? Explain why.

−3	+2	+1
+4	0	−4
−1	−2	+3

3. Sam found another card with these two magic squares. He thinks they must be related. Solve each magic square and explain how they are related.

		$5 + 1$
$5 + 4$	5	
$5 - 1$		

	$x + 2$	
$x + 4$	x	
	$x - 2$	

4. Create at least one original magic square. Use some positive and some negative integers. Try to develop another using a variable.

Chapters 9–12 Review

Chapter 9 Data Management: Collection and Display

1. Ariel counted the number of coloured candies in a package.

Colour	Tally	Frequency
Red	|	
Green	|||	
Blue	||||	
Yellow	||||	
Orange	|||	

a) Copy and complete the frequency table.
b) Draw a bar graph to show the data.
c) Draw a pictograph to show the data.
d) Identify one advantage of each type of graph.

2. a) Organize the following test scores using a stem-and-leaf plot.

82	77	54	66	82	69	71	90
84	62	70	55	74	83	94	96

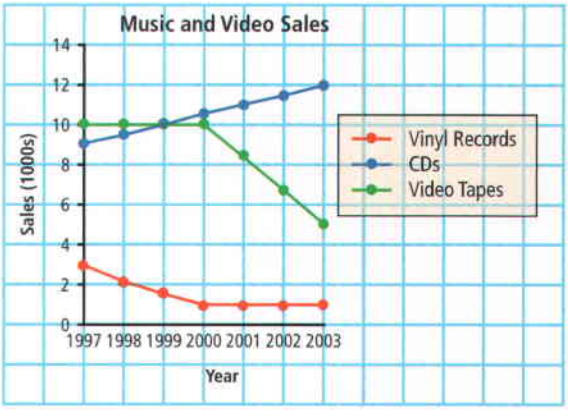
b) How many scores are in the 70s?
c) Which was the most common score?

3. Four friends worked together on a presentation. Kelly worked 4 h, Hai worked 3 h, Julio worked 3 h, and Annapola worked 2 h. Draw a circle graph that shows how the work was divided.

4. a) What is a database? Give an example.
b) What is a spreadsheet? Describe a situation in which you would use one.

Chapter 10 Data Management: Analysis and Evaluation

5. a) Describe each sales trend.
b) What product had the greatest sales in 2000?
c) Predict CD sales in 2005. Explain your prediction.

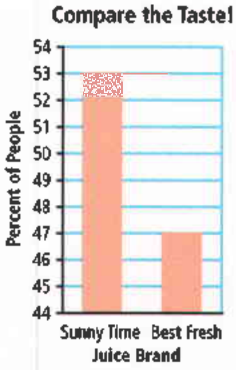


6. Shelley records the number of friends that attend her pool party each year: 45, 36, 13, 40, and 36.

a) Find the mean, median, and mode.
b) Which measure of central tendency best describes this set of data? Explain.

7. The results of a taste test are used in an advertisement.

a) Which company do you think created this graph? Explain.
b) Explain why this graph is misleading.
c) Draw a new graph for the data that is not misleading.
d) Compare the two graphs. Describe what you notice.



Chapter 11 Integers

8. Temperatures can be converted between degrees Celsius (°C) and degrees Fahrenheit (°F) using the table.

Degrees Celsius	Degrees Fahrenheit
−20	−4
−15	5
−10	14
−5	23
0	32
5	41
10	50
15	59
20	68

a) Describe the pattern using words and integers.

b) Convert −25°C to degrees Fahrenheit. Explain your strategy.

c) Convert −40°F to degrees Celsius. Explain your strategy.

9. Model each statement using integer chips or a number line. State the result of each.

a) (−5) + (−3) **b)** (+4) + (−9)
c) (−8) − (−3) **d)** (+7) − (+13)

10. Evaluate.

a) (+4) + (−10) **b)** (−12) − (−5)
c) (−8) + (+17) **d)** (+22) − (−21)

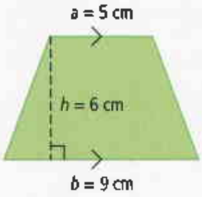
11. Use a calculator to help evaluate each expression.

a) 30 − 45 − 15 **b)** 74 − (−16) − 50
c) 33 + 44 − 87
d) (−100) − 200 + 300
e) (−250) + (−350) − (−450)

12. A scuba diver is at an elevation of 6 m above sea level. She dives to a depth of 8 m below the surface and then a further 7 m. How far below sea level did the diver end up?

Chapter 12 Patterning and Equations

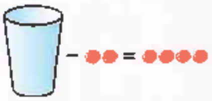
13. The formula for the area of a trapezoid is $A = \frac{1}{2}(a + b)h$. Use the formula to find the area of the trapezoid shown.



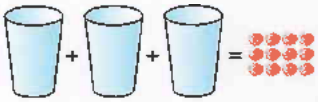
14. For what whole number values of *k* is 2*k* + 6 greater than 3*k*?

15. Find the number that makes each situation true.

a)



b)

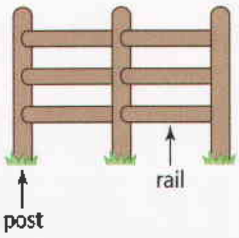


16. Solve.

a) $2a = 10$
b) $m - 5 = 9$
c) $y + 6 = 30$
d) $3x + 1 = 16$
e) $11w - 13 = 64$

17. The formula for the perimeter of an equilateral triangle is $P = 3s$. What side length is needed to make an equilateral triangle with a perimeter of 36 cm?

18. The diagram shows the posts and rails of a fence. How many posts are needed for a fence with 72 rails?



Geometry and Spatial Sense

- Explore transformations of geometric figures.
- Understand, apply, and analyse key concepts in transformational geometry.
- Recognize the image of a two-dimensional shape under a translation, a reflection, and a rotation.
- Create, analyse, and describe designs that include congruent, translated, rotated, and reflected two-dimensional images.
- Identify whether a figure will tile a plane.
- Construct and analyse tiling patterns with congruent tiles.

Key Words

frieze pattern
transformation
translation
rotation
reflection
image
tiling pattern
tessellation
tiling the plane

CHAPTER 13

Geometry of Transformations

A **tessellation** is another name for a mosaic. A mosaic is a picture or design made of small shapes of different colours. Ancient Romans made the world's most famous mosaics from small tiles. The mosaics covered and decorated surfaces, such as floors and walls. The tiles in a Roman mosaic were quadrilaterals, most often squares.

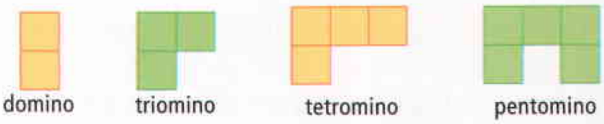
A tile used to make a mosaic is called a "tessera." This word comes from the ancient Greek word *tessares*, which means four. The tiles used to make ancient mosaics had four corners.

Think of some examples of tessellations or mosaics that you have seen in your home, school, or community. What shape are the tiles that are used?

In this chapter, you will explore transformations. You will use transformations to create, analyse, and describe designs.

Chapter Problem

Square tiles can be combined to make different shapes.

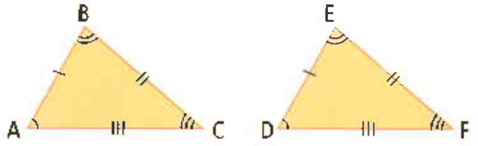


What other shapes can be made from two or more square tiles? How can you create tessellations using shapes made up of 2, 3, 4, or 5 squares?

Get Ready

Congruent Figures

The equal angles and equal sides in these **congruent figures** are called corresponding parts.



The corresponding parts are

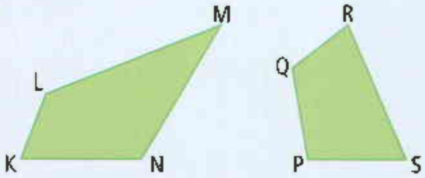
$\angle A = \angle D$ $AB = DE$

$\angle B = \angle E$ $BC = EF$

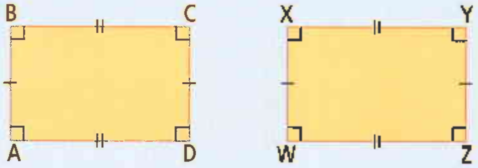
$\angle C = \angle F$ $AC = DF$

1. Are the figures in each pair congruent? Explain your reasoning.

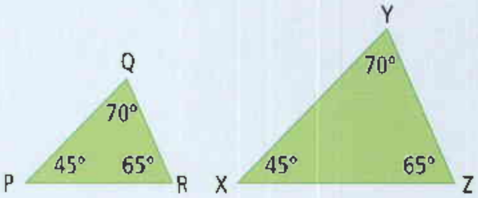
a)



b)



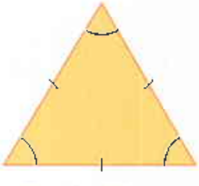
c)



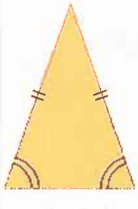
2. For any congruent figures you found in question 1, list the corresponding parts.

Regular and Irregular Polygons

A polygon with all sides equal and all angles equal is a **regular polygon**. An equilateral triangle is an example of a regular polygon. A polygon that is not regular is called an **irregular polygon**. An isosceles triangle is an example of an irregular polygon.



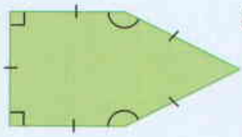
Regular Polygon



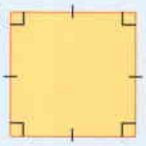
Irregular Polygon

3. Decide if each polygon is regular or irregular. Give reasons for your decisions.

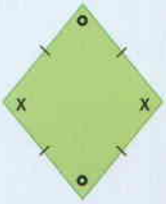
a)



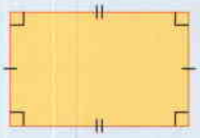
b)



c)

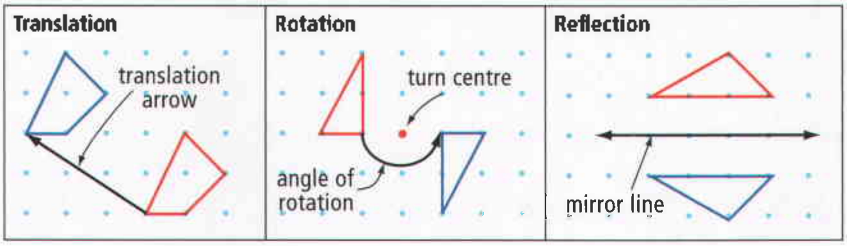


d)



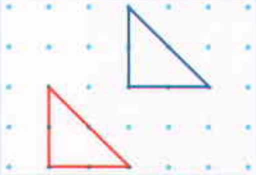
Transformations

A **transformation** moves one geometric figure onto another. Three common types of transformations are **translations**, **rotations**, and **reflections**.



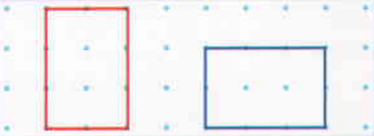
4. Name a transformation that moves the red figure onto the blue figure in each pair. Explain your reasoning.

a)



b)

c)



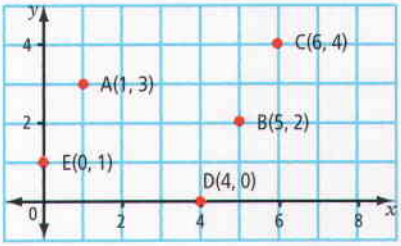
5. In each part of question 4, are the red and blue figures congruent? Explain.

Graphing Skills

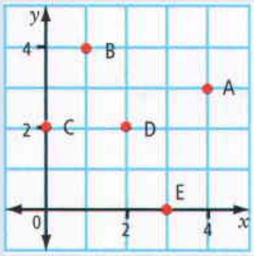
The points A(1, 3), B(5, 2), C(6, 4), D(4, 0), and E(0, 1) can be plotted on a **coordinate grid** as shown.

Each point is named with an **ordered pair**.

A(1, 3)
↑ ↑
x-coordinate **y-coordinate**



6. State an ordered pair for each point on the grid.



7. Plot these points on a coordinate grid. Join the points in alphabetical order. What letter shape is formed?

a) A(2, 6) **b)** B(3, 2) **c)** C(4, 5)
d) D(5, 2) **e)** E(6, 6)

13.1

Explore Transformations

Focus on...

- exploring transformations
- creating and analysing designs

frieze pattern

- a design pattern that repeats in one direction
- the word "frieze" sounds the same as the word "freeze"

A strip pattern or **frieze pattern** repeats in one direction. Frieze patterns are often used as decoration in arts, crafts, and architecture. You may have seen frieze patterns on textiles, pottery, and buildings. Describe the frieze pattern you see in this section of stone architecture.

Discover the Math

Materials

- grid paper

How can you use frieze patterns to explore transformations?

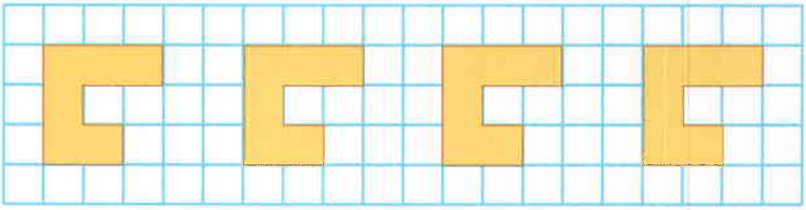
You can create a frieze pattern by transforming a design. Here is an example of a design.

transformation

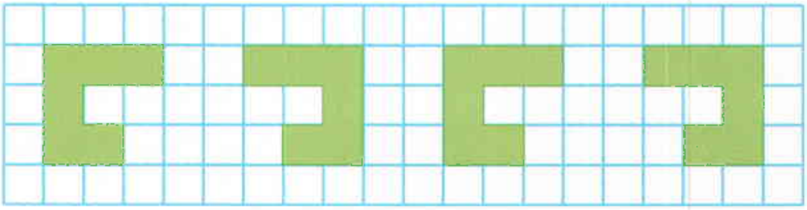
- a change in a figure that results in a different position, orientation, or size

1. The diagrams show ways of using the design to create frieze patterns. Describe how **transformations** are used in each case.

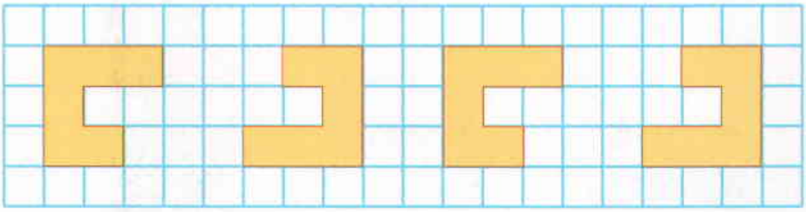
a)



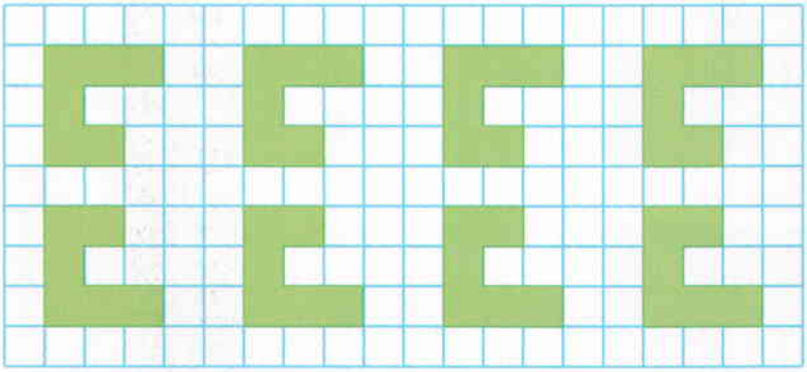
b)



c)



d)



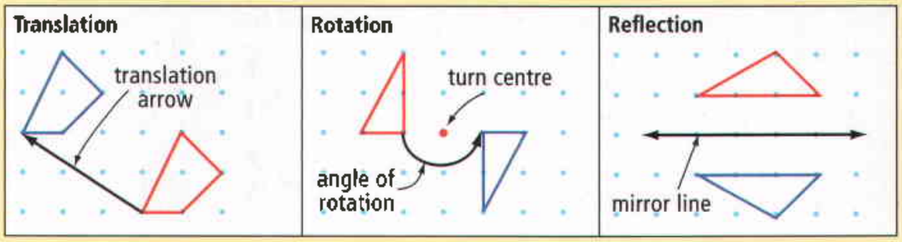
2. Create your own frieze pattern on grid paper. Describe how you created it.

3. Compare your pattern with a classmate's pattern. Describe how you think your classmate's pattern was created.

4. **Reflect** Explain why transformations are often used in art and design.

Key Ideas

- Three common types of transformations are **translations**, **rotations**, and **reflections**.



Translation **Rotation** **Reflection**

- A translation, rotation, or reflection **image** is congruent to the original figure.

translation
- a slide along a straight line

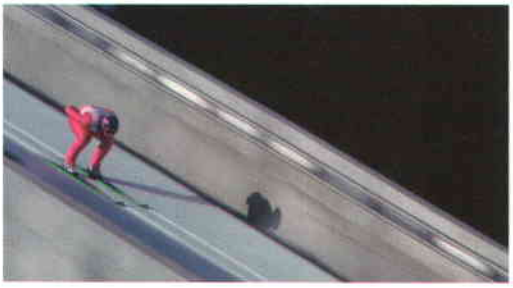
rotation
- a turn about a fixed point called the turn centre

reflection
- a flip over a mirror line

image
- a figure resulting from a transformation

Communicate the Ideas

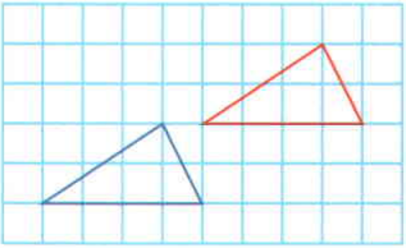
1. Name the transformation in each picture.

a) 

b) 

2. List examples of translations, reflections, and rotations you see in your classroom. Discuss your list with your classmates.

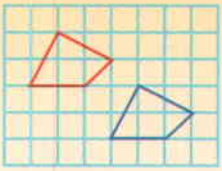
3. The blue figure is the translation image of the red figure. Describe the translation.



Literacy Connections

Describing Translations

Describe the movement left or right first. Then, describe the movement up or down.

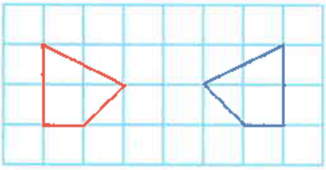


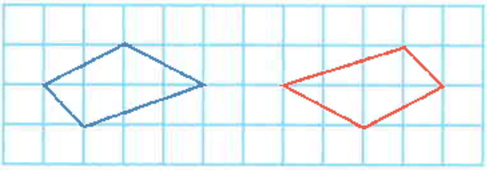
The red figure is translated 3 units right and 2 units down.

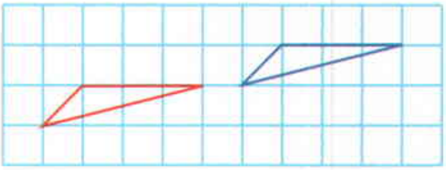
Check Your Understanding

Practise

4. Name the type of transformation that relates each pair of figures.

a) 

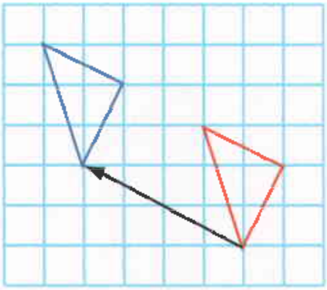
b) 

c) 

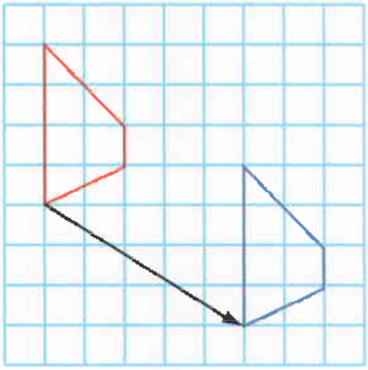
5. Create a frieze pattern by translating an irregular shape.

6. The blue figure is the translation image of the red figure. Describe the translation in words.

a)

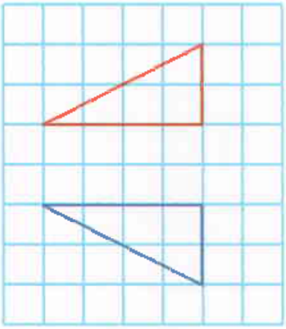


b)

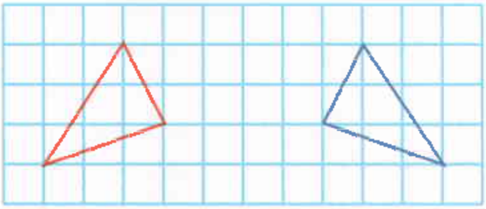


7. The blue figure is the reflection image of the red figure. Copy the diagram. Show the location of the mirror line.

a)

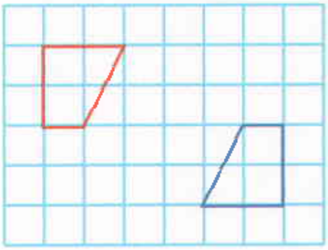


b)

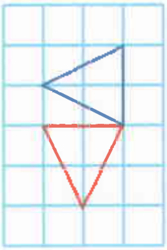


8. The blue figure is the rotation image of the red figure. Copy the diagram. Show the turn centre and the angle of rotation.

a)



b)



Apply

9. Explain why translations, rotations, and reflections can be called "congruency transformations."

10. Frieze patterns are used for decoration in many cultures. To find out more, go to **www.mcgrawhill.ca/links/math7** and follow the links. Choose a frieze pattern you like. Describe how and where it is used.

11. When you use a combination lock, you use rotations and translations. Describe them.

12. How do you use transformations when you ride a bicycle?

13. **a)** Describe a type of window that is opened and closed using translations.

b) Describe a type of window that is opened and closed using rotations.

14. Can you change the shape or size of a figure by translating, rotating, or reflecting it? Use diagrams to show how you know.

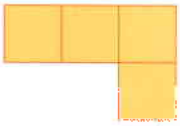
15. A domino is a figure made by joining two congruent squares along whole sides.

a) Where could you place a turn centre to rotate one square onto the other? Explain your reasoning.
b) What would the angle of rotation be?

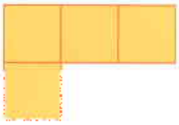
Chapter Problem

16. You can make or draw a tetromino by joining 4 congruent squares along whole edges.

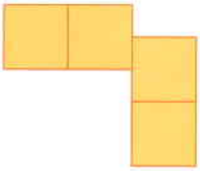
This is a tetromino:



This is the same tetromino:

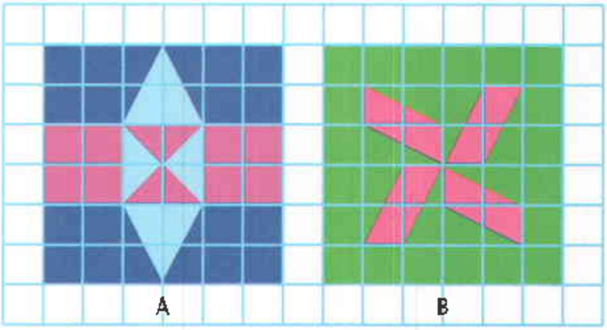


This is not a tetromino:



Two figures related by a transformation count as one tetromino. How many different tetrominoes can you make? Draw all the possible different tetrominoes on grid paper.

17. Some quilts are made from square blocks of material that are stitched together. The diagrams show two possible designs for a square block.

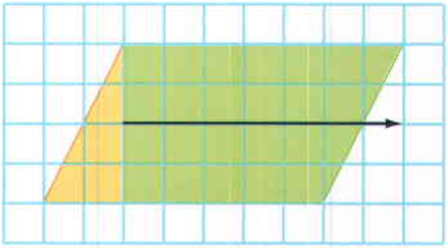


a) Describe the transformations that relate the congruent parts in each design. Is there more than one possible answer in some cases? Explain.
b) Describe reflections or rotations that would move each whole block onto itself.

18. a) Design your own square block for a quilt.
b) Describe the transformations you used to create the block.

Try This!

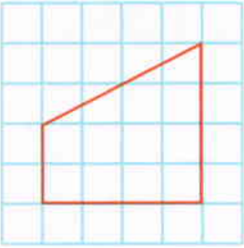
19. The diagram shows how a triangle can be translated across a parallelogram.



a) Name the type of figure that results.
b) How do the areas of the parallelogram and the resulting figure compare? Explain.
c) How do the perimeters of the parallelogram and the resulting figure compare? Explain.
d) Explain why the translation does not result in a figure that is congruent to the parallelogram.

Extend

20. Figures with the same shape but different sizes are known as similar figures.



a) Draw a figure on grid paper that is similar to the red figure by doubling the length of each side.

b) Draw another figure on grid paper that is similar to the same red figure by tripling the length of each side.

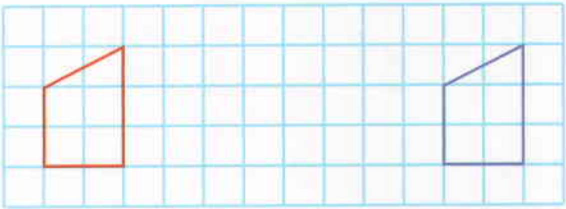
c) The figures you have drawn are known as enlargements of the original figure. What does "enlargement" mean?

d) Draw a figure on grid paper that is similar to the red figure, but with sides that are half as long.

e) Explain why the figure you drew in part e) is called a "reduction" of the red figure.

21. A figure is rotated twice in the same direction about the same turn centre. The angle of rotation is 180° for each rotation. How do the locations of the original figure and the final image compare? Draw a diagram to show why.

22. The blue figure is the translation image of the red figure.



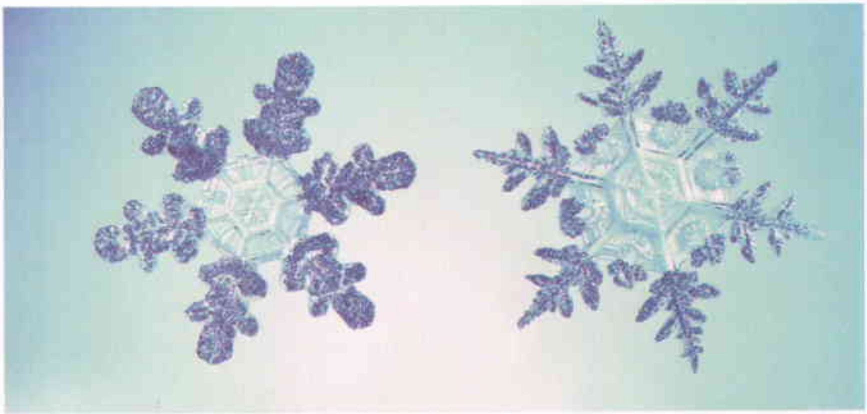
You can also transform the red figure onto the blue figure by using two reflections, one after the other. Copy the diagram onto grid paper. Draw the two mirror lines on grid lines between the two figures. Explain your reasoning.

Making Connections

What does math have to do with snow?

Everyone who lives in Canada knows something about snow. When you walk or ski across a thick blanket of snow, you stand on huge numbers of snowflakes packed together. But have you ever looked at the shape of just one snowflake?

You may have heard that no two snowflakes are exactly alike. This means that no snowflake shape is the translation, reflection, or rotation image of another snowflake shape.



Look at the pictures of the snowflakes. Can you reflect or rotate each shape onto *itself*? If so, show how you can do this for each snowflake.

Use your research skills to find out if you could transform all snowflake shapes onto themselves in the same ways. Explain why or why not.

13.2

Investigate Frieze Patterns With *The Geometer's Sketchpad®*

Focus on...

- creating designs using *The Geometer's Sketchpad®*

Discover the Math

Materials

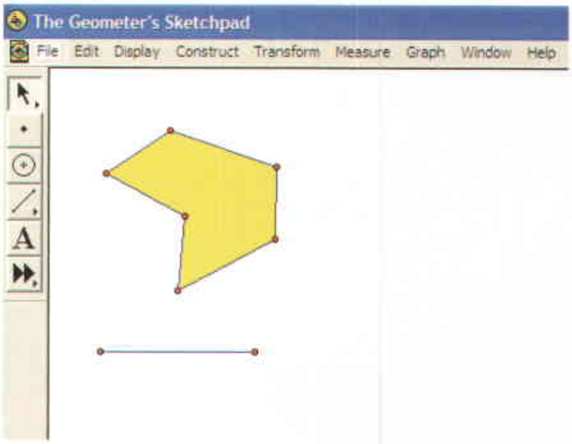
- computer
- *The Geometer's Sketchpad®* software (GSP 4)

Alternatives:

- TECH 13.2A Construct a Frieze Pattern (GSP 4)
- TECH 13.2B Construct a Frieze Pattern (GSP 3)

How do you construct frieze patterns with The Geometer's Sketchpad®?

1. Open *The Geometer's Sketchpad®* and begin a new sketch.



2. a) Near the top left corner of the screen, construct an irregular polygon and its interior.

b) To colour the interior, from the **Display** menu, choose **Color**. Click on a colour.

c) Beneath the polygon, construct a horizontal segment.

d) Select the left endpoint and the right endpoint of the segment in that order. From the **Transform** menu, choose **Mark Vector.**

e) Select the polygon interior. From the **Transform** menu, choose **Translate**. Describe what happens.

f) From the **Transform** menu, choose **Translate**. Repeat a few more times. Describe what happens.

g) Drag points on the original polygon and the line segment to change the pattern. When you like the result, save it as a Sketchpad document using the filename **frieze1.gsp**.

h) Describe the frieze pattern you made and the transformation you used.

Technology Tip

- Another way to mark a segment as a mirror is to double click on the segment.

3. a) Work with the pattern you saved in step 2g). Select the segment. From the **Transform** menu, choose **Mark Mirror**.

b) Select the interiors of all the polygons in your pattern. From the **Transform** menu, choose **Reflect**. Describe what happens.

c) Drag points on the original polygon and the line segment to change the pattern. When you like the result, save it using the filename **frieze2.gsp**.

d) In the first row of your pattern, select the interiors of the second, fourth, and sixth polygons, and so on. In the second row of your pattern, select the interiors of the first, third, and fifth polygons, and so on. From the **Display** menu, choose **Hide Polygon Interiors**. Describe the result. What combination of transformations produces this frieze pattern?

e) Save your pattern from step 3d) as a Sketchpad document using the filename **frieze3.gsp**. Experiment with the pattern by dragging points. Describe what you see.

4. a) Reopen the document **frieze1.gsp**. Construct a point near the centre of the pattern.

b) From the **Transform** menu, choose **Mark Center**.

c) Select all the polygon interiors. From the **Transform** menu, choose **Rotate**. In the dialogue box, specify an angle of 180°. Then, click *Rotate*. Describe what happens.

d) Drag the point you constructed in step 4a) until you like the frieze pattern. What combination of transformations produces this frieze pattern?

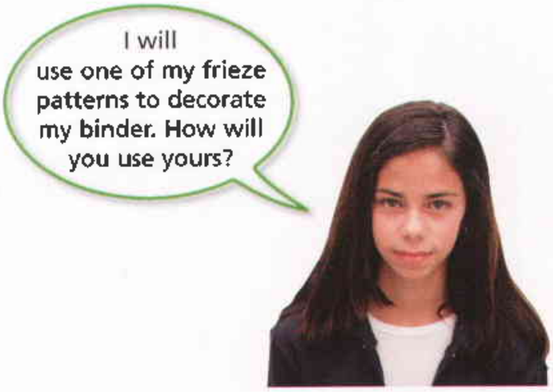
e) Save the pattern as **frieze4.gsp**.

Technology Tip

- Another way to mark a point as a turn centre is to double click on the point.

5. Experiment by creating more frieze patterns. Save the results and share them with your classmates.

6. Reflect Describe how you created your patterns in step 5.



Making Connections

How can you animate a pattern using *The Geometer's Sketchpad*®?

Reopen any frieze pattern file you saved in the above activity. Select the interior of the original polygon you constructed. From the **Display** menu, choose **Animate Polygon Interior**. Describe what happens.

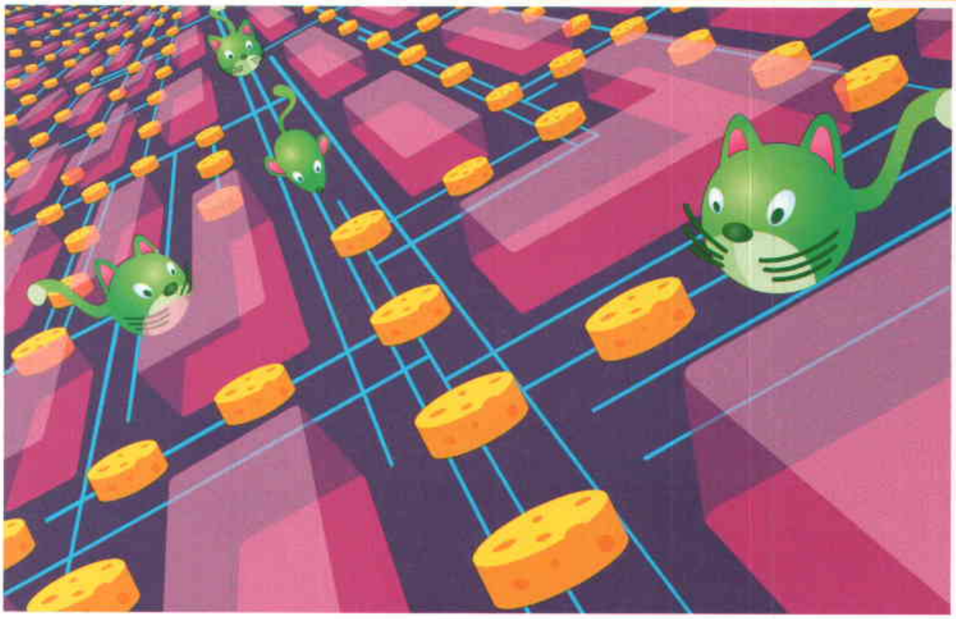
Internet Connect "Dueling Pinwheels" is an activity that uses *The Geometer's Sketchpad*®. The activity involves transformations and animation. To try the activity, go to www.mcgrawhill.ca/links/math7 and follow the links. Write a summary of what you learned from the activity.

13.3

Extension: Translations on a Coordinate Grid

Focus on...

- translating shapes on a coordinate grid



Many video games involve the movement of geometric shapes on a screen. The movements are programmed into the game as transformations on a coordinate grid. What transformations do you see in this screen display?

Discover the Math

Materials

- grid paper

Literacy Connections

Reading Figure Names

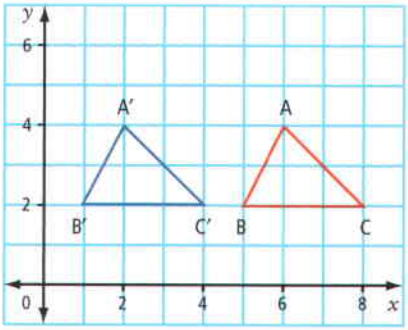
The image of a point can be named using prime symbols.

Read A′ as "A prime," B′ as "B prime," and so on.

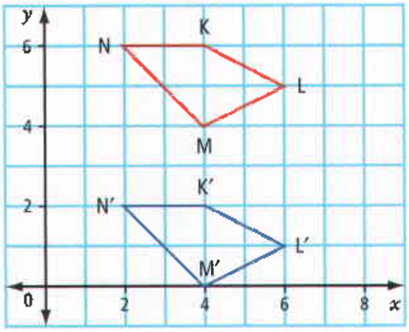
How can you translate shapes on a coordinate grid?

1. Each diagram shows a translation. Describe in words how far the figure moves horizontally and vertically.

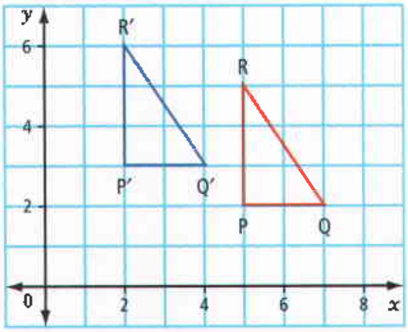
a) $\triangle ABC$ and its image, $\triangle A'B'C'$



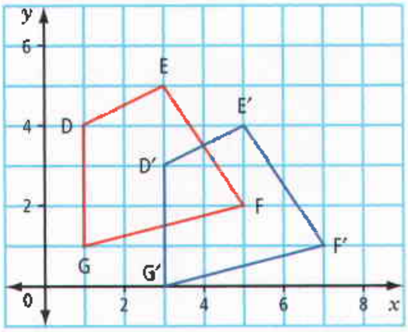
b) quadrilateral KLMN and its image, quadrilateral K′L′M′N′



c) △PQR and its image, △P′Q′R′



d) quadrilateral DEFG and its image, quadrilateral D′E′F′G′

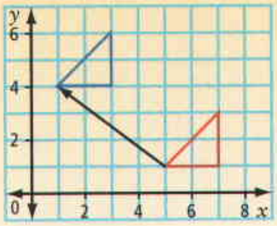


2. Reflect Describe how you found your answers in step 1.

Literacy Connections

Describing a Translation

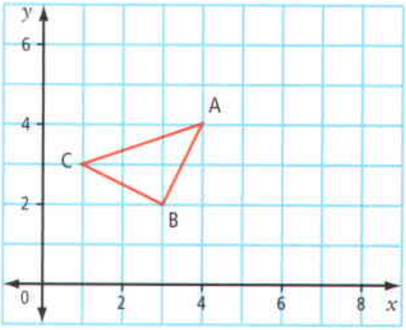
A translation arrow can be used to describe the translation of a figure on a coordinate grid. The figure shown in the diagram has been translated 4 units left and 3 units up.



Describe the movement along the *x*-axis (left or right) first. Then, describe the movement along the *y*-axis (up or down).

Example: Translations on a Coordinate Grid

△ABC is translated 3 units right and 2 units down. Draw its translation image, △A′B′C′.



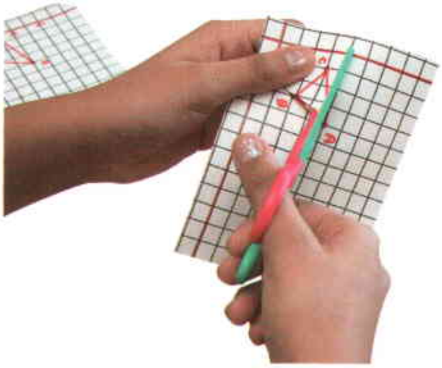
Solution

Method 1: Make a Model

Copy the diagram onto centimetre grid paper.

Trace △ABC on a separate sheet of paper.

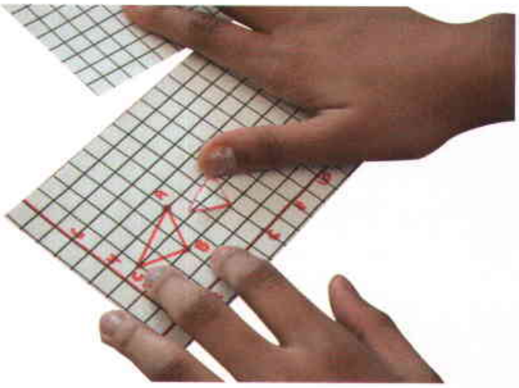
Cut out the triangle you traced.



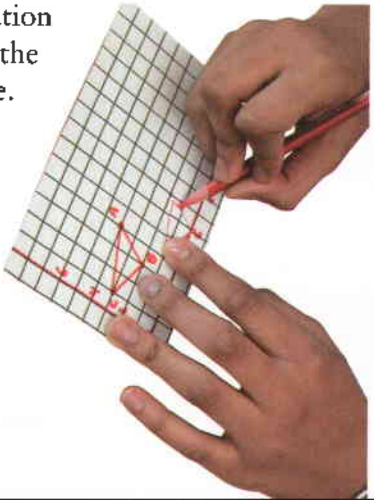
Strategies
Make a model

Place your cut-out △ABC on top of △ABC on the grid paper.

Move vertex B three grid squares right.



Now move vertex B two units down. The location you end up at is the image after both parts of the translation. Trace around your cut-out triangle. Label the image vertices A′, B′, and C′.

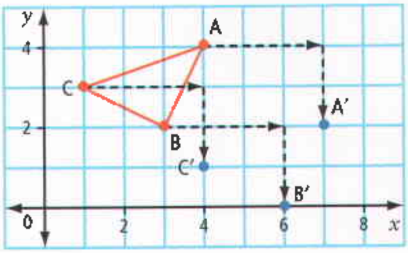


Method 2: Draw a Diagram

Draw the diagram on grid paper.

Count squares to move each vertex 3 units right and 2 units down.

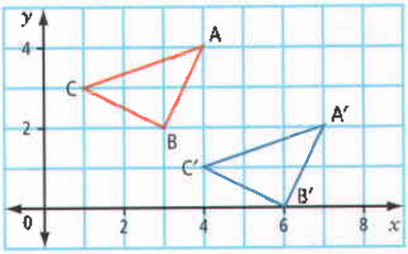
Label the image vertices A′, B′, and C′.



Strategies

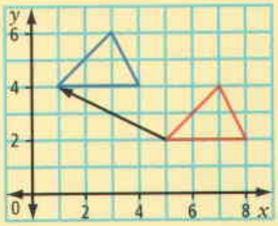
Draw a picture or diagram

Join A′, B′, and C′ to form the translation image.

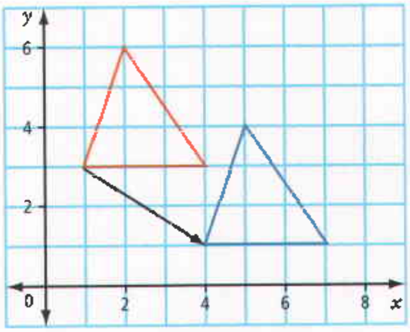


Key Ideas

- Translations can be carried out on a coordinate grid.
- A translation arrow can describe the translation of a figure on a coordinate grid. The translation arrow in the diagram shows a translation of 4 units left and 2 units up.



Communicate the Ideas



1. In the diagram shown, describe the translation that moves the red figure onto the blue figure.

2. A point and its translation image have the same first coordinate. Describe the direction of the translation. Show how you know.

3. The point P(4, 1) is translated 2 units left and 4 units up. How can you find the coordinates of the image without using a diagram?

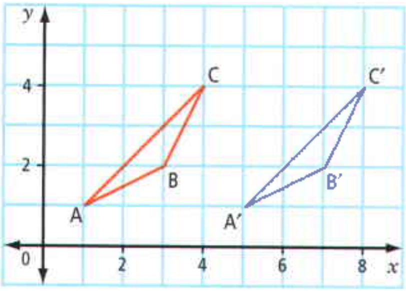
4. A translation moves all the points on a figure to new positions. Why is only one translation arrow needed to show the distance and direction of a translation?

Check Your Understanding

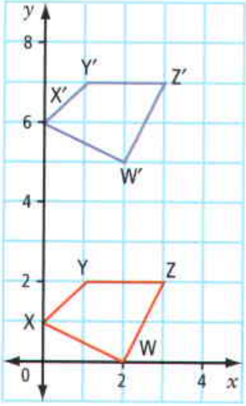
Practise

5. Describe the translation that moves each figure onto its image.

a) △ABC and its image, △A′B′C′

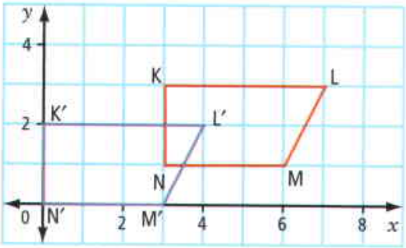


b) quadrilateral WXYZ and its image, quadrilateral W′X′Y′Z′

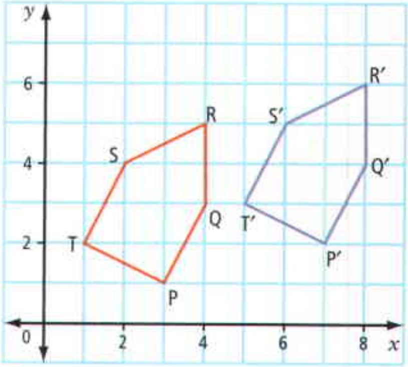


6. Describe the translation that moves each figure onto its image.

a) quadrilateral KLMN and its image, quadrilateral K′L′M′N′



b) pentagon PQRST and its image, pentagon P′Q′R′S′T′



For help with questions 7 to 10, refer to the Example.

7. Draw the image of each point after the given translation. State the coordinates of the image point.

a) A(3, 4); 2 units up

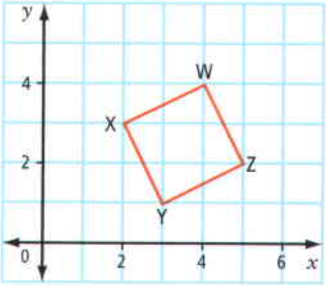
b) B(2, 1); 1 unit left

8. Draw the image of each point after the given translation. State the coordinates of the image point.

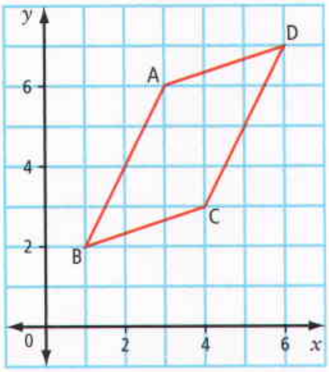
a) C(4, 6); 4 units left and 6 units down

b) D(0, 2); 3 units right and 2 units up

9. Square WXYZ is translated 2 units left and 3 units up. Draw its translation image, square W′X′Y′Z′.



10. Parallelogram ABCD is translated 1 unit right and 2 units down. Draw its translation image, parallelogram A′B′C′D′.



Apply

11. Fareeha drew the translation image of a figure on a coordinate grid. She described the translation to her friend, Michel. Michel could not see the figure or the image. But he told her how she could translate the image back onto the original figure.

a) What did Michel say?

b) How did Michel know?

12. $\triangle XYZ$ with vertices X(1, 1), Y(3, 5), and Z(4, 3) is translated 4 units right and 1 unit down. The image is then translated 5 units left and 3 units up.

a) Find the coordinates of the vertices of the final image.

b) Describe a single translation that would move the original figure onto the final image.

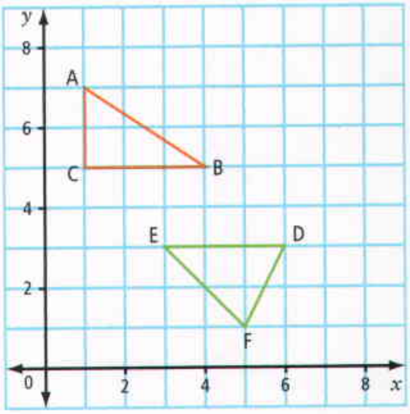
13. $\triangle ABC$ has vertices A(4, 1), B(1, 1), and C(0, 4). $\triangle DEF$ has vertices D(8, 2), E(5, 2), and F(4, 4).

a) Draw the two triangles on a coordinate grid.

b) Is one triangle the translation image of the other? Explain how you know.

Extend

14. $\triangle ABC$ and $\triangle DEF$ are shown on a grid. You read A, B, and C in alphabetical order clockwise around the figure. We say that the "sense" of this triangle is clockwise. For $\triangle DEF$, the sense is counterclockwise.



Maya thinks that translations do not change the sense of a triangle. Do you agree or disagree with her? Explain your reasoning.

Making Connections

In high school, you will learn more about transformations on a coordinate grid.

13.4

Identify Tiling Patterns and Tessellations

Focus on...

- identifying whether a figure will tile the plane
- constructing and analysing tiling patterns

In Section 13.1, you learned that a frieze pattern repeats in one direction. A wallpaper pattern repeats in two directions. Wallpaper patterns are not just found on wallpaper. They are common on carpets, fabrics, and baskets, for example. Describe the repeating pattern in the picture.

Many wallpaper patterns are made from shapes that cover the plane without overlapping or leaving gaps. These patterns are called **tiling patterns** or **tessellations**. Covering the plane in this way is called **tiling the plane**.

tiling pattern

- a pattern that covers a plane without overlapping or leaving gaps
- also called a **tessellation**

tiling the plane

- using repeated congruent shapes to cover a region completely

Discover the Math

Materials

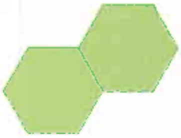
- set of pattern blocks or cardboard cutouts of pattern block shapes
- cardboard cutouts of a regular pentagon and a regular octagon
- cardboard
- scissors
- ruler

Alternative:

- BLM 13.4A Figures for Tiling the Plane

Which regular and irregular figures can you use to tile the plane?

1. Use a pattern block hexagon or make a cardboard cutout of a regular hexagon.



2. a) Draw around the regular hexagon. Move the hexagon to a new position, so that the two hexagons share a common side. Draw around the hexagon again. Continue to see if a regular hexagon tiles the plane.

b) Use the same method to find out if a regular (equilateral) triangle tiles the plane.

c) How could you predict the answer to part b) without drawing any triangles?

d) Does a square tile the plane? How do you know?

3. Try to tile the plane with each of the following. Describe your findings.

a) a regular pentagon **b)** a regular octagon

4. Find out if the following irregular shapes in a set of pattern blocks will tile the plane. Explain how you decided.

a) trapezoid **b)** rhombus

5. Cut out the shape of any irregular quadrilateral.

a) Predict whether the shape will tile the plane. Justify your prediction.
b) Try to tile the plane with the shape. What did you find?
c) Share your findings with your classmates.
d) Use the class results to write a rule.

Strategies
What strategy are you using here?

6. Repeat step 5, but use an irregular triangle instead of an irregular quadrilateral.

7. Try to tile the plane with the following shapes. What did you find?

a) an irregular pentagon **b)** an irregular hexagon

8. **Reflect**

a) What regular figures tile the plane? Explain why some regular shapes tile the plane but others do not. Hint: Look at the angles inside each shape. Can you find a pattern?
b) Explain why some irregular figures tile the plane but others do not.

Key Ideas

- A tiling pattern or tessellation is a pattern that covers a plane without overlapping or leaving gaps.
- Only three types of regular figures tile the plane.
- Some types of irregular figures tile the plane.

Communicate the Ideas

1. Draw the three types of regular figures that tile the plane. Justify your choices.

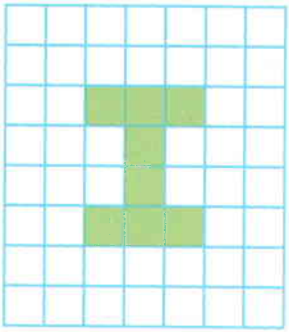
2. Identify two types of irregular figures that tile the plane. Explain why they do.

3. Ana has to choose paving stones to pave her driveway. Should she choose paving stones in the shape of a regular octagon? Show how you know.

Check Your Understanding

Practise

4. Use this shape to tile the plane. Show and colour the result on grid paper.

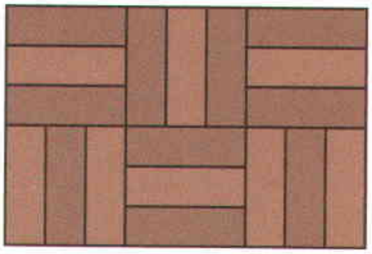


5. Tile the plane with an isosceles triangle. Use colours or shading to create an interesting design.

6. List tiling patterns you see at home or at school. Describe the shapes used to tile the plane. Share your descriptions with your classmates.

Apply

7. Rectangular hardwood strips are often used to tile floors. Aligning the strips in different ways can create attractive designs. An example is shown.



On grid paper, create a different floor design from congruent rectangular hardwood strips.

Chapter Problem

8. a) In question 16 on page 432, you made all the different tetrominoes. The simplest one is shown.

Choose any one except the simplest one. Make the shape from construction paper or cardboard. Use your shape to tile the plane.

b) Describe the transformations you used to tile the plane.

c) Colour your design and create a display to explain what you did.

9. There are many Web sites that deal with tessellations. To learn more, go to www.mcgrawhill.ca/links/math7 and follow the links. Find a tessellation you like and find out how it was created.

Internet Connect

10. A wealth of information is available on historical quilts. The art and craft of quilt making continues today. So, you can also find information about modern quilts. To learn more about quilts, go to www.mcgrawhill.ca/links/math7 and follow the links. Choose a design that you like and explain why you like it. Describe how it uses transformations.

Internet Connect

Try This!

11. a) In question 18 on page 432, you designed a square block for a quilt. Explain why square blocks are often used to make quilts.

b) Predict if blocks of other shapes could be used to make quilts. Justify your prediction.

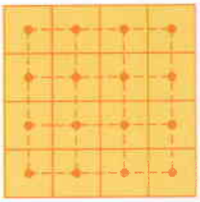
c) Design a quilt block that is not a square. Explain why you chose this shape.

d) Can your block be used to make a quilt? Explain why or why not.

e) If your block can be used to make a quilt, does the shape of the block have any disadvantages? Explain.

Extend

12. The diagram shows a tessellation of squares. A point has been added in the centre of each square. The points are joined by dashed segments perpendicular to common sides. The result is another tessellation. It is called the "dual" of the original tessellation.



a) Describe the dual of the original square tessellation.

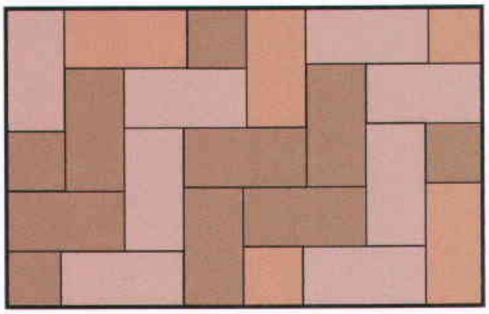
b) Draw a tessellation of regular hexagons. Draw and describe its dual.

c) Draw a tessellation of equilateral triangles. Draw and describe its dual.

Making Connections

What does math have to do with patios?

Some patios are made from interlocking bricks. In many cases, the bricks are all the same shape, such as squares or rectangles. Sometimes two or more different shapes are used.



Materials

- set of pattern blocks or cardboard cutouts of the shapes
- cardboard cutout of a regular octagon
- cardboard or construction paper
- scissors
- ruler
- pencil crayons

Alternative:

- BLM 13.4A Figures for Tiling the Plane

1. a) Draw the result when you try to tile the plane with a regular octagon.

b) Identify another regular shape you could use with a regular octagon to tile the plane.

c) Design a patio using bricks in the shape of a regular octagon and the shape you found in part b).

2. Design patios using combinations of bricks with the following regular shapes. Use colours to create attractive designs.

a) regular hexagons and equilateral triangles

b) squares and equilateral triangles

c) regular hexagons, squares, and equilateral triangles

Use Technology

This is a way to create tiling patterns using *The Geometer's Sketchpad®*.

Create Tiling Patterns Using *The Geometer's Sketchpad®*

Materials

- computer
- *The Geometer's Sketchpad®* software (GSP 4)

Alternatives:

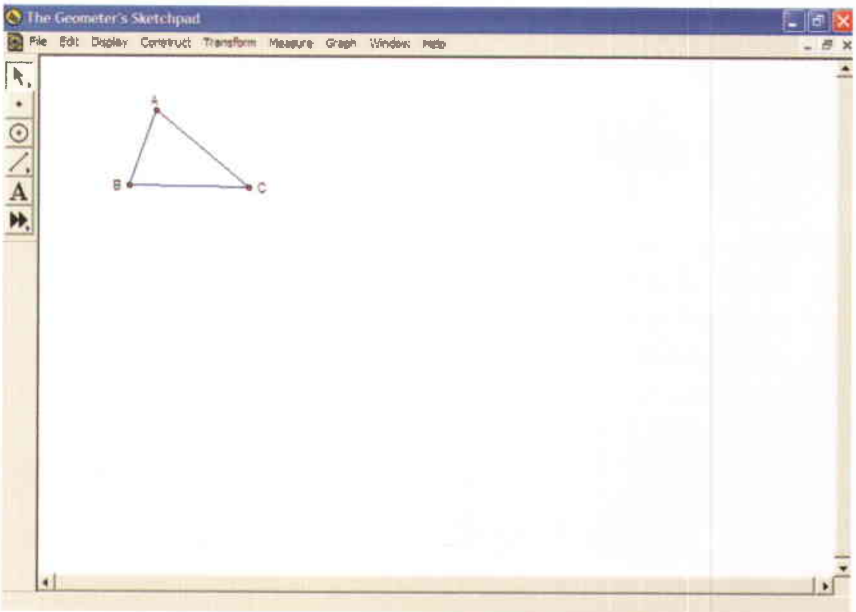
- TECH 13.4A Create a Tiling Pattern (GSP 4)
- TECH 13.4B Create a Tiling Pattern (GSP 3)

Create a Tiling Pattern

In this activity, you will construct an irregular triangle. You will use transformations to create a tiling pattern.

1. Open *The Geometer's Sketchpad®* and begin a new sketch.

2. a) Near the top left corner of the screen, construct an irregular △ABC, as shown.



b) Select point B and point A in that order. From the **Transform** menu, choose **Mark Vector**.

c) Select point C. From the **Transform** menu, choose **Translate**. Name the new point as point D.

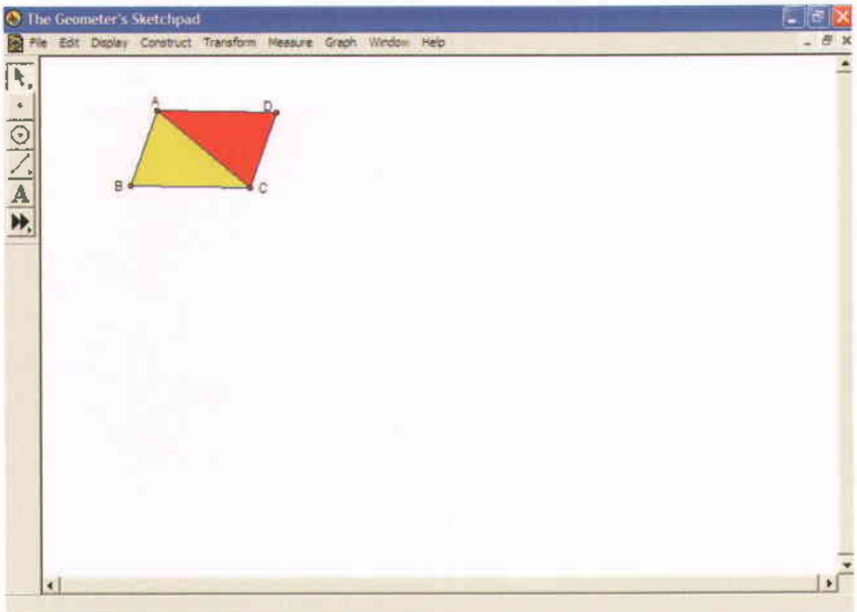
d) Construct segments AD and CD.

e) Construct the interior of △ABC.

f) Construct the interior of △ACD.

g) How do △ABC and △ACD compare? Hint: If you are not sure, measure side lengths and angles.

h) Select one of the triangles. From the **Display** menu, choose **Color.** Click on a colour. Then, select the other triangle and give it a different colour.



i) Select point B and point C in that order. From the **Transform** menu, choose **Mark Vector.**

j) Select the interiors of both triangles. From the **Transform** menu, choose **Translate.** Describe what happens.

k) From the **Transform** menu, choose **Translate.** Repeat a few more times. Describe what happens.

l) Select point A and point B in that order. From the **Transform** menu, choose **Mark Vector.**

m) Select all the triangles. From the **Transform** menu, choose **Translate.** Describe what happens.

n) From the **Transform** menu, choose **Translate.** Repeat a few more times. Describe what happens.

o) Drag points on △ABC. When you like the result, save it as a Sketchpad document.

p) Select the interior of △ABC. From the **Display** menu, choose **Animate Triangle.** Describe what happens.

3. Reflect Does any triangle tile the plane? Explain how you know.

13.5

Construct Translational Tessellations

Focus on...

- constructing tiling patterns
- creating designs using translated images

Did you know that tessellations have been used to create some beautiful works of art? The Dutch artist M.C. Escher (1898–1972) is famous for his use of tessellations. His work is in art galleries around the world. It also appears on T-shirts, posters, and other consumer products.

M.C. Escher's "Symmetry Drawing E105" © 2004 The M.C. Escher Company – Baarn – Holland. All rights reserved.

The picture shows one of Escher's works. How did he use translations to create it?

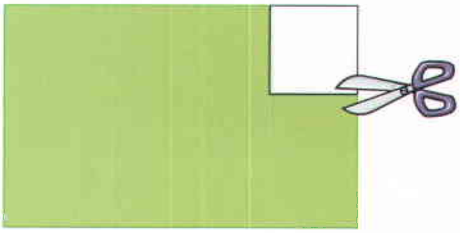
Discover the Math

Materials

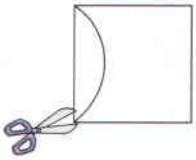
- paper
- tracing paper
- glue stick
- adhesive tape
- cardboard or construction paper
- scissors

How can you tessellate using translations?

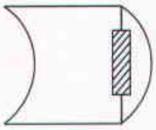
1. a) Draw a square with a side length of about 3 cm on a piece of paper. Cut out the square and glue it to a sheet of cardboard or construction paper. Cut out the square again.



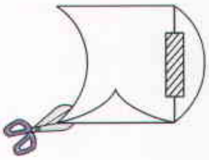
b) Inside the square, draw a curve that connects two adjacent vertices. Cut along the curve to remove a piece from one side of the square.



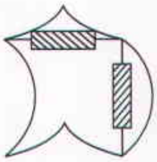
c) Translate the piece to the opposite side of the square. Tape the piece in place.



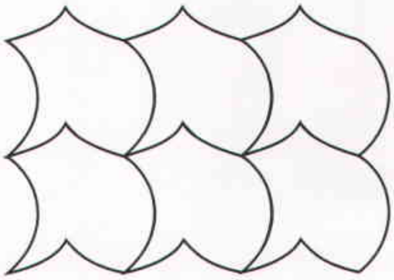
d) Now, draw a different curve on the third side of the square. Cut along this curve to remove another piece.



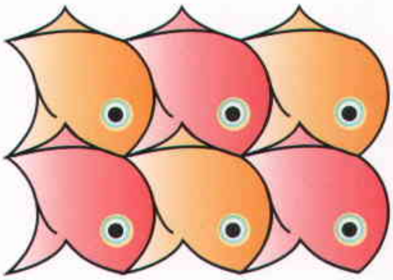
e) Translate the piece from part d) to the opposite side of the square. Tape the piece in place to complete your tile.



f) To tessellate the plane, draw around the tile on a piece of paper.



g) Add colour and designs to the tessellation to make a piece of art.



2. Start with another square. Use it to create your own work of art.

3. Experiment by starting with a rectangle, a parallelogram, or a rhombus, instead of a square.

4. Experiment by starting with a regular hexagon. Create an interesting tile by using the three pairs of opposite sides. Use the tile to create a work of art.

5. **Reflect** Describe how to use translations to create tessellations.

Did You Know?

The leading geometer of the 20th century was a professor at the University of Toronto. Donald Coxeter (1907–2003) was a friend of Escher's and gave him some ideas for his art. Professor Coxeter wrote some complex explanations of Escher's work. But Escher was not trained in math, so he could not understand them.

Use Technology

This is a way to tessellate with translations using *The Geometer's Sketchpad®*.

Tessellate by Translation Using *The Geometer's Sketchpad®*

Tessellate by Translation

In this activity, you will construct a rectangle. You will use translations to change it into a different irregular shape. You will then perform translations on this shape to tile the plane.

1. Open *The Geometer's Sketchpad®* and begin a new sketch.

2. Follow these steps to construct a rectangle ABCD.

a) Near the top left corner of the screen, construct two points, A and B. Construct the segment that joins these points.

b) Select point A and the segment. From the **Construct** menu, choose **Perpendicular Line.**

c) Select point B and the segment. From the **Construct** menu, choose **Perpendicular Line.**

d) Construct point C on the line that passes through B.

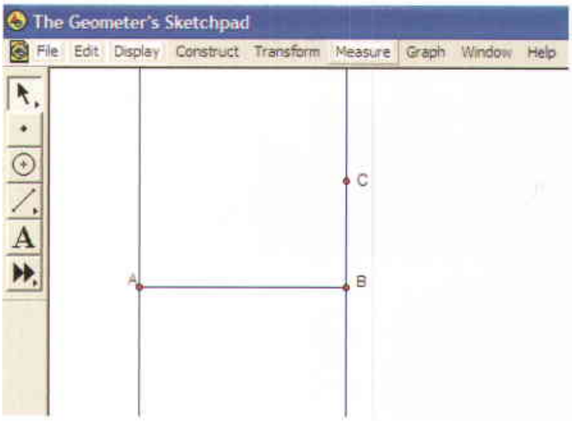
e) Select point C and the segment AB. From the **Construct** menu, choose **Parallel Line.**

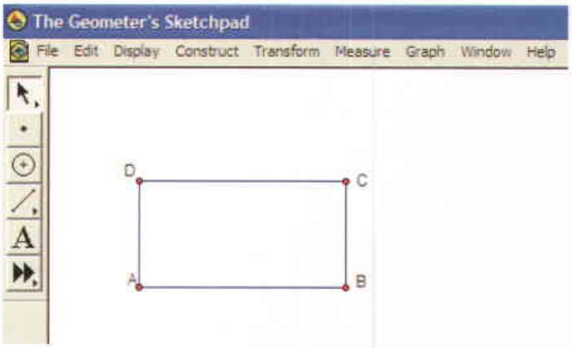
f) Construct point D at the fourth vertex of the rectangle.

g) Select the three lines, but not the segment AB. From the **Display** menu, choose **Hide Lines.**

h) Construct three segments joining B and C, C and D, and D and A. Rectangle ABCD is now complete.

i) Why does this method produce a rectangle?



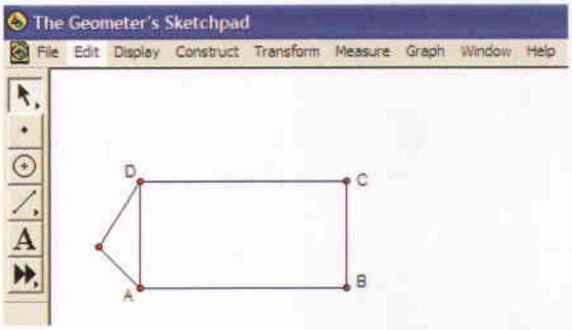


Materials

- computer
- *The Geometer's Sketchpad®* software (GSP 4)

Alternatives:

- TECH 13.5A Tessellate by Translation (GSP 4)
- TECH 13.5B Tessellate by Translation (GSP 3)



3. a) Construct a point close to side AD. Construct the segments from A and D to the new point.

b) Select point A and point B in order. From the **Transform** menu, choose **Mark Vector**.

c) Select the point and the two segments you constructed in step 3a). From the **Transform** menu, choose **Translate**. Describe what happens.

d) Construct a point close to side CD. Construct the segments from C and D to the new point.

e) Select point D and point A in order. From the **Transform** menu, choose **Mark Vector**.

f) Select the point and the two segments you constructed in step 3d). From the **Transform** menu, choose **Translate**. Describe what happens.

g) Select all the vertices of the irregular polygon you have created. From the **Construct** menu, choose **Polygon Interior**.

h) From the **Transform** menu, choose **Translate**. Repeat at least twice more. Describe what happens.

i) Select point A and point B in order. From the **Transform** menu, choose **Mark Vector**.

j) Select the interiors of all the polygons. From the **Transform** menu, choose **Translate**. Repeat at least twice more. Describe what happens.

k) Select individual polygons and use different colours to show the tessellation. Save the tessellation.

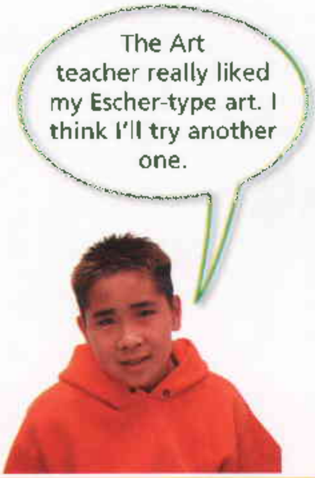
l) Drag points on the original polygon and describe what you see.

m) Reopen the file you saved in step 3k). Select the interior of the original irregular polygon. From the **Display** menu, choose **Animate Polygon Interior**. Describe what happens.

4. Investigate how to construct a parallelogram that is not a rectangle. Use translations to transform the parallelogram into another irregular polygon. Translate this polygon to tile the plane.

5. Experiment to produce some Escher-type art using translations. Share your art with your classmates.

6. **Reflect** Describe how to use translations to create tessellations.



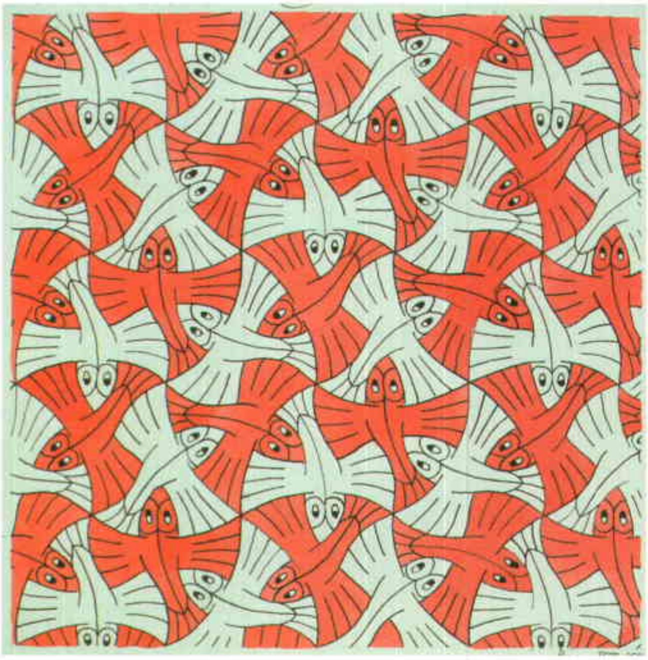
13.6

Construct Rotational Tessellations

Focus on…

- constructing tiling patterns
- creating designs using rotated images

The picture shows a piece of art created by the Dutch artist M.C. Escher. How did he use rotations to create it?



M.C. Escher's "Symmetry Drawing E99"
© 2004 The M.C. Escher Company –
Baarn – Holland. All rights reserved.

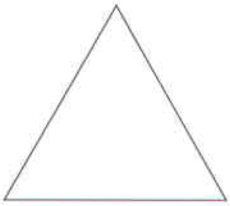
Discover the Math

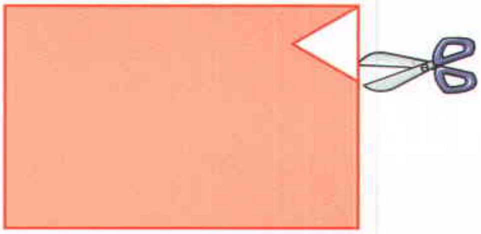
Materials

- paper
- tracing paper
- glue stick
- adhesive tape
- cardboard or construction paper
- scissors

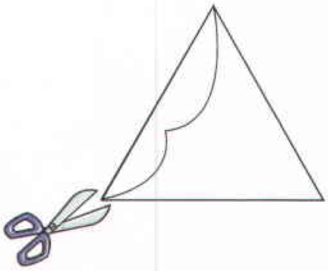
How can you tessellate using rotations about vertices?

1. a) Trace the equilateral triangle shown or draw an equilateral triangle with a side length of about 3 cm on a piece of paper. Cut out the triangle you drew and glue it to a sheet of cardboard or construction paper. Cut out the triangle again.

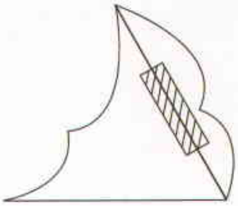




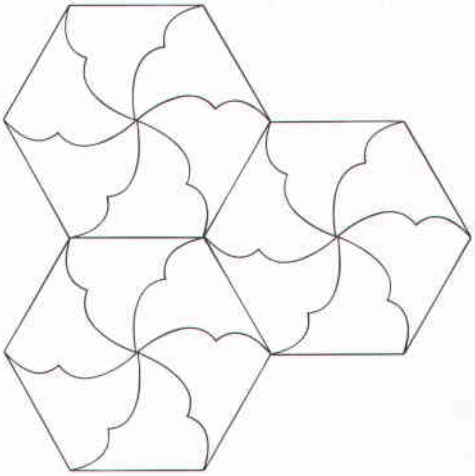
b) Inside the triangle, draw a curve that connects two adjacent vertices. Cut along the curve to remove a piece from the triangle.



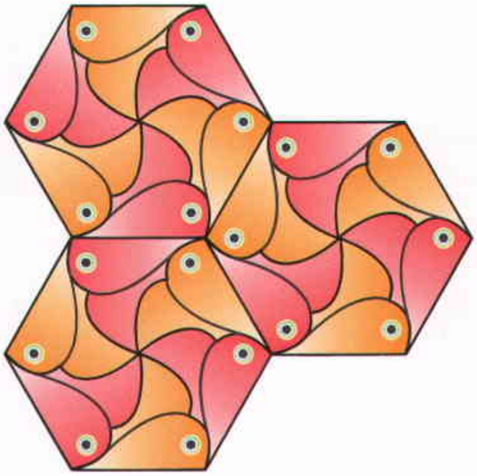
c) Rotate the piece you removed 60° about a vertex at one end of the curve. This rotation moves the piece to another side of the triangle. Tape the piece in place to complete your tile.



d) To tessellate the plane, draw around the tile on a piece of paper. Then, rotate and draw around the tile over and over until you have a design you like.



e) Add colour and designs to the tessellation to make a piece of art.

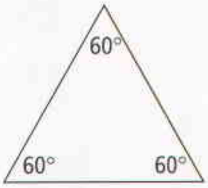


2. Experiment by starting with a square or a regular hexagon. Use rotations about vertices to create a tile that will tessellate the plane. Use the tile to create a piece of art using rotations.

3. **Reflect** How can you tessellate using rotations about vertices?

Did You Know?

The angles in an equilateral triangle are all 60°.



Making Connections

Escher Art

There are many Web sites that describe Escher's life and work. To learn more, go to **www.mcgrawhill.ca/links/math7** and follow the links. Find a piece of Escher's art that you like. Think about the transformations he used to create it. Share your findings with your classmates.

Use Technology

This is a way to tessellate with rotations using *The Geometer's Sketchpad®*.

Tessellate by Rotation Using *The Geometer's Sketchpad®*

Materials

- computer
- *The Geometer's Sketchpad®* software (GSP 4)

Alternatives:

- TECH 13.6A Tessellate by Rotation (GSP 4)
- TECH 13.6B Tessellate by Rotation (GSP 3)

Tessellate by Rotation

In this investigation, you will construct an equilateral triangle. You will use a rotation about a vertex to change the triangle into an irregular shape. You will then perform rotations on this shape to tile the plane.

1. Open *The Geometer's Sketchpad®* and begin a new sketch.

2. The first step is to construct an equilateral △ABC.

a) Near the centre of the screen, construct two points, A and B. Construct the segment that connects them.

b) Select point A. From the **Transform** menu, choose **Mark Center**.

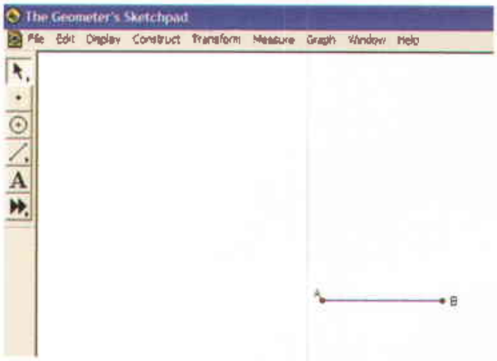
c) Select point B and the segment AB. From the **Transform** menu, choose **Rotate**. In the dialogue box, set the angle of rotation at 60°. Then, click on *Rotate*.

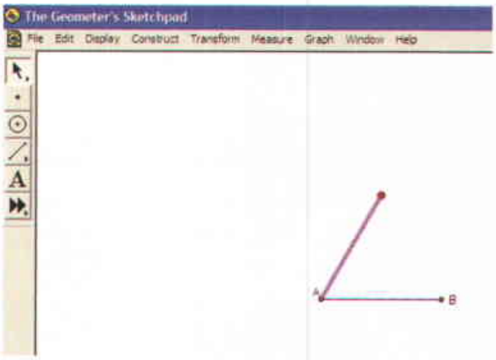
d) Rename the new point, B′, as point C.

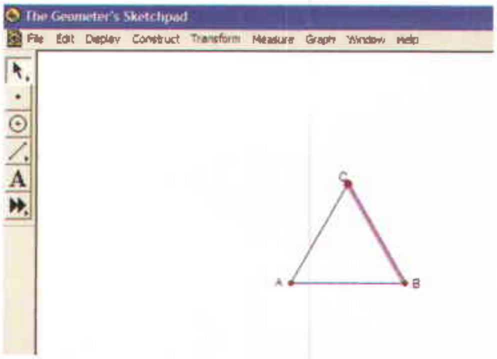
e) Select point C. From the **Transform** menu, choose **Mark Center**.

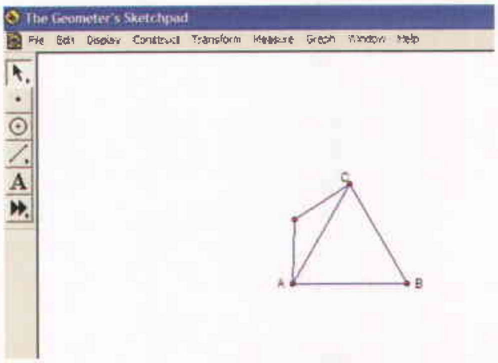
f) Select segment AC. From the **Transform** menu, choose **Rotate**. In the dialogue box, keep the angle of rotation at 60°. Then, click on *Rotate*.

g) Explain why this method produces an equilateral triangle.









3. a) Construct a point close to side AC. Construct the segments from A and C to the new point.

b) Select point C. From the **Transform** menu, choose **Mark Center**.

c) Select the point and the two segments you constructed in step 3a). From the **Transform** menu, choose **Rotate**. In the dialogue box, set the angle of rotation at 60°. Then, click on *Rotate*. Describe what happens.

d) Select all the vertices of the irregular pentagon you have created. From the **Construct** menu, choose **Pentagon Interior**.

e) From the **Transform** menu, choose **Rotate**. Repeat four more times. Describe what happens.

f) Select point A. From the **Transform** menu, choose **Mark Center**.

g) Select the interiors of all the polygons. From the **Transform** menu, choose **Rotate**. In the dialogue box, set the angle of rotation at 120°. Then, click on *Rotate*. Rotate once more by the same angle. Describe what happens.

h) Select point B. From the **Transform** menu, choose **Mark Center**.

i) Select the interiors of all the polygons. Rotate by 120° about point B.

j) Select individual polygons and use different colours to show the tessellation. Save the tessellation.

k) Drag points on the original polygon and describe what you see.

l) Reopen the file you saved in step 3j). Select the interior of the original irregular polygon. From the **Display** menu, choose **Animate Pentagon**. Describe what happens.

4. Investigate how to construct a square. Experiment by rotating shapes about the vertices to produce an irregular polygon. Create a tessellation pattern. Describe your results.

5. Experiment to produce some Escher-type art using rotations about vertices. Share your art with your classmates.

6. **Reflect** Describe how to use rotations about vertices to create tessellations.

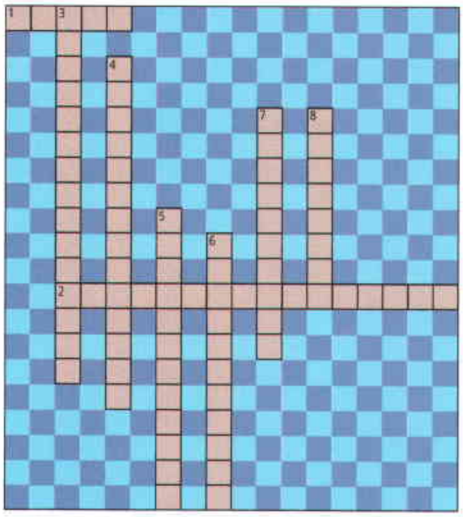
We could use these designs to decorate the gym for the end of the year concert. Let's see how many different designs we can make?

CHAPTER 13

Review

Key Words

Use the clues to help you solve the puzzle.



Across

1. a figure resulting from a transformation
2. shows the distance and direction of a translation (2 words)

Down

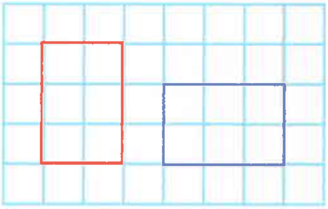
3. the angle through which a figure turns (3 words)
4. moves one geometric figure onto another
5. a tiling pattern that covers a plane without overlapping
6. a slide along a straight line
7. a flip over a mirror line
8. a turn about a fixed point called the turn centre

13.1 Explore Transformations, pages 428–433

9. a) Design a frieze pattern that involves the translation and rotation of an irregular figure.

b) Describe how you created your design.

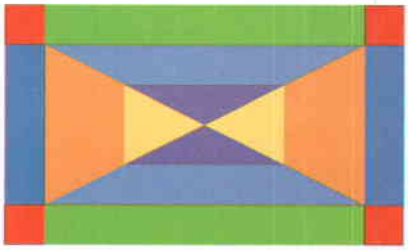
10. The blue figure is the rotation image of the red figure. Copy the diagram and show the turn centre and the angle of rotation.



11. Johan drew a square on a piece of paper. He then used a transparent mirror to reflect the square onto itself. Where could he have placed the mirror? Explain.

12. A figure is flipped so that its image is horizontally beside it. Describe the mirror line.

13. The diagram shows a design for a stained-glass window.



a) Describe the transformations that relate the congruent parts in the design.

b) Describe three ways to transform the whole window so that the original and the image are in the same position.

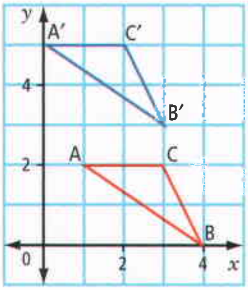
c) If a classmate told you that the window was "upside down," what would you say? Explain.

14. a) Design your own stained-glass window on grid paper.

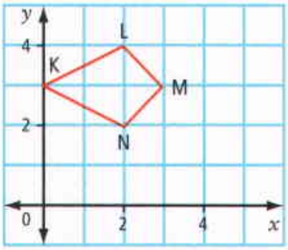
b) Describe the transformations you used to create the window.

13.3 Translations on a Coordinate Grid, pages 436–441

15. Describe the translation that moves △ABC onto its translation image, △A′B′C′.

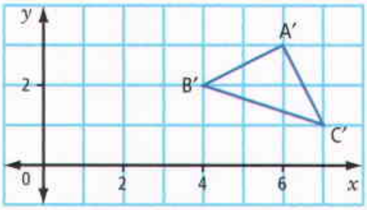


16. Kite KLMN is translated 3 units right and 2 units down. Draw the translation image, kite K′L′M′N′.



17. A point and its translation image have the same second coordinate. Describe the direction of the translation.

18. △ABC is translated 3 units right and 1 unit down to give its image, △A′B′C′. The image is shown in the diagram. What are the coordinates of points A, B, and C?



19. Describe the effect of the following transformation on any figure drawn on a coordinate grid.

Subtract 2 from the first coordinate of every point, and add 3 to the second coordinate of every point.

13.4 Identify Tiling Patterns and Tessellations, pages 442–445

20. Does a regular hexagon tile the plane? Justify your response.

21. Does a parallelogram tile the plane? Justify your response.

22. A triomino is a figure made by joining three congruent squares along whole sides.

a) How many different triominoes are there? Draw them.

b) Does each triomino tile the plane?

23. The diagram shows a garden path made from irregular 12-sided bricks.

a) Explain why the 12-sided brick will tile the plane.

b) Design an irregular 10-sided brick that could be used to make a path.

c) Explain why your 10-sided brick will tile the plane.

d) Design an irregular 6-sided brick that could be used to make a path.

e) Explain why your 6-sided brick will tile the plane.

24. Create two different tiling patterns using rectangles.

CHAPTER 13

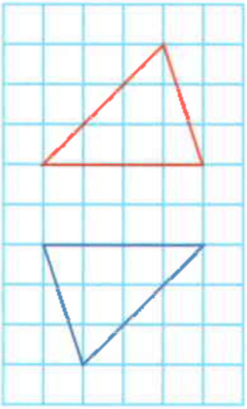
Practice Test

Strand	NSN	MEA	GSS	PA	DMP
Questions			1–13		

Multiple Choice

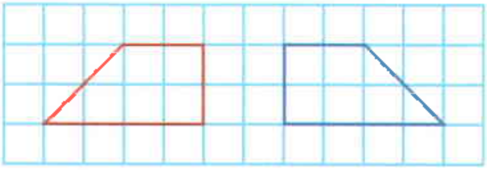
For questions 1 to 4, select the correct answer.

1. The transformation that relates the figures is



A a translation
B a reflection
C a rotation
D a translation and a reflection

2. The transformation that relates the figures is



A a translation
B a reflection
C a rotation
D a translation and a rotation

3. Point P(3, 4) is translated 1 unit left and 2 units down. The coordinates of its translation image, point P′, are

A (2, 2)
B (1, 3)
C (4, 2)
D (5, 5)

4. You can tile the plane using

A any figure
B any irregular figure
C any regular figure
D some figures, but not others

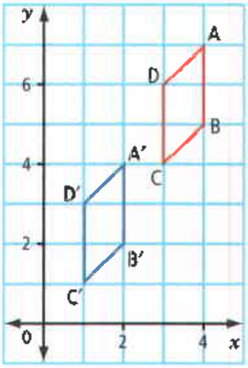
Short Answer

5. a) Design a frieze pattern that involves the translation and reflection of an irregular figure.

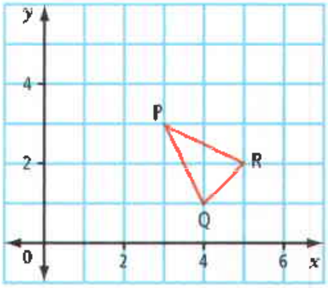
b) Describe how you used the transformations to create your design.

6. What effect does the translation, reflection, or rotation of a figure have on the side lengths and angle measures of the figure? Explain why.

7. Describe the translation that moves parallelogram ABCD onto its image, parallelogram A′B′C′D′.



8. $\triangle PQR$ is translated 2 units left and 3 units up. Draw the translation image, $\triangle P'Q'R'$.

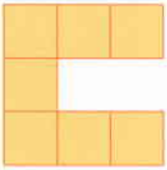


9. Does a regular pentagon tile the plane? Explain why or why not.

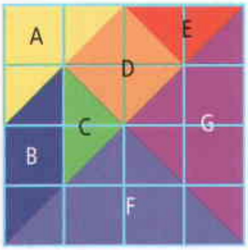
10. Does a scalene triangle tile the plane? Explain why or why not.

11. Create a tiling pattern using parallelograms that are not rectangles or squares. Colour or shade the pattern to create a design.

12. Seven square tiles have been arranged in a C-shape. Use grid paper to find out if this shape will tile the plane.



Extended Response



13. A tangram is an ancient Chinese puzzle. It includes 7 geometric pieces.

a) Can you find pairs of pieces that are related by a translation? If so, describe the translation that relates the pieces in each pair.

b) Can you find pairs of pieces that are related by a rotation? If so, describe the turn centre and turn angle for each pair.

c) Can you find pairs of pieces that are related by a reflection? If so, describe the mirror line for each pair.

Chapter Problem Wrap-Up

In question 16 on page 432, you used identical squares to make all the possible tetrominoes. In question 8 on page 444, you chose a tetromino and used it as a tessellation tile to make a design.

Now, make or draw a pentomino by joining 5 congruent squares along whole edges. The simplest one is shown.

1. How many different pentominoes are there? Draw them on grid paper.
2. Choose any pentomino except the simplest one. Make the shape from construction paper or cardboard. Use your shape to tile the plane.
3. Describe the transformations you used to tile the plane.
4. Colour your design and create a display to explain what you did.

Answers

GET READY FOR GRADE 7

1 Fractions, Metric Units, Estimation, pages 2–3

1. a) $\frac{1}{3}$ **b)** $\frac{5}{4}$ **c)** $\frac{7}{9}$ **d)** $\frac{10}{3}$ **e)** $\frac{4}{5}$ **f)** $\frac{2}{1}$
3. B
5. C
7. B
9. B
11. a) Answers may vary. It gets smaller by half every time.
13. 128
15. Answers may vary. A fraction represents part of a whole object or a share of a group of objects.
17. Answers will vary. You might tell the classmate to picture a similar object that they know more about.

2 Multiplying and Dividing Decimals, Estimation, pages 4–5

1. a) 320 **b)** 3200 **c)** 3.2 **d)** 0.32
3. Using 32: 1a): move decimal one place to the right; 1b): move decimal two places to the right; 1c): move decimal one place to the left; 1d): move decimal two places to the left; 2a): move decimal one place to the left; 1b): move decimal two places to the left; 1c): move decimal one place to the right; 1d): move decimal two places to the right
5. a) 64.1 **b)** 64.1 **c)** 6.41 **d)** 6.41
7. C
9. C
11. C
13. Carriff: 7.8 km, Jeremy: 6.4 km, Len: 7.2 km, Meghan: 7.8 km, Amy: 6.3 km
15. Answers may vary. Organize the data for each event from best to worst.
17. Multiplying: count the total number of decimal places in the two numbers you start with. There will be this many decimal places in the answer. Dividing: there is no distinct relationship.
19. Try to estimate first to see roughly what your answer will be.

3 Patterns With Natural Numbers, Fractions, and Decimals, pages 6–7

1. a) 9, 11, 13 **b)** 10, 13, 16 **c)** 33, 43, 53 **d)** 25, 36, 49 **e)** 35, 46, 57 **f)** 32, 47, 65
3. a) increased by 3 **b)** multiplied by 2 and then increased by 3 **c)** multiplied by 3 **d)** multiplied by 2 and then increased by 1
5. Answers may vary. **a)** A = 0, B = 100, C = 120, D = 180, E = 250 **b)** Just after point C, but before point D. **c)** 180 **d)** It should be greater than 100 since it is the same distance from point A to 60 as from 60 to point C (C = 120).
7. Answers may vary.
9. Answers may vary. Multiples of 5.
11. They are prime numbers.
13. They are multiples of 2, 3, 6, and 9.
14.–18. Answers may vary.

CHAPTER 1

Get Ready, pages 10–11

1. a) 40 mm **b)** 24 m
3. a) 9 km **b)** 18 km **c)** 1.2 km **d)** 0.7 km
5. a) 16 cm^2 **b)** 8 cm^2 **c)** 15 cm^2

1.1 Perimeters of Two-Dimensional Shapes, pages 15–17

5. a) 22 cm **b)** 400 cm
7. 220 cm
9. a) 96 cm **b)** 9 m **c)** 6.9 cm **d)** 90 mm
11. Answers will vary.
13. Convert to the same units. 450 cm
15. a) 1200 m **b)** 24 km
17. Sasha; Anders forgot to change the measurements to the same units.
19. A loonie is a regular polygon with 11 sides, 7.7 cm
21. Answers may vary slightly. **a)** longer sides 2.4 cm, shorter sides 1.2 cm **b)** 7.2 cm **c)** Use a formula; $P = 2 \times (l + w)$.
23. No. Each side of the octagon measures 2.25 m.

1.2 Area of a Parallelogram, pages 20–21

3. a) 12 cm^2 **b)** 6 cm^2 **c)** 6 cm^2 **d)** 3 cm^2
5. Answers may vary slightly. **a)** 3 cm^2 **b)** 1 cm^2
7. a) No; the height is not given. **b)** Measure the distance between equal sides.
9. Joel measured a side instead of the height. He has to measure the perpendicular distance between two equal sides.
11. a) Monica is correct. **b)** Answers will vary. A rectangle is a special kind of parallelogram.

1.3 Area of a Triangle, pages 24–25

5. a) 6 cm^2 **b)** 24.5 mm^2 **c)** 13 m^2 **d)** 2.5 m^2
7. a) 60 cm^2 **b)** 18 m^2 **c)** 30 km^2 **d)** 7.5 mm^2
9. a) $b \times h$ comes from the formula for the area of a parallelogram. **b)** The area of a parallelogram divided by 2 is the area of a triangle. **c)** The area of a triangle is half the area of a parallelogram with the same base and height.
11. no
13. a) all equal areas **b)** Answers may vary slightly. Each base is 1.3 cm, each height is 1.2 cm. **c)** 0.72 cm^2

15. a) Diagrams may vary. Any triangle with base 4.0 m and height 2.5 m. **b)** No, the perimeters do not remain constant. The lengths of the sides change as the height changes.

1.4 Apply the Order of Operations, pages 28–29

5. 31.75
7. a) 11 **b)** 6 **c)** 15 **d)** 14
9. a) multiplication **b)** subtraction inside brackets **c)** division
11. a) In line 2, Vanya divided 16 by 4 first, she should have done 64 + 16 first. In line 4, she subtracted 3 – 2, she should have done the multiplication 2 × 2 first. **b)** 0
13. a) 11.4 **b)** 2.1 **c)** 0 **d)** 8 **e)** 5
17. a) $A = 2 \times (4 \times 10) + 5 \times 6 \div 2$; 95 cm^2 **b)** Assume that the shape is symmetric.

1.5 Area of a Trapezoid, pages 32–33

5. a) a = 8 cm, b = 18 cm, h = 6 cm **b)** a = 8 mm, b = 10 mm, h = 6 mm **c)** a = 2 m, b = 6.5 m, h = 3 m **d)** a = 5 cm, b = 6.4 cm, h = 1.4 cm
7. a) a = 1 cm, b = 2.1 cm, h = 1.3 cm **b)** a = 1.4 cm, b = 2.8 cm, h = 0.8 cm
9. Answers may vary slightly. 6. a) 2.4 cm^2 **b)** 1.8 cm^2 7. a) 2.0 cm^2 b) 1.7 cm^2
11. Answers may vary.
13. 75 375 cm^2 (or about 7.54 m^2)
15. Both trapezoids with height 3 m have equal area. The sum of their parallel sides is the same too, 12 m.
19. a) and **b)** Answers may vary. **c)** Use the formula for the area of a parallelogram and then divide by 2.

1.6 Draw Trapezoids, page 36

1.–8. Check your measurements with a good ruler.

1.7 Composite Shapes, pages 43–45

5. a) 6 m, 8 m **b)** 8 cm, 16 cm, 4 cm, 4 cm
7. 48 m
9. Answers may vary.
11. 13.6 m
13. a) Answers may vary. Split the logo into smaller shapes to calculate the area. **b)** $1275 **c)** Answers may vary. Find the area of the rectangle and subtract the four triangles.
15. Answers may vary.
17. a) $A = (17 \times 8) - (4 \times 5)$ **b)** 116 m^2 **c)** The answers are the same.
21. a) $75 **b)** Answers may vary.

Review, pages 46–47

1. D **3.** E **5.** A
7. 28 m
9. Diagrams may vary. The parallelogram should have base 6 and height 3.
11. 0.56 m^2
13. a) Performed addition before division. **b)** Divided from right to left.
15. a) trapezoid, has one pair of parallel sides **b)** 850 cm^2
17. a) no **b)** no
19. Answers may vary. An example could have a = 10 cm, b = 14 cm, and h = 4 cm.
21. Answers may vary slightly. **a)** 6.21 cm^2 **b)** 16.0 cm

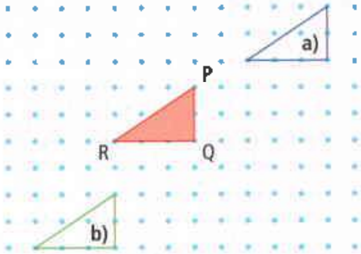
Practice Test, pages 48–49

1. D **3.** B **5.** B
7. a) 8 **b)** 9 **c)** 21
9. Answers may vary. Use a ruler to check that the perimeter is 26 cm.
11. a) Use a ruler, measure the sides and the height. Make sure a = 15 cm, b = 9 cm, and h = 4 cm.
b) Answers may vary.
13. a) 130 m **b)** Answers may vary. **c)** 592 m^2, both areas same. Total area does not change if shape is split differently.

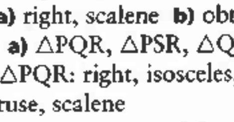
CHAPTER 2

Get Ready, pages 52–53

1. Answers may vary. Name two points that are joined by a line segment. AB, GH, BE, AD
3. Answers may vary. AB = BC
5. Answers may vary. **a)** 77° **b)** 100° **c)** 80°
7. a) acute angle **b)** right angle **c)** obtuse angle
9.



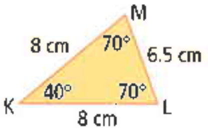
11.

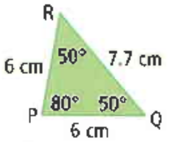


2.1 Classify Triangles, pages 57–59

5. a) isosceles **b)** scalene
7. a) right **b)** acute
9. a) right, scalene **b)** obtuse, scalene
11. a) △PQR, △PSR, △QSR
b) △PQR: right, isosceles, △PSR: acute, scalene, △RSQ: obtuse, scalene
13. a) PR = 7.3 cm, PQ = 5.0 cm, RQ = 4.4 cm
b) ∠Q = 105°, ∠R = 40°, ∠P = 35° **c)** obtuse, scalene

15. a) acute, isosceles **b)** acute, isosceles

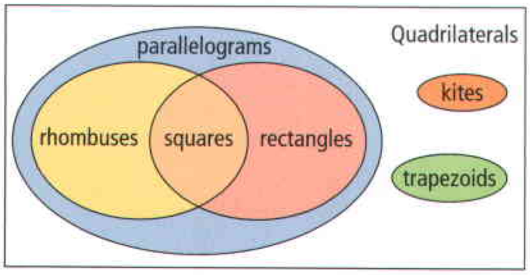




17. a) XY and YZ are of equal length, so △XYZ is an isosceles triangle.
b) Yes, ∠X = ∠Z.
19. a) ∠S = 60°, RS = 5 cm, TS = 5 cm **b)** △RST is equilateral and acute
21. a) There are right, obtuse, isosceles, and scalene triangles. **b)** Answers will vary.
23. Equilateral triangles are used in bridges because they are more rigid than squares, rectangles or other shapes. This makes bridges using them able to support heavier loads and stronger winds.

2.2 Classify Quadrilaterals, pages 63–65

5. a) parallelogram **b)** rhombus
7. JKLO is a trapezoid, OLNM is a rectangle.
9. a) C rhombus **b) D** trapezoid **c) B** kite **d) A** square
11. a) Piece 4 is a square, 6 is a parallelogram **b)** Pieces 3 and 4, 4 and 5, or 5 and 6 form trapezoids.
13. a) trapezoid **b)** rhombus
17.



2.3 Congruent Figures, pages 68–69

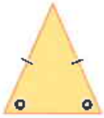
3. a) Yes, they are each hexagons of the same size. **b)** No, although they are all parallelograms, they are all different sizes. **c)** Yes, they are all identical trapezoids.
5. △DEF and △GHI are congruent.
7. ∠A = ∠D, ∠B = ∠E, ∠C = ∠F, AC = DF, BC = EF, AB = DE
9. a) △AJG and △AJD **b)** △AIJ and △ABJ, △JIF and △JBE, △CBE and △HIF
c) △DCE and △GHF, △DBJ and △GIJ
11. No. For example, two squares with different side lengths. They are the same shape but different sizes.
13. No. For example, a square with side 2 cm has area 4 cm^2. A rectangle 4 cm by 1 cm also has area 4 cm^2.

2.4 Congruent and Similar Figures, pages 73–74

3. a) yes **b)** no
5. KLMN and WXYZ
7. yes
9. a) C **b)** E; each side of A is twice the length of the corresponding side of E.
11. a) Yes; pieces 1 and 2 are congruent, pieces 3 and 5 are congruent. **b)** Yes; pieces 1, 2, 3, 5, and 7 are all similar right isosceles triangles.
13. The diagonal cuts the rectangle into two sets of similar triangles (if the rectangles are similar).
17. It is a square.

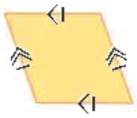
Review, pages 76–77

1. a)

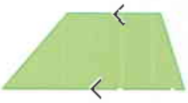


b)

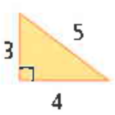
c)

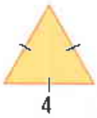


d)



3. a) acute and equilateral **b)** scalene and obtuse
5. a) acute, isosceles **b)** obtuse, scalene
7. Squares, rectangles, trapezoids, rhombuses, and irregular quadrilaterals.
9. Answers may vary.
11. Not always.

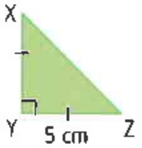




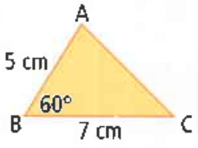
13. DEFG is similar to HIJK and LMNO is similar to PQRS. There are no congruent figures.
15. a) No, △ABC is taller than △DEF so they are not the same size. **b)** No, they are not the same shape either. FE is about as long as CB but DF is much shorter than AC.

Practice Test, pages 78–79

1. B **3.** D **5.** B
7. a)



b)



9. In order for triangles to be similar their angles must be identical. In a right triangle the other two angles are both acute. Because of this a right triangle can never be similar to an obtuse triangle.
11. ABCD: side length 4 cm, perimeter 16 cm, EFGH: side length 8 cm, perimeter 32 cm. These figures are similar.

CHAPTER 3

Get Ready, pages 84–85

1. a) $\frac{3}{2}, 1\frac{1}{2}$ **b)** $\frac{7}{4}, 1\frac{3}{4}$ **c)** $\frac{16}{6}, 2\frac{4}{6}$

3. Diagrams may vary.

a) $\frac{1}{4} > \frac{1}{8}$ **b)** $\frac{1}{3} > \frac{1}{4}$ **c)** $\frac{2}{3} > \frac{5}{8}$

5. a) 2, 4, 6, 8, 10 **b)** 4, 8, 12, 16, 20 **c)** 5, 10, 15, 20, 25

7. a) $\frac{4}{12} = \frac{1}{3}$ **b)** $\frac{2}{8} = \frac{1}{4}$

3.1 Add Fractions Using Manipulatives, pages 88–89

5. a) $\frac{5}{6}$ **b)** $\frac{1}{2}$

7. a) $\frac{1}{6} + \frac{1}{3} = \frac{3}{6}$ or $\frac{1}{2}$ **b)** $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$

c) $\frac{1}{2} + \frac{1}{6} = \frac{4}{6}$ or $\frac{2}{3}$ **d)** $\frac{1}{2} + \frac{1}{3} + \frac{1}{6} = \frac{6}{6}$ or 1

9. a) $\frac{6}{6}$ or 1 **b)** $\frac{6}{6}$ or 1 **c)** $\frac{5}{6}$

11. a) $\frac{1}{2} + \frac{1}{2} + \frac{1}{3} = \frac{4}{3}$

b) $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{6} + \frac{1}{6} - \frac{1}{6} + \frac{1}{6} = \frac{5}{3}$

c) $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{2} = \frac{5}{2}$

13. a) $\frac{1}{2} + \frac{1}{2}$ **b)** $\frac{1}{2} + \frac{1}{2}$ **c)** $\frac{1}{2} + \frac{1}{4} + \frac{1}{4}$

15. Answers may vary.

17. Diagrams may vary. $\frac{1}{3} + \frac{1}{3} + \frac{1}{6} = \frac{5}{6}$

19. $1\frac{5}{6}$

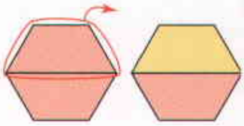
3.2 Subtract Fractions Using Manipulatives, pages 92–93

3. a) $1 - \frac{1}{3} = \frac{2}{3}$ **b)** $\frac{1}{3} - \frac{1}{6} = \frac{1}{6}$ **c)** $\frac{5}{6} - \frac{1}{3} = \frac{1}{2}$

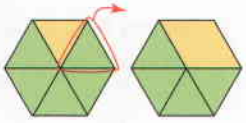
5. a) $\frac{1}{2} - \frac{1}{6} = \frac{1}{3}$ **b)** $\frac{2}{3} - \frac{1}{6} = \frac{1}{2}$ **c)** $\frac{1}{2} - \frac{1}{3} = \frac{1}{6}$

7. Diagrams may vary.

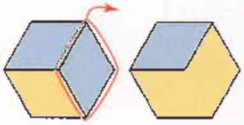
a)



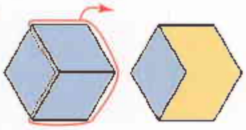
b)



c)

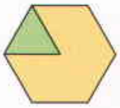


d)

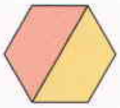


9. Diagrams may vary.

a)

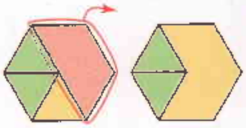


b)

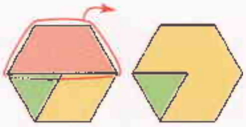


11. Diagrams may vary.

a)

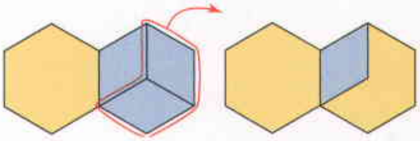


b)



13. a) $1 - \frac{1}{2} = \frac{1}{2}$ **b)** $1 - \frac{1}{4} = \frac{3}{4}$

17. a) Diagrams may vary.



b) In this representation, each fraction is represented by pattern block pieces that are twice the size as they are when 1 hexagon = 1 whole. $\frac{1}{2}$ is represented by a whole hexagon and $\frac{1}{3}$ is represented by two blue rhombuses.

c) The numerical answer to part a) of $\frac{1}{6}$ is the same, but it is represented by pattern blocks twice the size as those that represent the answer when 1 hexagon = 1 whole.

3.3 Find Common Denominators, page 97

5. Answers may vary. **a)** 15 **b)** 21 **c)** 20 **d)** 24

7. Answers may vary. **a)** 6 **b)** 24 **c)** 15 **d)** 12

9. Answers may vary. **a)** 15, 30 **b)** 12, 24

11. 12, 24, 36

13. Answers may vary. **a)** 10, $\frac{3}{5} = \frac{6}{10}$, $\frac{1}{2} = \frac{5}{10}$

b) 8, $\frac{5}{8}$, $\frac{1}{4} = \frac{2}{8}$

15. Answers may vary. For example, you could find common multiples of 2, 3, and 4. **a)** 12 or 24 **b)** 60

3.4 Add and Subtract Fractions Using a Common Denominator, pages 101–103

5. a) $\frac{2}{6} + \frac{3}{6} = \frac{5}{6}$ **b)** $\frac{1}{4} - \frac{2}{4} = \frac{3}{4}$ **c)** $\frac{2}{8} + \frac{3}{8} = \frac{5}{8}$

d) $\frac{1}{6} + \frac{3}{6} = \frac{4}{6}$

7. a) $\frac{3}{4}$ **b)** $\frac{4}{3}$ or $1\frac{1}{3}$ **c)** $\frac{4}{5}$

9. Answers may vary. **a)** $\frac{2}{10} + \frac{5}{10} = \frac{7}{10}$

b) $\frac{3}{12} + \frac{10}{12} = \frac{13}{12}$ or $1\frac{1}{12}$

c) $\frac{6}{15} + \frac{10}{15} = \frac{16}{15}$ or $1\frac{1}{15}$

d) $\frac{1}{6} + \frac{3}{6} = \frac{4}{6}$ or $\frac{2}{3}$

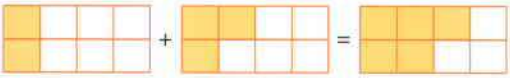
11. a) $\frac{2}{2} - 1$, $\frac{3}{3} = 1$, $\frac{4}{4} = 1$, $\frac{5}{5} = 1$

b) They all equal one. **c)** 1

13. Answers may vary. Total number of pieces: $8 \times \frac{1}{8} = 1$;
Number of pieces that did not fall out: $3 \times \frac{1}{8} = \frac{3}{8}$

15. a) Answers may vary. The sections being added are not the same size.

b)



17. $\frac{2}{5} + \frac{1}{2}$ is greater. $\frac{2}{5} + \frac{1}{2} = \frac{9}{10}$, $\frac{2}{3} + \frac{1}{6} = \frac{5}{6}$, $\frac{9}{10} > \frac{5}{6}$

19. a) $\frac{13}{12}$ or $1\frac{1}{12}$ **b)** $\frac{19}{20}$

21. Diagrams may vary. $1\frac{3}{4}$

23. $\frac{2}{5} + \frac{1}{4} + \frac{3}{10} = \frac{19}{20}$; Since $\frac{19}{20}$ is less than 1, the addition shows that together they did not clean all the windows. They should not be paid the full amount.

3.5 More Fraction Problems, pages 106–107

5. a) $\frac{17}{4}$ or $4\frac{1}{4}$ **b)** $\frac{17}{6}$ or $2\frac{5}{6}$ **c)** $\frac{27}{8}$ or $3\frac{3}{8}$

7. $\frac{5}{4}$ or $1\frac{1}{4}$

9. $\frac{7}{2}$ or $3\frac{1}{2}$

11. $\frac{3}{2} = 1\frac{1}{2}$, $\frac{8}{6} = 1\frac{2}{6}$ or $1\frac{1}{3}$

13. a) blue: $\frac{5}{16}$, purple: $\frac{3}{16}$, white: $\frac{2}{16}$ or $\frac{1}{8}$, green: $\frac{6}{16}$ or $\frac{3}{8}$ **b)** $\frac{11}{16}$ **c)** $\frac{13}{16}$

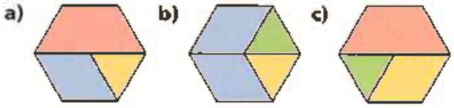
15. a) $\frac{1}{5}$ **b)** $\frac{1}{3}$ or $\frac{1}{7}$

19. 6

Review, pages 108–109

1. D **3.** H **5.** A **7.** G

9. Diagrams may vary.

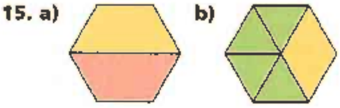


11. a)

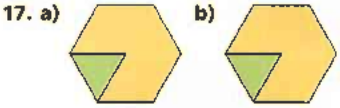
b) $\frac{1}{2} + \frac{1}{3} + \frac{1}{6} = 1$

13. a) $2 \times \frac{1}{3}$ **b)** $5 \times \frac{1}{6}$

15.



17.



19. Answers may vary. **a)** 12 **b)** 20

21. Answers may vary. **a)** 12 **b)** 10

23. a) $\frac{1}{2} - \frac{1}{3} = \frac{1}{6}$ **b)** $\frac{1}{3} + \frac{1}{6} + \frac{1}{6} = \frac{4}{6}$ or $\frac{2}{3}$

25. a) $\frac{7}{30}$ **b)** $\frac{13}{12}$ or $1\frac{1}{12}$

27. a) red: $\frac{6}{24}$ or $\frac{1}{4}$, blue: $\frac{4}{24}$ or $\frac{1}{6}$, white: $\frac{9}{24}$ or $\frac{3}{8}$, grey: $\frac{5}{24}$ **b)** $\frac{10}{24}$ or $\frac{5}{12}$ **c)** $\frac{15}{24}$ or $\frac{5}{8}$

Practice Test, pages 110–111

1. B **3.** B **5.** C

7. a) $4 \times \frac{1}{5} = \frac{4}{5}$ **b)** $5 \times \frac{2}{7} = \frac{10}{7}$ or $1\frac{3}{7}$

9. 12, 24

11. a) $1 - \frac{3}{10}$ **b)** Strategies may vary. You could subtract and compare your answers.

13. 16

CHAPTER 4

Get Ready, pages 114–115

1. a) 3 **b)** 165.6 cm **c)** 53.5 kg **d)** 15.2 mm **e)** 12.4 m **f)** 34.6 jellybeans

3. a, d

5. a) $\frac{8}{24}$ and $\frac{9}{24}$; $\frac{3}{8}$ **b)** $\frac{4}{10}$ and $\frac{5}{10}$; $\frac{1}{2}$ **c)** $\frac{20}{30}$ and $\frac{21}{30}$; $\frac{7}{10}$

7. a) yellow: $\frac{4}{8} = \frac{1}{2}$, blue: $\frac{3}{8}$, red: $\frac{1}{8}$ **b)** red, blue, yellow

9. a) 0.5 **b)** 0.375 **c)** 0.4

4.1 Introducing Probability, pages 118–120

5. Frequency: 13, 16, 11; Total trials: 40

7. a) $\frac{2}{7}$ **b)** $\frac{4}{400} = \frac{1}{100}$ **c)** $\frac{37}{40}$

9. a) It will stay the same. **b)** zippy zingers: $\frac{26}{50} = \frac{13}{25}$; stomach stirrers: $\frac{6}{50} = \frac{3}{25}$; tongue twisters: $\frac{4}{50} = \frac{2}{25}$; face freezers: $\frac{14}{50} = \frac{7}{25}$

11. a) $\frac{4}{13}$ **b)** tan; grey; The colour with the greatest number of pairs is tan. The colour with the least number of pairs is grey. **c)** The probabilities change because the number of favourable outcomes for each colour is different.

13. Answers may vary. She should conduct more trials or examine the spinner.

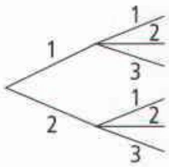
15. a) before: red = $\frac{7}{30}$, black: $\frac{6}{30} = \frac{1}{5}$, yellow: $\frac{4}{30} = \frac{2}{15}$, orange: $\frac{5}{30}$, $\frac{1}{6}$, green: $\frac{8}{30}$, $\frac{4}{15}$; after: red: $\frac{5}{20}$, $\frac{1}{4}$, black: $\frac{6}{20}$, $\frac{3}{10}$, yellow: $\frac{2}{20}$, $\frac{1}{10}$, orange: $\frac{4}{20}$, $\frac{1}{5}$, green: $\frac{3}{20}$ **b)** Answers may vary. She probably picked the colours she liked.

17. Answers will vary.

4.2 Organize Outcomes, pages 124–125

3. a) spinning red: $\frac{1}{3}$, spinning blue: $\frac{1}{3}$, spinning yellow: $\frac{1}{3}$ **b)** spinning red: $\frac{1}{6}$, spinning blue: $\frac{1}{6}$, spinning yellow: $\frac{1}{6}$, spinning pink: $\frac{1}{6}$, spinning orange: $\frac{1}{6}$, spinning brown: $\frac{1}{6}$

5. a)



b) $\frac{1}{6}$ **c)** $\frac{2}{6} = \frac{1}{3}$

7. Answers will vary.

9. $\frac{4}{6} = \frac{2}{3}$

13. a) Answers will vary. **b)** Answers may vary. Player 1 = $\frac{1}{2}$, Player 2 = $\frac{1}{2}$ **c)** Both players are equally likely to win because their number of favourable outcomes is equal.

4.3 Use Outcomes to Predict Probabilities, pages 128–130

5. a) $\frac{1}{40}$ **b)** $\frac{1}{40}$ **c)** $\frac{2}{40} = \frac{1}{20}$ **d)** $\frac{8}{40} = \frac{1}{5}$ **e)** $\frac{8}{40} = \frac{1}{5}$

7. a) 1, 2, 3, 4, 5, or 6 **b)** A, B, or C **c)** red, black, green, or yellow

9. H/white, H/yellow, H/green, T/white, T/yellow, T/green

11. a) $\frac{3}{6} = \frac{1}{2}$; 3 favourable outcomes, 6 possible outcomes **b)** $\frac{2}{6} = \frac{1}{3}$; 2 favourable outcomes, 6 possible outcomes

13. A, because it occurs most often.

15. a) Each side has a probability of $\frac{1}{8}$. **b)** 4, because it occurs most often.

19. $\frac{4}{8} = \frac{1}{2}$

4.4 Extension: Simulations, pages 132–133

3. a) 12 **b)** O, since it was only chosen once.

5. Answers may vary. Each item should have a choice for every possible outcome. **a)** a coin **b)** an eight-sided die **c)** a five-section spinner

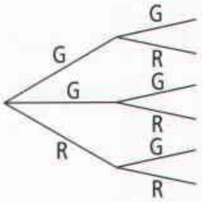
7. Answers will vary.

9. Answers may vary. **a)** chossing at random from five different types of pop **b)** choosing at random from one black, one green, and two red jellybeans. **c)** choosing at random from two hot dogs, two hamburgers, and two steaks

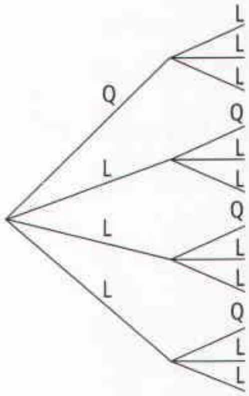
11. a) Answers may vary. You could collect the different letters in "WINNER" (need only one N) or you could spell "WINNER" (need two Ns). 1st way: $\frac{1}{5}$, 2nd way: $\frac{2}{6} = \frac{1}{3}$ **b)** Each side has a letter from "WINNER." This fits the second way. **c)** Answers will vary.

4.5 Apply Probability in Sports and Games, pages 137–139

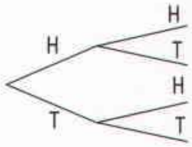
3. $\frac{2}{6} = \frac{1}{3}$



5. $\frac{6}{12} = \frac{1}{2}$



7. a)



Player 1	Player 2
1 point	0 points
1 point	0 points
1 point	0 points
0 points	2 points

b) No. *Heads* has an advantage.

9. a) $\frac{250}{1000} = \frac{1}{4}$ **b)** $\frac{750}{1000} = \frac{3}{4}$; 0.75 **d)** 6

11. a) $\frac{4}{10} = \frac{2}{5}$ **b)** $\frac{6}{10} = \frac{3}{5}$ **c)** 1, represents the probability of every possible outcome.

13. Answers will vary.

15. a) $\frac{6}{36} = \frac{1}{6}$ **b)** $\frac{30}{36} = \frac{5}{6}$ **c)** It is difficult to get out of jail by rolling doubles.

19. a) $\frac{1}{2}$ **b)** no **c)** $\frac{7}{8}$

Review, pages 140–141

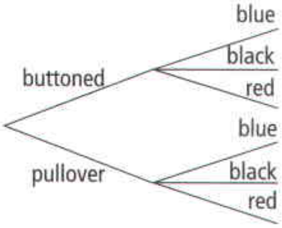
1. probab(i)lity, ta(l)ly chart, freque(n)cy table

3. (t)ree d(i)agra(m)

5. a) no **b)** $\frac{4}{16} = \frac{1}{4}$

7. a) most likely = orange, the most common colour; least likely = white, the least common colour **b)** There is a different number of marbles for each colour.

9. a)



b) 6 **c)** No, it depends on Connie's preference.

11. a) $\frac{3}{6} = \frac{1}{2}$; 3 favourable outcomes, 6 possible outcomes

b) $\frac{3}{12} = \frac{1}{4}$; 3 favourable outcomes, 12 possible outcomes

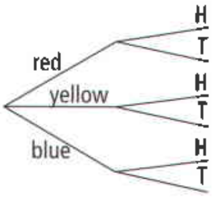
13. 6; probability of $\frac{1}{10}$

15. a) $\frac{9}{100}, \frac{42}{100} = \frac{21}{50}, \frac{49}{100}$ **b)** coloured; coloured squares have the highest probability

Practice Test, pages 142–143

1. A **3.** B **5.** C

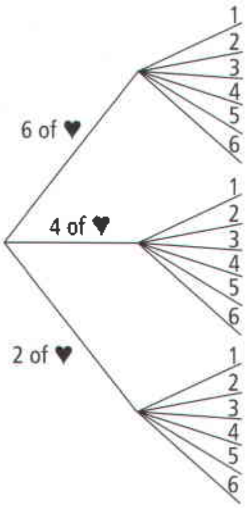
7. a)



b) red/H, red/T, blue/H, blue/T, yellow/H, yellow/T

c) $\frac{1}{6}$ **d)** $\frac{2}{6} = \frac{1}{3}$

9. a)



b) 6 of ♥/1, 6 of ♥/2, 6 of ♥/3, 6 of ♥/4, 6 of ♥/5, 6 of ♥/6, 4 of ♥/1, 4 of ♥/2, 4 of ♥/3, 4 of ♥/4, 4 of ♥/5, 4 of ♥/6, 2 of ♥/1, 2 of ♥/2, 2 of ♥/3, 2 of ♥/4, 2 of ♥/5, 2 of ♥/6

11. a) He should get two correct. **b)** Answers will vary.

c) 0 right = $\frac{1}{16}$; 1 right = $\frac{4}{16}$; 2 right = $\frac{6}{16}$; 3 right = $\frac{4}{16}$, 4 right = $\frac{1}{16}$ **d)** The number of trials is too small.

Chapters 1–4 Review, pages 146–147

1. a) 8 cm **b)** 6.8 cm

3. a) 23 **b)** 8 **c)** 9 **d)** 27 **e)** 15

5. Answers may vary. An example could have a = 3 cm, b = 6 cm, and h = 4 cm.

7. Diagrams may vary.

a) scalene triangle **b)** isosceles triangle

c) square or rhombus **d)** rectangle or parallelogram

9. a) $\frac{1}{2} + \frac{1}{6} = \frac{2}{3}$ **b)** $\frac{1}{3} + \frac{1}{6} = \frac{1}{2}$

11. a) 10 **b)** 12

13. a) $\frac{8}{24} = \frac{1}{3}$ **b)** $\frac{6}{24} = \frac{1}{4}$ **c)** 5 h, $\frac{5}{24}$

15. a) $\frac{1}{18}$ **b)** $\frac{2}{18} = \frac{1}{9}$ **c)** $\frac{3}{18} = \frac{1}{6}$ **d)** $\frac{6}{18} = \frac{1}{3}$

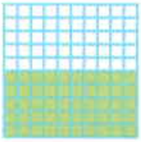
CHAPTER 5

Get Ready, pages 150–151

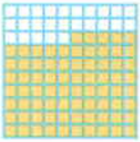
1. a) 0.3 **b)** 0.7 **c)** 0.01 **d)** 0.23

3. a) 0.2, 0.225, 0.25 **b)** 1.334, 1.34, 1.43

5. a)



b)



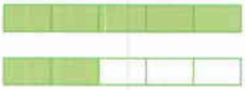
7. a) 43% **b)** 60% **c)** 5% **d)** 2%

5.1 Fractions and Decimals, pages 156–157

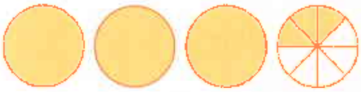
5. a)

b)

c)



d)



7. a) 1.301, 1.3, 0.34, 0.3 **b)** 0.489, 0.29, 0.2, 0.06

9. a) $\frac{1}{3}, \frac{4}{9}, \frac{1}{2}$ **b)** $1\frac{3}{8}, 1\frac{2}{3}, 1\frac{3}{4}, 1\frac{5}{6}$

11. a) $0.1\overline{6}$ **b)** $1.\overline{3}$ **c)** $3.\overline{6}$ **d)** $2.8\overline{3}$

13. $\frac{250}{1000}$ or $\frac{1}{4}$

15. a) $\frac{21}{27}, \frac{17}{20}$, 0.87, $\frac{23}{25}$, 1.04, $\frac{6}{5}$

b) $\frac{319}{25}$, 12.84, $12\frac{5}{6}$

17. a) $\frac{8}{8}$, $\frac{14}{16}$, $\frac{9}{12}$ **b)** Answers will vary.

19. a) 6.7, $6\frac{9}{20}$, $6\frac{2}{5}$, $6\frac{3}{8}$, $6\frac{1}{3}$, 6.05

b) Express all the numbers in decimal form or write them all in fraction form.

21. a) $\frac{3}{4}$ **b)** $\frac{4}{5}$ **c)** $\frac{1}{2}$ **d)** $\frac{13}{20}$

23. a) Yes, the digits begin to repeat on the calculator.
b) More digits of the number would aid in a decision.
c) Answers will vary, numbers like $\frac{4}{7}$ have sequences of numbers that repeat.

5.2 Calculate Percents, pages 160–161

5. a) 70% **b)** 85% **c)** 80%
7. Diagrams may vary. Use number lines, hundred charts, or circles.
9. 10
11. a) 75% **b)** Answers will vary. Think of a clock—45 min is $\frac{3}{4}$ of an hour (60 min). $\frac{3}{4}$ is 75%.
13. No, Amir mixed a percent with a score out of 100. He should say "I got 65 out of 100."
15. a) No, the decimal point should move 2 places to the right. **b)** Answers will vary.
17. a) Answers will vary. A minority government will have less than half of the 300 seats. For example, Liberal 140, Conservative 110, NDP 40, Bloc Quebecois 10.
b) Answers will vary. The party with the most seats will be the governing party. **c)** and **d)** Answers will vary.

5.3 Fractions, Decimals, and Percents, pages 164–165

3. a) $\frac{1}{4}$ **b)** $\frac{1}{2}$ **c)** $\frac{1}{10}$ d) $\frac{1}{5}$ **e)** $\frac{3}{4}$ **f)** 1
5. a) 0.32 **b)** 0.64 **c)** 0.7 **d)** 0.83 **e)** 0.05
7. $\frac{1}{2}$, $\frac{3}{4}$, $\frac{1}{10}$, $\frac{1}{4}$
9.

Fraction	Percent	Decimal
$\frac{3}{20}$	15%	0.15
$\frac{13}{20}$	65%	0.65
$\frac{1}{50}$	2%	0.02

11. A: 20, B: 17, C: 14
13. a) CD $\frac{3}{5}$, DVD $\frac{1}{5}$, Cassettes $\frac{3}{20}$, Other $\frac{1}{20}$
b) CD 0.60, DVD 0.20, Cassettes 0.15, Other 0.05
15. a) "Nearly two-thirds of children …" **b)** Answers will vary. You would need information related to the water supply in the developing world and census statistics for the same areas where you conducted your research.

17. a) 0.53 **b)** 424 **c)** 376
19. Answers will vary. For example, Winning percent = $\frac{\text{number of wins} \times 100\%}{\text{number of total games}}$

5.4 Apply Fractions, Decimals, and Percents, pages 169–171

5. a) 18.75% **b)** 65% **c)** $83.\overline{3}\%$ **d)** 68%
7. a) $\frac{2}{5}$ **b)** 0.4 **c)** 40%
9. a) 20% **b)** 19%
11. 40%
13. a) $\frac{39}{50}$ **b)** $\frac{21}{100}$ **c)** $\frac{1}{100}$
15. Percent of Total Sales: Koala Cola 26%, Lizard Lime 10%, Lemur Lemon 16%, Gorilla Grape 6%, Roary Root Beer 10%, Oliphant Orange 20%, Jumping Ginger 7%
Visuals may vary. A vertical bar graph with bottle shapes coloured appropriately to match each flavour would be good. The height of each bottle would show the sales of that flavour.
17. a) Yes. Cut at least 11.5 min and at most 25 min to get between 50% and 65% of 90 min.
19. a) $105 **b)** In Ontario the jacket costs $120.75.

Review, pages 172–173

1. st(a)tistic
3. (p)ercent
5. a)  **b)** 
c) 
7. a) $0.58\overline{3}$ **b)** $0.\overline{5}$ c) 0.548
9. a) 65% **b)** 40% **c)** 23.7% **d)** 0.8%
11. a) 0.33 **b)** 0.06 **c)** $0.\overline{7}$ **d)** $0.1\overline{2}$
13. a) 30% **b)** .300 **c)** 54
15. 88%, 0.88, $\frac{22}{25}$; 36%, 0.36, $\frac{9}{25}$; 20%, 0.2, $\frac{1}{5}$; 42%, 0.42, $\frac{21}{50}$; 4%, 0.04, $\frac{1}{25}$
17. a) Answers may vary. Engineering on shelf 1, Math on shelf 2, and Science and other on shelf 3. **b)** Between 54 (30%) and 63 (35%) books will be on each shelf. Assume that each book is about the same thickness and weight.

Practice Test, pages 174–175

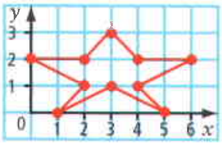
1. A **3.** C **5.** A
7. $\frac{7}{25}$, 0.28, 28%; $\frac{3}{8}$, 0.375, 37.5%; $\frac{9}{20}$, 0.45, 45%; $\frac{1}{20}$, 0.05, 5%; $\frac{24}{240}$, 0.1, 10%
9. a) Test, Lab, Quiz **b)** Answers will vary. Convert all the results into decimal form and then compare them.
11. a) Answers will vary. Blue is in the middle and may be easiest to hit. **b)** blue $\frac{9}{25}$, yellow $\frac{12}{25}$, red $\frac{4}{25}$
c) yellow, blue, red

CHAPTER 6

Get Ready, pages 178–179

1. a) Pattern adds a vertical paper clip, followed by a horizontal paper clip, alternating at the top and bottom. **b)** Increasing by 2 each term. 10, 12, 14 **c)** Decreasing by 5 each term. 80, 75, 70 **d)** Each term is double the previous term. 48, 96, 192 **e)** Add alternating yellow and green right triangles, placed to form connecting squares. **f)** Each denominator is double the previous denominator. $\frac{1}{16}, \frac{1}{32}, \frac{1}{64}$ **g)** Every three terms, the pattern is ab, abc, abcd, ab, abc, abcd **h)** Each step, one more happy face is added to the beginning of the pattern.

3. a)–c)



d) a five-pointed star

5. Ferris Wheel (0, 5), First Aid (2, 3), Washrooms (3, 2), Basketball Throw (4, 0), Battle of the Bands (7, 7), Petting Zoo (8, 1), Food Court (10, 6)

6.1 Investigate and Describe Patterns, page 182

3. Answers may vary. **a)** Starting at 1, multiply each counting number by 5. **b)** 30, 35, 40

5. a) 0, 0 + 6 = 6, 6 + 8 = 14, 14 + 10 = 24, 24 + 12 = 36, 36 + 14 = 50,... **b)** 66, 84

7. Answers will vary.

9. The top half of each triangle is cut horizontally in half. The new triangle formed alternates between a yellow and green colour.

11. A spiral pattern with rectangles 1 unit longer than the previous rectangle added in the pattern.

13. a) 5, 10, 15, 20, ... **b)** 3, 6, 12, 24, ... **c)** 18, 16, 14, 12, ... **d)** 400, 200, 100, 50, ...

15. a) 9, 11, 13, 15 **b)** Start with 9 and increase by 2. **c)** 17, 19, 21 **d)** Yes they will.

17. Answers may vary.

19. a) Start with an equilateral triangle pointing up and add a triangle pointing the opposite way of the previous triangle. **b)** A triangle is added at each stage, so the figure is growing larger. **c)**

21. a) Rows of dots, arranged in order of length 5, 4, 3, 2, 1. Then the longest row is removed. **b)**

23. No, there are many answers.

6.2 Organize, Extend, and Make Predictions, pages 187–189

3. a) 4, 8, 12, 16 **b)** Each perimeter is 4 times each natural number, starting at 1.

5. a) Each number is 5 times each natural number.
b) Each number is the square of each natural number.
c) The pattern is the odd natural numbers.
d) The pattern is $\frac{1}{\text{each natural number} + 1}$.

7. a)

Number of Cubes	Number of Horizontal Faces	Operation
1	2	1 × 2
4	8	4 × 2
9	18	9 × 2

b) $n \times 2$ **c)** 200

9. a) 8 **b)** 15 **c)** 103 **d)** $k + 3$

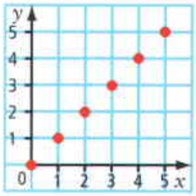
11. a) Starts at 4 and increases by 3. 4, 7, 10 **b)** 13
c)

d) 16 **e)** Answers may vary. Build a model.

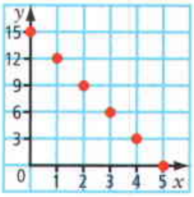
17. a) 602 **b)** 114

6.3 Explore Patterns on a Grid or in a Table of Values, pages 193–194

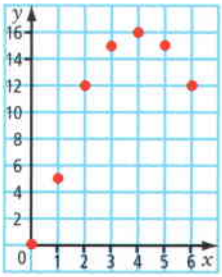
3. a) Points go up and to the right in a straight line.



b) Points go down and to the right in a straight line.



5. a)



b) The points form a hill, or upside-down U-shape. The other graphs were all straight lines.

7. length = 2 × width

9. The first design has 9 toothpicks. Each one after that has 3 toothpicks more than the one before it. The pattern rule is six plus three times the stage number, $6 + 3n$.

11. Start at (5, 7) and move in each direction: North (5, 7), (5, 8), (5, 9), (5, 10); North-East (5, 7), (6, 8), (7, 9), (8, 10); East (5, 7), (6, 7), (7, 7), (8, 7); South-East (5, 7), (6, 6), (7, 5), (8, 4); South (5, 7), (5, 6), (5, 5), (5, 4); South-West (5, 7), (4, 6), (3, 5), (2, 4); West (5, 7), (4, 7), (3, 7), (2, 7); North-West (5, 7), (4, 8), (3, 9), (2, 10)

15. a) The x-coordinate is 5 less than the y-coordinate. **b)** Answers may vary. **c)** $y = x + b$, $x = y - b$

6.4 Express Simple Relationships, pages 197–199

3. a) (1, 5), (2, 10), (3, 15), (4, 20); y-value is 5 times the x-value.
b) (0, 4), (1, 5), (2, 6), (3, 7) ; y-value is 4 more than the x-value.

5. a) (2, 3), (3, 4), (4, 5), (5, 6) **b)** The C-value is 1 more than the z-value. The cost (in dollars) is the number of zones + 1.

7. a)

x	y
0	0
1	2
2	4
3	6

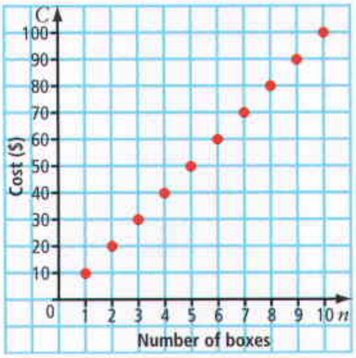
c) Each y-value is twice the x-value.

9. Answers may vary.

11. a)

Number of Boxes, n	Cost, C ($)
1	10
2	20
3	30
4	40
5	50
6	60
7	70
8	80
9	90
10	100

b)



c) $C = 10n$ **d)** $150

13. a) Each y-value is 1 less than the x-values. $y = x - 1$ **b)** Each y-value is 6 times the x-value. $y = 6x$ **c)** The x-values are always 4 greater than the y-values. $y = x - 4$ **d)** Each y-value is 3 less than the x-value. $y = x - 3$

15. a)

Side Length, s (cm)	Area, A, (cm^2)
1	1
2	4
3	9
4	16
5	25
6	36

b) Area is the square of the side length.

Review, pages 200–201

1. b) natural numbers

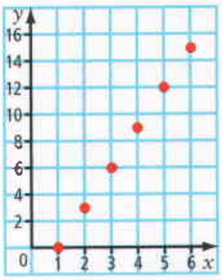
3. a) relationship

5. a) variable expression **b)** ordered pair **c)** variable **d)** algebraic equation

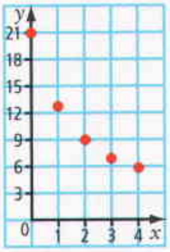
7. a) Start with an equilateral triangle. Join the midpoint of each side. This makes 4 congruent triangles. Colour the centre triangle. Repeat the process to divide each small white triangle into 4 smaller triangles. This process can continue indefinitely.

9. a) 9 cm **b)** 17 cm **c)** $n + (n - 1)$ cm **d)** 35 cm

11. a)



b)



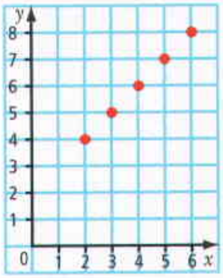
13. a) (1, 3), (2, 6), (3, 9), (4, 12) **b)** Each y-value is 3 times the x-value.

15. a) (1, 13), (2, 26), (3, 39), (4, 52), (5, 65), (6, 78) **b)** The number of computers sold equals 13 times the month number. **c)** 91

Practice Test, pages 202–203

1. A **3.** B **5.** B

7. a)

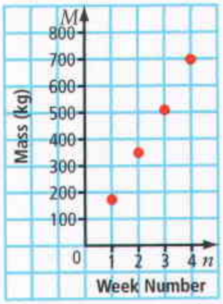


b) Each y-value is 2 more than the x-value. **c)** $y = x + 2$ **d)** 17

9. a) 4 **b)** 1 **c)** 5

11. a) diagram 1: 4 unit squares + 1 two by two square; diagram 2: 9 unit squares + 4 two by two squares + 1 three by three square. Extending the pattern, in diagram 3: 1 + 4 + 9 + 16 = 30. **b)** 30

13. a)



b) There is an increase of 175 kg of paper collected for each week. **c)** Assume 44 weeks in a school year. 7700 kg

CHAPTER 7

Get Ready, pages 208–209

1. a) 1, 2, 4, 8 **b)** 1, 17 **c)** 1, 2, 3, 4, 6, 8, 12, 24

3. a) 4, 8, 12, 16 **b)** 8, 16, 24, 32 **c)** 6, 12, 18, 24
5. a) $13 < 14$ **b)** $13.6 > 13.5$ **c)** $8 \times 3 = 2 \times 12$
7. a) 2000 m **b)** 2 m **c)** 300 mm
9. 64 m^2
11. 27 cm^3

7.1 Understand Exponents, pages 212–213

5. a) 36 square units **b)** 144 square units
c) 121 square units
7. a) 125 cubic units **b)** 1728 cubic units
c) 8000 cubic units
9. a) 9×9 **b)** $7 \times 7 \times 7$ **c)** $12 \times 12 \times 12$
11. a) 1.69 **b)** 5.76 **c)** 16.81 **d)** 1.728 **e)** 32.768 **f)** 15.625
13. $10 \times 10 \times 10$, 25×25, 8^3, 20^2
15. Because the units are squared along with the number.
17. 96 cm^2
19. a) 1^2 , 3^2 , 6^2 , 10^2 , … or 1^2, $(1 + 2)^2$, $(1 + 2 + 3)^2$, $(1 + 2 + 3 + 4)^2$, …
b) 2^3, 3^3, 4^3, 5^3, … or $(3 - 1)^3$, $(6 - 3)^3$, $(10 - 6)^3$, $(15 - 10)^3$, … Each number in the sequence is the next natural number cubed.

7.2 Represent and Evaluate Square Roots, pages 216–217

5. a) 3 **b)** 5
7. a) 1.3 cm **b)** 3.5 m **c)** 0.2 mm **d)** 1.4 cm
9. a) 8 **b)** 12 **c)** 20
11. a) 1.2 **b)** 1.5 **c)** 2.4 **d)** 0.5
13. Answers may vary. **a)** Enter 81. Press the square root key.
15. a) 12 m **b)** 48 cm
17. $\sqrt{41}, \sqrt{38}, \sqrt{45}$
21. You might try systematic trial. **a)** 3 **b)** 5 **c)** 100

7.3 Understand the Use of Exponents, pages 221–223

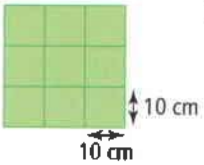
5. a) base: 2, exp: 4 **b)** base: 1, exp: 6 **c)** base: 4, exp: 3
7. a) 32 **b)** 1 **c)** 216
9. a) 4^6 **b)** 9^3 **c)** 2^8
11. a) 3^6 **b)** 5^4 **c)** 9^7
13. a) 0.000 32 **b)** 3.8416 **c)** 0.000 729
15. a) 4^3 **b)** 4^4 **c)** 4^1
17. 19.5, 18^1 , 17, 2^4, $\sqrt{225}$, 1^{18}
19. a) Notice that $10^3 = 1000$, and $10^6 = 1\ 000\ 000$, the number of zeros is the same as the exponent. **b)** 100
21. 81, 729

7.4 Fermi Problems, page 227

3. A loonie has a diameter of 2.6 cm. A square of side length 2.6 cm has area of approximately 7 cm^2. For a classroom 11m by 10 m about 160 000 loonies are needed.
5. Estimate that there are 200 words on a page, so on about 500 pages there are 100 000 words.
7. Assume the football field is 60 m by 136 m, and the textbooks are 20 cm by 25 cm. This means you need approximately 163 000 textbooks.
9. If the flat bag is 100 cm by 60 cm, it has two sides so the total area is 12 000 cm^2. This gives a cube of side about 45 cm and volume about 91 000 cm^3. Approximate the banana by a rectangular prism 4 cm by 4 cm by 20 cm. Its volume is 320 cm^3. So, about 280 bananas fit in the bag.

Review, pages 228–229

1. POWER
3. BASE
5. SQUARE ROOT
7. a) 9 square units **b)** 36 square units
9. a) 256 **b)** 1.69 **c)** 512 **d)** 1331
11. a) $3^3 > 5^2$ **b)** $14^2 < 6^3$ **c)** $3.2^2 < 2.2^3$
13. a)



b) 100 cm^2
15. $3 \times 3 = 9$
17. 120 cm
19. 124 cm
21. a) 5^4 **b)** 10^9 **c)** 3^5
23. a) 1024 **b)** 1 **c)** 0.0001
25. $2^8 = 256$
27. Answers may vary. Assume the basketball court is 15 m × 29 m, and a phonebook is 21.5 cm by 28 cm. You need approximately 7000 phonebooks.
29. Answers may vary. **a)** Assume you receive 5 coins in change a day. This means you will receive approximately 1825 coins in a year. **b)** Count the number of mixed coins needed to fill a small box. Divide 1825 by this number of coins. Multiply this answer by the volume of the box.

Practice Test, pages 230–232

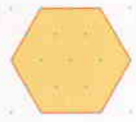
1. D **3.** B **5.** A
7. a) 64 cubic units **b)** 512 cubic units
9. a) 8 **b)** 20 **c)** 1.2 **d)** 1.5
11. a) $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 512$
b) $3 \times 3 \times 3 \times 3 \times 3 \times 3 = 729$
c) $4 \times 4 \times 4 \times 4 \times 4 = 1024$
13. 72 cm^3
15. a) Yes, 2^6 is greater than 2^5 because when a number is multiplied by 2 it gets bigger.
b) No, $1^6 = 1^5$ because any number times 1 equals itself.
17. 20 cm
19. Answers may vary. **a)** Estimate the distance from Earth to the moon and your average walking speed, then use these to find the time to get to the moon. **b)** Estimate the volume of a cell phone, then estimate the volume of a backpack, then use these to estimate the number of cell phones that would fit in the backpack. **c)** Estimate how many soft drinks one student drinks in a day, then multiply by the number of students and the number of days in a year.

CHAPTER 8

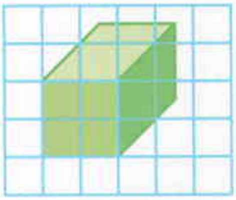
Get Ready, pages 234–235

1. a)

b)

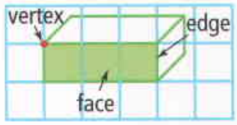


3. a)



b)

5.



7. $A = l \times w$ **a)** 15 m^2 **b)** 32 cm^2

9. 1 cm by 64 cm, 2 cm by 32 cm, 4 cm by 16 cm, and 8 cm by 8 cm

8.1 Explore Three-Dimensional Figures, pages 239–241

3. a) rectangular prism **b)** square-based pyramid **c)** triangular prism **d)** cylinder

5. Answers may vary. **a)** sugar cube **b)** hockey puck **c)** brick **d)** skateboard ramp **e)** tent **f)** globe **g)** skyscraper

7. a) one hexagon and six triangles **b)** hexagonal pyramid

9. a) Answers may vary. AB = AC = DE = DF **b)** Answers may vary. △ABC and △DEF are congruent, ACFD is congruent to ABED. **c)** triangular prism

11. A sphere, cylinder, and rectangular prism. Answers may vary. Drinking mugs can be a cylinder or hexagonal prism.

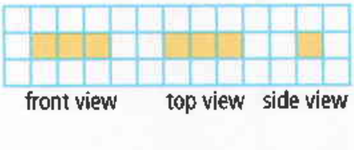
13. a) cube, any prism, or any pyramid except for a triangular pyramid **b)** cylinder **c)** triangular prism **d)** cube, triangular pyramid (tetrahedron), octahedron, icosahedron, dodecahedron **e)** sphere

15. Rectangles have four right angles, opposite sides with equal length, and two pairs of opposite sides that are parallel. Squares have four right angles, all four sides with equal length, and two pairs of opposite sides that are parallel. Since a square fits all the descriptions of a rectangle, it can be thought of as a rectangle with the same length and width. So, a rectangular prism can also be thought of as a square-based prism.

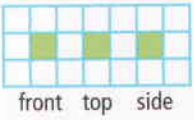
17. Answers may vary.

8.2 Sketch Front, Top, and Side Views, page 245–246

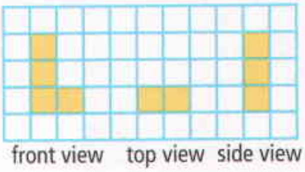
3. a)



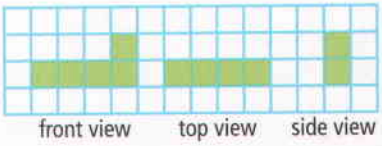
b)



5. a)



b)



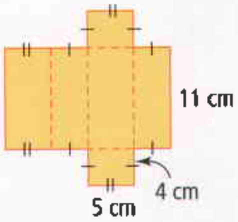
7. a) Diagrams may vary. **b)** Answers will vary.

9.

11. a)–d) Answers may vary. **e)** yes

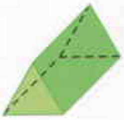
8.3 Draw and Construct Three-Dimensional Figures Using Nets, pages 249–251

3. Answers may vary.



5. rectangular prism

7. a)

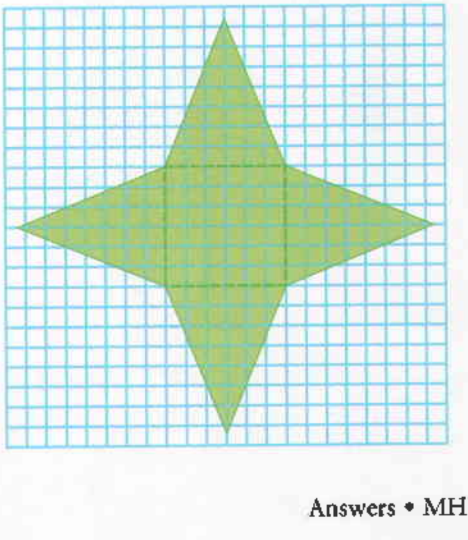


b) triangular prism

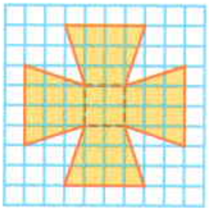
c) Answers may vary. A Toblerone chocolate bar.

9. triangular pyramid

11.



13. a) trapezoid **b)**



8.4 Surface Area of a Rectangular Prism, pages 255–257

3. 16 cm^2

5. a) 230 cm^2 **b)** 412 cm^2

7. 1000 cm^2

9. 532 cm^2

11. 1.198 m^2

13. cube; S.A. = $6s^2$

15. No. You can only form a 2 by 4 by 1 prism, a 1 by 8 by 1 prism, or a 2 by 2 by 2 prism. Their surface areas, in square units, are 28, 34, and 24 respectively.

19. 456 cm^2

8.5 Volume of a Rectangular Prism, pages 260–261

3. a) 175 cm^3 **b)** 240 cm^3

5. a) 288 cm^3 **b)** 3600 m^3

7. B, because washing machines are close to 1 m by 1 m by 1 m cubes.

9. 4320 cm^3

11. a) 30 000 cm^3 **b)** 15 L

13. 4000 cakes

15. 11.5 cm

Review, pages 262–263

1. three, polygons

3. pyramid

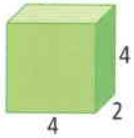
5. space

7. a) square-based pyramid **b)** cube

c) pentagonal pyramid or triangular prism

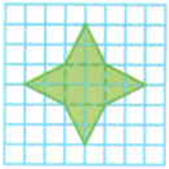
9. a) handle and wheels are cylinders, wagon box and handle pole are rectangular prisms **b)** Answers may vary.

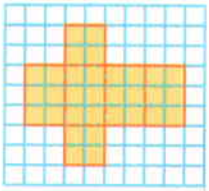
11.



13. Answers may vary.

a) **b)**





15. 1192 cm^2

17. 4500 cm^3

19. No. The volume of drawer is 107 250 cm^3 but the volume of the suitcase is only 90 000 cm^3.

Practice Test, page xxx

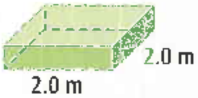
1. C **3.** D

5.

7. 972 cm^2

9. a) **b)** no

11. 0.4 m; 2 × 2 × 0.4 = 1.6



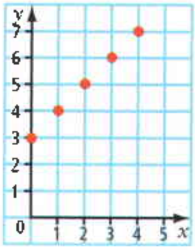
Chapters 5–8 Review, pages 268–269

1. a) 70%, 75% **b)** science test

3. 25%

5. a) 6, 10, 14, 18 **b)** 45, 34, 23, 12 **c)** 7, 14, 28, 56 **d)** 128, 64, 32, 16

7. a) **b)** (5, 8)



9. a) 3^4 **b)** 8^3

11. a) 5 **b)** 7 **c)** 30

13. a) 16 **b)** 144

15. a) rectangular prism **b)** cylinder **c)** triangular pyramid **d)** cube

17. a) 952 cm^2 **b)** 1760 cm^3

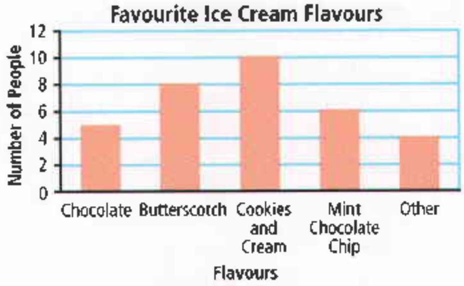
CHAPTER 9

Get Ready, pages 272–273

1. a) 4 **b)** 6 **c)** golf; 2 people

3. Answers will vary.

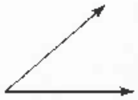
5.



Favourite Ice Cream Flavours	
Chocolate	☺ ☺ ◖
Butterscotch	☺ ☺ ☺ ☺
Cookies and Cream	☺ ☺ ☺ ☺ ☺
Mint Chocolate Chip	☺ ☺ ☺
Other	☺ ☺

☺ represents 2 people

7. a)



b)

c)

9. a) 90° **b)** 120°

9.1 Collect and Organize Data, pages 277–279

5.

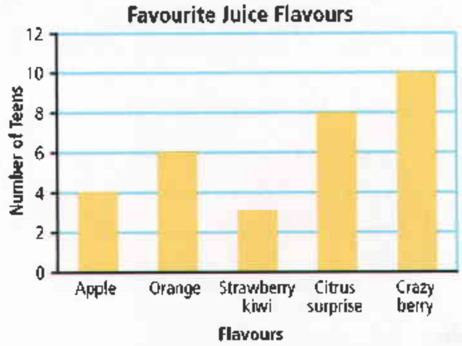
Favourite Insect	Tally	Frequency
Butterfly	𝍸 𝍸	10
Spider	𝍸	5
Fly	I	1
Ant	IIII	4

7. a) five **b)**

Method	Tally	Frequency
Bus	𝍸 𝍸 IIII	14
Car	𝍸 III	8
Bike	𝍸 III	8
Walk	IIII	4
Other	II	2

9. a) Frequency: 4, 6, 3, 8, 10

b)



c) Crazy berry **d)** Strawberry-kiwi **e)** 31

11.

Favourite Ice Cream Flavours	
Chocolate	🍦 🍦 🍦 🍦
Strawberry	🍦 🍦 🍦
Cookies and Cream	🍦 🍦 🍦 🍦 🍦
Bubble Gum	🍦 🍦
Vanilla	🍦

🍦 represents 2 people

13. a) primary **b)** secondary **c)** primary **d)** secondary

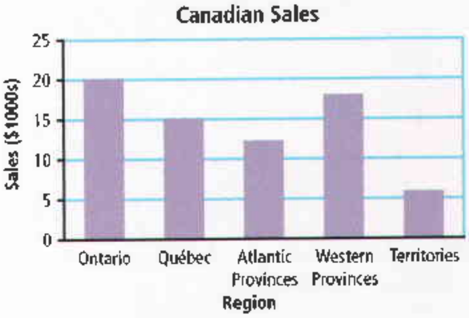
15. a) Since each number is equally likely to be rolled, each number would probably occur about the same number of times. **b) c) d)** Answers will vary.

17. Answers will vary.

19. Answers will vary. Primary data consists of information you collect by surveying or counting. Secondary data consists of information obtained from other sources.

21.

Region	Sales ($1000s)
Ontario	20
Québec	15
Atlantic Provinces	12
Western Provinces	18
Territories	6
TOTAL	71



9.2 Stem-and-Leaf Plots, pages 283–285

3. a) four **b)** 1, 3 **c)** 2 **d)** 14, 17, 20, 23, 23, 28, 31, 35, 42

5. a) two children, ages 7 and 9 **b)** three teenagers, ages 13, 13, 17 **c)** 58, 59, 62, 63

7. a) tens **b)** ones **c)**

Stem (tens)	Leaf (ones)
1	0 2 4 8
2	1 2 5
3	2 4 6
4	7 7

d) Answers will vary.

9. a)

Stem (tens)	Leaf (ones)
5	9
6	4 6 7
7	3 3
8	1 2

b) $73 **c)** $565

11. a)

Stem (tens)	Leaf (ones)
4	5 9
5	3 7
6	1 8
7	3 4 7
8	0 0 2 5 5 5 8
9	0 2 5
10	3

b) three **c)** 85

13. a)

Stem (ones)	Leaf (tenths)
0	7 8 8
1	0 0 2 3 4

b) The stem represents the ones digit, and the leaf represents the tenths digit. **c)** 0.7 g

15. a) 11 **b)** \$290 **c)** \$348 **d)** \$3303 **e)** \$337; no, one employee is paid \$348.

9.3 Circle Graphs, pages 290–291

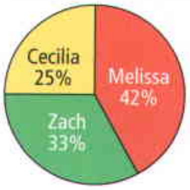
5. a)

b)

7. a)

Person	Hours	Fraction	Decimal	Section Angle
Melissa	5	$\frac{5}{12}$	$0.41\overline{6}$	150°
Zach	4	$\frac{4}{12}$	$0.\overline{3}$	120°
Cecilia	3	$\frac{3}{12}$	0.25	90°
TOTAL	12			360°

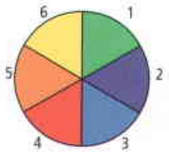
b) **Science Project Work**



9. a) meat and alternatives **b)** 400 g **c)** 200 g

11. a) squirrels; $\frac{1}{4}$ **b)** Squirrels, chipmunks, and raccoons.

13. **Probability of Rolling Each Number on a Number Cube**



15. Answers will vary.

9.4 Use Databases to Find Data, pages 295–297

5. a) alphabetically by type of business **b)** business name, location, phone number, sometimes advertising of services offered

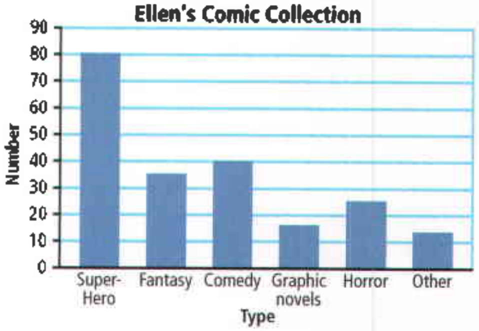
7. – 19. Answers will vary.

9.5 Use a Spreadsheet to Display Data, pages 302–303

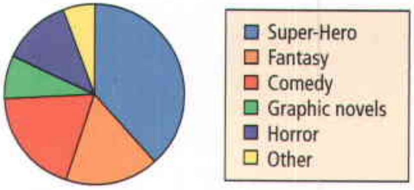
5. The bar graph clearly identifies the top students. In the pie graph, it is difficult to tell which student has the highest standing because the sections are all very close in size.

7. The bar graph clearly shows how Marisa spent her \$50 because each bar identifies exactly how much was spent on each item. The pie graph only gives a rough idea of what portion of the whole \$50 was spent on each item.

9. b) **Ellen's Comic Collection**



Ellen's Comic Collection



c) Answers will vary. The pie graph is a good choice because it compares the number of comic books of each type with the total number of comics Ellen owns. The bar graph is a good choice because the bars display exactly how many comic books of each type Ellen owns.

11. Answers will vary. A database is any organized collection of information. A spreadsheet is a software tool used for organizing and displaying numeric data

13. Answers will vary.

Review, pages 304–305

1. database, secondary data

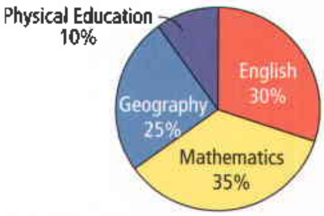
3. frequency table, primary data

5. pie chart

7. a) 60 **b)** O'Connor; about 75 **d)** about 47; Ziffareto has about 28 hours and O'Connor has about 75.

9. a) 34 **b)** 43; The highest number in the data set is 43. **c)** 15

11. **Drago's Study Time**



13. a) 1, B1 **b)** 25; Add up the numbers in the B column.

15. a) Answers will vary. A spreadsheet is just one way of organizing data in a database. A database is any collection of information. **b)** and **c)** Answers will vary.

Practice Test, pages 306–307

1. D **3.** D **5.** B

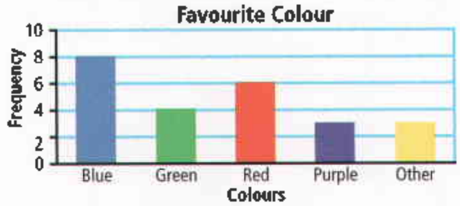
7. a)

Stem (tens)	Leaf (ones)
1	9
2	2 4 5 7 7 7 8 9
3	0 1 1 3 4

b) 27; three

9. a) Frequency: 8, 4, 6, 3, 3

b)



c)

d) Answers will vary. Both charts display the data well. The best one to use depends on what you want the graph to show. If you want the graph to show how each colour compares to the whole then use the pie chart. If you want to compare the colours to each other and know their exact values, then use the bar graph.

CHAPTER 10

Get Ready, pages 310–311

1. a) 7, 4, 4, 3 **b)** play a sport **c)** other

3. a) Population is increasing. **b)** 7 years **c)** Answers may vary. Since it has increased for 7 years, it will probably keep increasing.

5. a) 12 **b)** 34.29 **c)** 70 **d)** 2.2

10.1 Analyse Data and Make Inferences, pages 315–317

3. Answers may vary. You can see the most common, least common, and range of temperature.

5. a) It increases, stays the same, then decreases.
b) It stays the same, decreases, then stays the same.

7. a) Moe and Sable 2 weeks, Lucky 3 weeks **b)** Lucky **c)** Moe

9. The heights are between 153 cm and 157 cm. 155 cm is the most common height.

11. a)

Earnings ($)	Tally	Frequency
18	|	1
19		0
20	||||	4
21		0
22	|	1
23		0
24	|	1

b) Dale earns between $18 and $24 each week. Dale earns $20 most often.

13. a) Its position increases, stays the same, then decreases. **b)** Week 3 **c)** 2 weeks **d)** Number 1 is the highest (best) position.

15. a) Riverside: The population is increasing. Short Branch: The population stayed the same and is now decreasing. **b)** Short Branch **c)** Riverside **d)** 2001 **e)** Answers may vary. Riverside: about 1600, Short Branch: about 700.

17. Answers will vary.

10.2 Measures of Central Tendency, pages 322–325

3. Diagrams may vary. **a)** median = 4, mode = 4, mean = 5 **b)** median = 10, mode = 11, mean = 9 **c)** median = 5, mode = 5, mean = 6

5. a) Diagrams may vary. **b)** median = 7, mode = 7, mean = 6.5; median: half the girls shoot worse and half shoot better than 7 baskets out of 10, mode: most girls sink 7 baskets out of 10, mean: on average, a girl will sink 6.5 (about 6 or 7) baskets out of 10.

7. a) median = 47, mode = 41 **b)** median = 76, mode = 83

9. a)

Stem (tens)	Leaf (ones)
3	0 5 7
4	2 5 7

b)

Stem (tens)	Leaf (ones)
4	1 1 9
5	2 3
6	3 7

11. a)

Stem (tens)	Leaf (ones)
5	5 8
6	2 3 4 5 6 8
7	0 1 3 3 3 5 6
8	2 3 4 8
9	2

b) median = 72, mode = 73, mean = 72.05 **c)** Any one can be used because they are very close in value.

13. a) median = 36, mode = 36, mean = 36.7
b) The mode is most important since those are the jackets sold most often.

15. a) 39.25 **b)** 43 **c)** Answers may vary.

10.3 Bias, pages 328–330

3. Since the students might not want to hurt Wes' feelings, the results might contain bias.

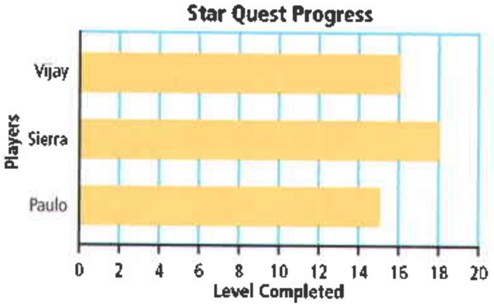
5. The question says "Do you *really* like…", and Faye is asking, so the results might contain bias.

7. a) cat and dog **b)** Since cat and dog are listed, it is easier to answer them.

9. Yes; students would probably not want to do homework.
11. a) No; it only lists a few groups. **b)** No; people will have a different definition of what *rocks*. **c)** Answers may vary. "What is your favourite rock band?", with no choices given.
13. a) contains bias; Answers may vary. "Do you think the manager should be fired?" **b)** no bias **c)** contains bias; Answers may vary. "Who is the most famous Prime Minister of all time?", with no choices given.
15. a) It only lists a few shows. **b)** Answers may vary. If a broadcasting station's shows are listed, it makes them seem more popular than shows that are not listed.
c) Answers may vary. "What is your favourite T.V. show?", with no choices given.
17. Answers will vary.
19. Answers will vary.

10.4 Evaluate Arguments Based on Data, pages 333–335

3. a) The first graph's scale starts at 0 and its scale goes up by 5s. The second graph's scale starts at 20 and goes up by 4s. **b)** The second graph make the price increase look greater than on the first.
5. a) The first graph's scale starts at 15 and increases by 1s. The second graph's scale starts at 0 and increases by 4s. **b)** The first graph make the difference in raked leaves look greater than on the second.
7.



9. a) The first graph compares the ratings of the two shows for year 4. The second graph compares the ratings of the two shows over a 4-year period. **b)** The first graph because it makes the ratings look much better for Happy Times. **c)** The second graph because it shows how Buddies' ratings are increasing and Happy Times' ratings are decreasing.
11. No. The scale starts at 0 and is evenly spaced.
13. No. The scale starts at 0 and is evenly spaced.

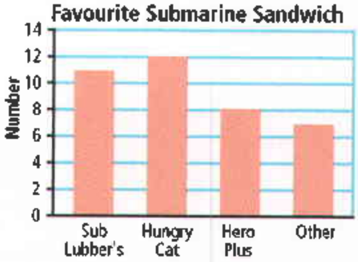
Review, pages 336–337

1. median
3. bias
5. mode
7. Forrest Hills Collegiate; it is decreasing, but should stay above the other two schools

9. a)

Tomatoes	Tally	Frequency
6	I	1
7	I	1
8	IIII	4
9	II	2
10	I	1
11	I	1

b) Oswald picks between 6 and 11 tomatoes from each plant. The plants produce 8 tomatoes the most often.
11. a) mean 10, median 13, mode 3; mean: the average number of appointments per day is 10, median: the dentist is busier half of the time, and less busy half of the time than 13 appointments, mode: the most common number of appointments on each day is 3. **b)** Answers may vary. Since there are usually many more than 3 appointments, either the mean or median can be used.
13. a) It assumes that *Big Barney Burger* is good and popular. **b)** The owner of *Big Barney Burger*. **c)** Answers may vary. "What is your favourite fast-food burger restaurant?"
15. a) It only lists a few breakfast foods. **b)** Answers may vary. "What is your favourite breakfast food?", with no choices given.
17. a) Frequency: 11, 12, 8, 7 **b)** Graphs may vary.
c) Graphs may vary.
d) Answers may vary. The owner might use the bias graph to makes Hungry Cat look more popular.



Practice Test, pages 338–339

1. D **3.** A
5. a) Hawks: decreasing; Dancers: increasing, decreasing, then increasing **b)** Answers may vary. Hawks about 600, Dancers about 800
7. a) Yes; the difference in the graph size exaggerates the difference in price by using a vertical scale that does not start at 0. **b)** People will think that Bubbles pop is much cheaper.
9.

Test Score	Tally	Frequency
64	I	1
66	I	1
68	I	1
72	I	1
73	II	2
74	I	1
75	III	3
80	I	1
81	I	1

b) 73 **c)** median = 73.5, mode = 75

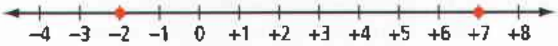
CHAPTER 11

Get Ready, pages 344–345

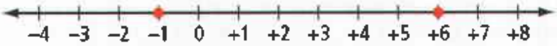
1. a) –6 **b)** –5 **c)** 0 **d)** +2
3. A –6, B –3, C –1, D 0, E +4, F +6
5. a) 12 < 15 **b)** 32 > 23 **c)** 20 > 0 **d)** 33 < 42
e) 29 < 30 **f)** 4 < 40
7. a) 3 **b)** 30 **c)** 25 **d)** 8.4
9. a) 3 **b)** 12 **c)** 50 **d)** 60

11.1 Compare and Order Integers, pages 349–351

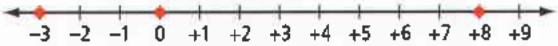
5. a) +1, +5 **b)** –5, –2 **c)** –5, –2, +1, +5 **d)** –5, +5
7. a) –5°C, 0°C, 8°C, 15°C, 20°C
b) –30°C, –21°C, 0°C, 8°C, 12°C, 17°C
c) –8°C, –2°C, –1°C, 11°C, 19°C, 32°C
9. a) –2, +1 **b)** –9, 0, +5, +11
11. a) –30 **b)** –12 **c)** –5 **d)** –7 **e)** –43 **f)** –14
13. a) –2, +7



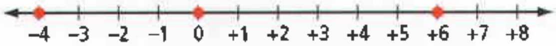
b) –1, +6



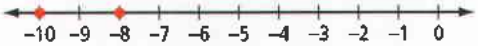
c) +8, –3, 0



d) 0, +6, –4



15. Answers may vary. **a)** –10°C today, –8°C yesterday
b) –10 < –8



17. a) The 3 means the third game. The 0 is his plus/minus rating. **b)** In the first game, Jake had a plus/minus rating of –1. **c)** The ordered pairs (6, 2), (7, 2), and (8, 1) represent games 6, 7, and 8. Jake can say, "In the last three games, I have improved to a positive plus/minus rating."
19. b) February 3, 5, 6, and 7 **c)** February 1, 2, 3, and 4 **d)** It was 6°C colder. **e)** –3°C on day 3 **f)** The trend was toward colder temperatures.

11.2 Explore Integer Addition, pages 354–355

3. a) (+2) + (–2) = 0 **b)** (+5) + (–5) = 0
5. a) (+3) + (–5) = –2 **b)** (–6) + (+3) = –3
7. a) (+3) + (–4) = –1 **b)** (+2) + (–1) = +1
c) (–7) + (+4) = –3
9. a) 0 **b)** 0 **c)** 0 **d)** 0; This happens because the integers in each pair are opposite. Opposite integers add to 0.

11.3 Adding Integers, pages 359–361

5. a) (+2) + (+2) = +4 **b)** (–1) + (–3) = –4
c) (–2) + (+4) = +2 **d)** (+1) + (–4) = –3
7. a) +9 **b)** –8 **c)** –6 **d)** –3 **e)** 0 **f)** +6
9. a) positive **b)** 0 **c)** positive **d)** negative **e)** negative
11. a) +16 **b)** 0 **c)** 0 **d)** +2 **e)** –40 **f)** –5
13. a) (–20) + (+15) = –5 **b)** (+89) + (–95) = –6
c) (–3) + (+10) = +7 **d)** (+83) + (–23) = +60
15. Answers may vary. (–1) + (–2) = –3; (+2) + (–5) = –3; (+3) + (–6) = –3
17. a) (+12) + (–15) = –3 **b)** (+25) + (–32) = –7
c) (+18) + (–11) = +7
19. a) +21 **b)** –6 **c)** –2 **d)** –11 **e)** –20
23. a)

3	–4	1
–2	0	2
–1	4	–3

b)

0	–7	–2
–5	–3	–1
–4	1	–6

11.4 Explore Integer Subtraction, pages 366–367

3. a) (–5) – (–3) = –2 **b)** (+3) – (+2) = +1 **c)** (–4) – (–4) = 0
5. a) +4 **b)** –2 **c)** –12 **d)** +5
7. a) –2 **b)** –4 **c)** –7 **d)** –8 **e)** +9 **f)** –3
9. a) +7 **b)** +4 **c)** +13 **d)** –4 **e)** –5 **f)** 0
11. Start with –3
🔵🔵🔵
Add two zero pairs
🔵🔵🔵🔵🔵
🔴🔴
Take away –5
🔴🔴
The answer is +2

Start with –5
🔵🔵🔵🔵🔵
Take away –3
🔵🔵
The answer is –2

13. a) Vancouver 9°C, Edmonton 5°C, Ottawa 8°C, Trois-Rivières 7°C, Fredericton 2°C, Saint John 7°C.
b) Vancouver **c)** Fredericton
15. Answers may vary. **a)** (–3) – (+4) – (–5) + (+7) = +5
b) (–3) + (–5) – (+4) – (+7) = –19

11.5 Extension: Subtracting Integers, pages 371–373

5. a) +7 **b)** (+5) + (+2) = +7
7. a) +4 **b)** –7 **c)** –14 **d)** –17 **e)** +12 **f)** +20 **g)** +4 **h)** –5
9. a) –1 **b)** –14 **c)** –4 **d)** –1 **e)** –8 **f)** –6 **g)** –1 **h)** +9
11. a) +3 **b)** –5 **c)** +12 **d)** +16 **e)** –4 **f)** –4 **g)** +1 **h)** –5
13. a) (+8) – (+15) = –7 **b)** (–8) – (+15) = –23
c) (+4) – (+7) = –3 **d)** (–5) – (+4) = –9
e) (–100) – (+600) = –700 **f)** (–20) – (–5) = –15
15. a) +8 **b)** –12 **c)** –30 **d)** –20
21. a) $100 **b)** Answers may vary. You forgot the minus sign.

11.6 Integers Using a Calculator, pages 376–377

3. (–11) + 23 + 79 + (–18) = 23 + 79 – 11 – 18; Answer: 73
5. a) +1 **b)** +16 **c)** –5 **d)** 0
7. a) +233 **b)** –40 **c)** –57 **d)** –252 **e)** –196 **f)** 232
9. +1, –1, +2, –2, +3, –3, … ; You obtain an alternating sequence of positive and negative integers.
11. 10 under par
13. a) (–5) + (–4) + (–3) = –12 **b)** (–11) + (–10) + (–9) = –30
c) (–1) + (0) + (+1) = 0
15.

1	–6	–1
–4	–2	0
–3	2	–5

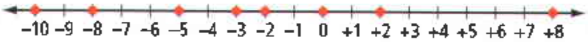
17. a) –26°C **b)** 108°C
19. a) 2 – 3 + 4 = 3 **b)** 2 + 4 – 3 = 3 **c)** 4 + 2 – 3 = 3
d) –3 + 2 + 4 = 3 **e)** 2 + (–3) + 4 = 3

f) 4 + 2 − (+3) = 3; The order in which the additions and subtractions are carried out does not affect the answer.

Review, pages 378–379

1. Term	**Example**
a) positive integers	**D)** +1 and +2
b) negative integers	**A)** −1 and −2
c) opposite integers	**C)** +2 and −2
d) zero principle	**B)** (+1) + (−1) = 0

3. a)



b) −10, −8, −5, −3, −2, 0, +2, +8

5. a) (+9) + (−3) = +6; The stock finished trading $6 higher. **b)** (−8) + (+6) = −2; The final temperature was −2°C. **c)** (+3) + (−4) = −1; Elizabeth's total score was 1 under par. **d)** (+7) + (−10) = −3; Keith still owed $3. **e)** (+6) + (−8) = −2. The candidate had 2 more votes against them than for them.

7. $390

9. a) −2 **b)** −10 **c)** −5 **d)** +8

11. Both have result +12. Diagrams may vary.

13. a) +10 **b)** −10 **c)** +9 **d)** −5

15. +1

17. a) (−6) + (+2) + (+3) + (−4) + (−7) + (−4) **b)** −16 kg

Practice Test, pages 380–381

1. C **3.** A **5.** C **7.** B

9. a) −9 **b)** +5 **c)** −7 **d)** +16 **e)** −10 − 10 − 9 − 7 + 5 + 16

11. a) −20 **b)** Add −10 (or subtract +10).

CHAPTER 12

Get Ready, pages 384–385

1. a) 13 **b)** 31 **c)** 26 **d)** 27

3. a) 3 + 13 = 16 **b)** 7 − 4 = 3 **c)** 2 × 6 = 12 **d)** 13 − 8 = 5 **e)** 8 × 9 = 72

5. P = 46 cm, A = 120 cm^2

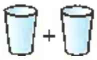
7. a) begin with 4, increase by 4; 16, 20 **b)** begin with 6, increase by 4; 18, 22 **c)** begin with 5, increase by 4; 17, 21

12.1 Variables and Expressions, pages 389–391

5. Let C represent the variable. **a)** $C + 6$ **b)** $2C + 2$ **c)** $3C + 1$ **d)** $2C + 4$

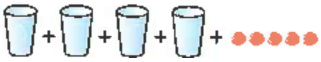
7. a)

b)

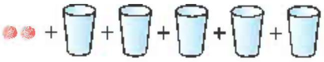


9. a)

b)



c)

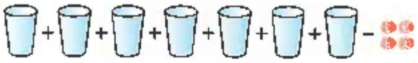


d)

11. a) 11 **b)** 2 **c)** 15 **d)** 13 **e)** 7 **f)** 19

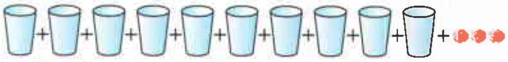
13. a)

b)



c)

d)



15. Variables may vary. **a)** $10 + p$ **b)** $8a$ **c)** $A + 10$ **d)** $2l$

17. a)

b) Sonja is paid $3 to come plus $5 for every hour she babysits. **c)** $28

19. a) 7, 10, 13, 16, 19 **b)** Begin at 7, increase by 3. Answers may vary. For example, points in a basketball game where the team has 7 points and keeps sinking 3 pointers.

23. a) 9, 10 **b)** $2x$ gives the greater result. This will not be true for all values of x. For example: when $x = 3$, $x + 4 = 7$, $2x = 6$

12.2 Solve Equations by Inspection, pages 395–397

5. a) 9 **b)** 6 **c)** 8

7. a) 7 **b)** 2 **c)** 3

9. a) 19 **b)** 50 **c)** 9 **d)** 4 **e)** 9 **f)** 5

11. 21

13. a) yes **b)** no **c)** no **d)** no **e)** no **f)** no

15. a) 3 **b)** 6 **c)** 38 **d)** 77 **e)** 5 **f)** 4

17. a) Answers will vary. For example, $x + 4 = 8$ or $3x - 7 = 8$ **b)** Answers will vary.

19. 15 min

23. a) $3 + x = -10$ **b)** −13

12.3 Model Patterns With Equations, pages 401–403

5. a)

Number of Rows	Perimeter	Number Equation
1	1 + 1 + 1	3 × 1 = 3
2	2 + 2 + 2	3 × 2 = 6
3	3 + 3 + 3	3 × 3 = 9

b) $3r = 27$

7. Equations may vary. **a)** $5 + n = 6$, $5 + n = 7$, $5 + n = 8$ **b)** $5 + n = 17$, there are 17 marbles at the nth diagram in the pattern. Solving for n shows that $n = 12$, or that the twelfth diagram in the pattern will have 17 marbles.

9. a) $37 = 5 + 4w$, $4w + 5 = 37$, or $5 + 4w = 37$ **b)** $2a + 6 = 26$, $26 = 2a + 6$, or $26 = 6 + 2a$

11. $2n - 2 = 20$, n is the number of blocks

13. Answers may vary. **a)** 6 cubes minus 2 cubes leaves 4 cubes. **b)** 11 cubes, each cube has 2 stickers on it but one of the cubes has an extra sticker on it. **c)** 4 cubes in the shape of a square, one sticker on each exposed face, but the two top faces have no stickers.

15. a) $102 = 3m + 2n$ **b)** Answers will vary.

12.4 Solve Equations by Systematic Trial, pages 408–409

5. a)

Diagram Number	Number of Dots	Pattern
1	2 on the left, 1 in the middle, 2 on the right = 5	$2 + 1 \times 1 + 2$
2	2 on the left, 2 in the middle, 2 on the right = 6	$2 + 2 \times 1 + 2$
3	2 on the left, 3 in the middle, 2 on the right = 6	$2 + 3 \times 1 + 2$
d	2 on the left, d in the middle, 2 on the right = $2 + d + 2$	$2 + d \times 1 + 2$

b) $2 + d \times 1 + 2 = 11$ **c)** $d = 7$; the seventh diagram has 11 dots.

7. a) 3 **b)** 3

9. a) 22 **b)** 4 **c)** 5 **d)** 7

11. a) $3 + 7n$ **b)** 6

13. a) 28 **b)** 26 **c)** 9 **d)** 6

15. a) $5n + 13 = 48$ **b)** $n = 7$ **c)** Answers may vary. Use systematic trial.

17. a) $1 + 3s = 52, s = 17$ **b)** $2 + 2s = 48, s = 23$

19. Answers will vary. **a)** $3n + 2 = 26$ **b)** $(n - 2) \div 3 = 2$

23. 49 no, 106 yes. The number of smiley faces is $4n + 2$ (the 4 side faces + 2 ends). This expression always gives an even number, so 49 is not possible. When there are 26 cubes in the rod, the number of smiley faces is $4 \times 26 + 2$, or 106.

12.5 Model With Equations, pages 413–415

5. a) $C \div 4 = 10$, $C = \$40$ **b)** $2A + 14 = 42$, $A = 14$ **c)** $S + 15 = 32$, $S = 17$ **d)** $h \div 2 - 10 = 70$, $h = 160$ cm

7. 17 cm

9. $h = 5$ cm

11. Answers will vary. **a)** You have 5 books but you want to have 7, how many more do you need? **b)** Each book costs $7, the total bill came to $28, how many books were bought? **c)** There are 3 shelves in a book case. On the first two shelves there are 14 books, there are 19 books in total on the bookcase, how many books are on the third shelf?

13. They are the same equation, just written in a different order.

15. a) $300 + 5s = C$ **b)** 140 **c)** Answers may vary. Use systematic trial.

17. a) Answers will vary. **b)** 11 cm

19. a) 80 m **b)** Answers may vary. Use systematic trial.

23. 56°

Review, pages 416–417

1. eq(u)a(t)i(o)n, var(i)able

3. solution

5. a) Three times a number is equal to 6. **b)** $3x = 6$

7. a) 5 **b)** 5

9. 70

11. $6t + 6 = 46$

13. a) 6 **b)** 12 **c)** 8 **d)** 2

15. a) $1 + 2n$ **b)** $1 + 2n = 51$ **c)** 25

17. a) $J = 2 \times 31 - 1$ **b)** 61

Practice Test, pages 418–419

1. A **3.** B **5.** C

7. a) $5h = 35$ **b)** $3h = 42$ **c)** $4a + 2 = 38$ **d)** $3a + 10 = 55$

9. a) 12 **b)** 3 **c)** 23 **d)** 12 **e)** 11

11. a) $10 + V = 75$ **b)** $65

13. $4(c - 1) + 6 = 86$, $c = 21$

Chapters 9–12 Review, pages 422–423

1. a)

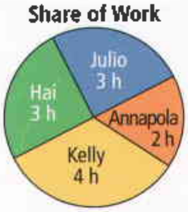
Colour	Tally	Frequency
Red	𝍸 I	6
Green	𝍸 III	8
Blue	IIII	4
Yellow	𝍸 IIII	9
Orange	III	3

b)

c)

d) The bar graph makes it clear exactly how many candies of each type were in the package. The pictograph looks like a candy and gives an impression of what fraction of the total each colour is.

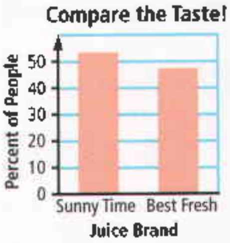
3.



5. a) Vinyl records: decreasing, then staying the same; CDs: increasing; Video tapes: staying the same, then decreasing. **b)** CDs **c)** 13 000; each year sales increase by 500.

7. a) Sunny Time; the graph makes their product look more popular. **b)** The scale starts at 44.

c)



d) The two juices are almost the same in popularity, but the first graph makes it look like Sunny Time is 3 times more popular.

9. a) −8 **b)** −5 **c)** −5 **d)** −6

11. a) −30 **b)** +40 **c)** −10 **d)** 0 **e)** −150

13. 42 cm^2

15. a) 6 **b)** 4

17. 12 cm

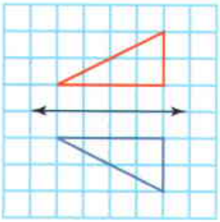
CHAPTER 13

Get Ready, pages 426–427

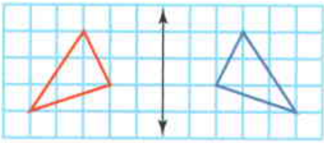
1. a) No; corresponding sides and angles are not equal. **b)** Yes; corresponding sides and angles are equal. **c)** No; corresponding sides are not equal.
3. a) irregular; all angles not equal **b)** regular; all sides equal and all angles equal **c)** irregular; all angles not equal **d)** irregular; all sides not equal
5. Yes; corresponding sides and angles are equal.
7. W

13.1 Explore Transformations, pages 430–433

5. Answers will vary.
7. a)



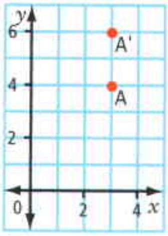
b)



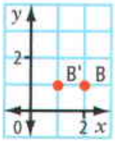
9. They do not change the size or the shape of the figure. After the transformation, the image is congruent to the original.
11. You use rotations when you turn the dial. You use translations when you open and close the lock.
13. Answers may vary. **a)** A window with two panes of glass. One pane is pushed over the other to open the window. **b)** A window with one pane of glass. As you rotate a handle, the glass rotates outward, opening the window.
15. a) In the middle of the common side. **b)** 180°
17. a) Design A: squares related by translation, reflection, or rotation; triangles related by rotation or reflection; trapezoids related by reflection or rotation. Design B: parallelograms related by rotation or translation.
b) Answers will vary. Design A: reflection, rotation 180°. Design B: rotation 90°.
21. The location of the original and the final image is the same.

13.3 Extension: Translations on a Coordinate Grid, pages 440–441

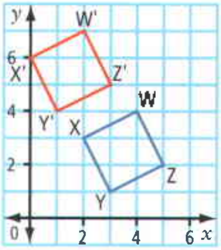
5. a) 4 units right **b)** 5 units up
7. a) (3, 6)



b) (1, 1)



9.



11. a) Michel said, "Translate the image 2 units right and 1 unit up. This brings the image back onto the original." **b)** He knew this by reversing Fareeha's instructions.

13.4 Identify Tiling Patterns and Tessellations, pages 444–445

5.–9. Answers will vary.

Review, pages 456–457

1. image
3. angle of rotation
5. tessellation
7. reflection
9. Answers will vary.
11. On any of the 4 axes of symmetry, that is, along the diagonals of the square or perpendicular to and halfway along any side of the square.
13. a) rectangles: translation, reflection, or rotation; trapezoids: reflection or rotation; triangles: reflection or rotation **b)** rotation: rotate about its centre 180° or 360°; reflection: place a mirror horizontally through the centre; reflection: place a mirror vertically through the centre.
c) It does not matter. The window is symmetrical so it looks the same upside down as it does right side up.
15. Translate the figure 1 unit left and 3 units up.
17. The image was translated horizontally either left or right.
19. translates the figure 2 units left and 3 units up
21. yes
23. a) Each brick is made up of three regular hexagons with the common sides removed. Regular hexagons tile the plane. **b) c) d) e)** Answers will vary.

Practice Test, pages 458–459

1. C **3.** A
5. Answers will vary.
7. Parallelogram ABCD is translated 2 units left and 3 units down.
9. no
11. Answers will vary.
13. a) no **b)** large triangles (F and G): turn centre is in the middle of the tangram puzzle, rotate F counterclockwise 90° or rotate G clockwise 90°; small triangles (C and E): turn centre is 1 unit right of the middle, rotate C clockwise 90° or rotate E counterclockwise 90° **c)** large triangles (F and G): reflect along the line between them; small triangles (C and E): reflect through a mirror line in centre of square parallel with the two triangles

Glossary

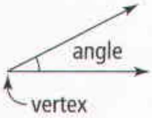
A

acute angle An angle whose measure is less than 90°.

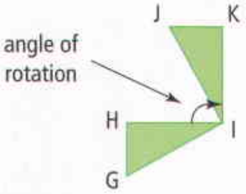
acute triangle A triangle in which each of the three interior angles is less than 90°.

algebraic equation An equation or formula that describes a relationship. Uses numbers and variables. $3x = 6$ and $C = 3d$ are algebraic equations.

angle The figure formed by two rays or two line segments with a common endpoint called the vertex.



angle of rotation The angle through which a figure turns.



area The number of square units contained in a two-dimensional region.

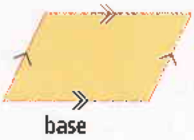
B

bar graph A graph that uses horizontal or vertical bars to represent data visually.

base (exponential form) The factor you multiply.

In 5^2, the base is 5.

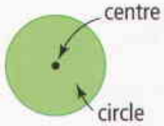
base (2-D geometry) A side of a polygon. Short form is b.



bias An emphasis on characteristics that are not typical of an entire population. Certain responses can be encouraged by the wording of a question.

C

circle The set of all points in the plane that are the same distance from a fixed point called the centre.



circle graph A graph in which a circle is used to represent a whole and is divided into sectors that show how data are divided into parts by percent. Also called a pie chart.

common denominator A number that is a common multiple of the denominators of a set of two or more fractions.

10 is a common denominator for $\frac{1}{2}$ and $\frac{1}{5}$.

composite shape A two-dimensional figure that can be split into two or more simpler figures.

concept map A diagram that places concepts or ideas in balloons. The balloons are linked together to show how concepts are related.

concrete materials Objects that can be used to help in understanding mathematical concepts and skills. Also called manipulatives.

Examples are base 10 blocks, centimetre cubes, pattern blocks, geoboards, number lines, hundred charts, spinners, number tiles, and so on.

cone A three-dimensional object with a circular base and a curved surface.

congruent figures Figures that have the same size and shape.

△ABC and △DEF are congruent.

coordinate grid The two-dimensional or (*x*, *y*) plane. Also known as the coordinate or Cartesian plane.

corresponding angles Angles that have the same relative position in geometric figures.

Corresponding pairs of angles are
∠A and ∠D
∠B and ∠E
∠C and ∠F

corresponding sides Sides that have the same relative position in geometric figures.

Corresponding pairs of sides are
AB and DE
BC and EF
AC and DF

cube A polyhedron with six congruent square faces.

cube (cubic number) The product of three equal factors. Represents the volume of a cube.

$2 \times 2 \times 2 = 2^3$

cylinder A three-dimensional object with two parallel circular bases.

D

data Facts or information.

database An organized collection of information. Often stored electronically.

denominator The number of equal parts in the whole or the group.

$\frac{3}{4}$ has denominator 4.

difference A number resulting from subtraction.

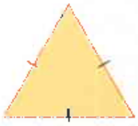
dodecahedron A polyhedron with 12 pentagonal faces.

E

edge Where two faces meet.

equation A mathematical statement that has equal expressions on either side of the equal sign.

equilateral triangle A triangle with all three sides equal.



equivalent fractions Fractions such as $\frac{1}{3}$ and $\frac{2}{6}$ that represent the same part of a whole or group.

estimate An approximate answer obtained using mental mathematics strategies. An estimate is used when an exact answer is not required or to check the reasonableness of a calculation.

exponent The number of factors you multiply.

exponential form A shorter method for writing numbers expressed as repeated multiplication.

expression Numbers and variables, combined by operations.

$3x + 2y$ is an expression.

F

face A flat or curved surface of an object.

factors The numbers that are multiplied to produce a specific product.

2 and 3 are factors of 6, since $2 \times 3 = 6$.

favourable outcome An outcome that counts for the probability being calculated.

Fibonacci sequence The sequence 1, 1, 2, 3, 5, 8, 13, …. Describes many patterns in nature.

formula A set of ideas, words, symbols, figures, characters, or principles used to state a general rule.

The formula for the area, A, of a rectangle with length l and width w is $A = l \times w$.

fractal A pattern that gets smaller as it repeats forever.

fraction A number that represents a part of a whole or a part of a group.

frequency table A table used to show the total numbers of occurrences in an experiment or survey.

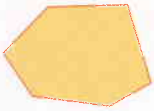
frieze pattern A design pattern that repeats in one direction.

H

height The perpendicular distance from the base of a polygon to the opposite side. Short form is *h*.

heptagon A polygon with seven sides.

hexagon A polygon with six sides.



hexagonal prism A prism whose bases are congruent hexagons.

I

icosahedron A polyhedron with 20 triangular faces.

image A figure resulting from a transformation.

improper fraction A fraction in which the numerator is greater than the denominator, such as $\frac{8}{5}$.

integer A number in the sequence ..., −3, −2, −1, 0, +1, +2, +3, ...

irregular polygon A polygon that is not regular.

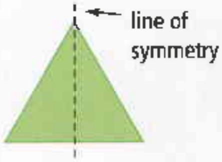
isosceles triangle A triangle with exactly two equal sides.

K

kite A quadrilateral with two pairs of adjacent sides equal.

L

line of symmetry A line that divides a shape into two parts that can be matched by folding the shape in half.



line segment The part of a line that joins two points.

M

manipulatives Objects that are can be used to help in understanding mathematical concepts and skills. Also called concrete materials.

Examples are base 10 blocks, centimetre cubes, pattern blocks, geoboards, number lines, hundred charts, spinners, number tiles, and so on.

mean The sum of a set of values divided by the number of values in the set.

measure of central tendency A value that represents the centre of a set of data. It can be the mean, median, or mode.

median The middle number in a set of data when the data are arranged in order from least to greatest. If there is an even number of pieces of data, the median is the average of the two middle values.

The median of 1, 1, 3, 5, 6, is 3.
The median of 1, 1, 3, 5, is 2.

mixed number A number made up of a whole number and a fraction, such as $3\frac{1}{2}$.

mode The value that occurs most frequently in a set of data. There can be more than one mode, or no mode.

For 1, 2, 3, 3, 8, the mode is 3.

model (noun) A physical model that can be used to represent a situation.

model (verb) To represent the facts and factors of, and the results of, a situation.

multiple The product of a given number and a natural number.

Multiples of 2 are 2, 4, 6, 8, and so on.

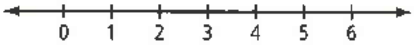
N

natural numbers The numbers 1, 2, 3, … and so on. Also called positive integers.

negative integer One of the numbers −1, −2, −3, ….

net A two-dimensional drawing that can be folded to form a three-dimensional object. A single pattern piece that shows all the faces of the figure.

number line A line that matches a set of points and a set of numbers one to one.



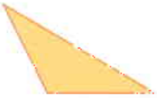
numerator The number of equal parts being considered in the whole or the group.

$\frac{3}{4}$ has numerator 3.

O

obtuse angle An angle that measures more than 90° but less than 180°.

obtuse triangle A triangle containing one obtuse angle.



octagon A polygon with eight sides.

octahedron A polyhedron with eight triangular faces.

opposite integers Two integers with the same numeral but opposite signs.

+2 and −2 are opposite integers.

order of operations Correct sequence of steps for a calculation. Use BODMAS to remember.

B	Brackets, then
O	Order:
D	Division and Multiplication,
M	from left to right
A	Addition and Subtraction,
S	from left to right

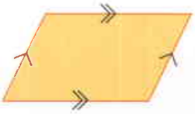
ordered pair A pair of numbers, such as (2, 5), used to locate a point on a coordinate grid.

outcome One possible result of a probability experiment.

P

parallel lines Lines in the same plane that never meet.

parallelogram A four-sided figure with both pairs of opposite sides parallel.

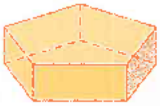


pattern An arrangement of shapes, lines, colours, numbers, symbols, and so on, for which you can predict what comes next.

pattern rule A simple statement that tells how to form or continue a pattern.

pentagon A polygon with five sides.

pentagonal prism A prism whose bases are congruent pentagons.



pentagonal pyramid A pyramid with a pentagonal base.

percent Out of 100.

50% means $\frac{50}{100}$ or 0.5.

percent circle A circle divided into 100 equal sections. Each section represents 1%.

perfect square A number whose square root is a natural number.

4 is a perfect square. Its square root is 2.

perimeter The distance around the outside of a two-dimensional shape or figure.

perpendicular lines Two lines that cross at 90°.

pictograph A graph that illustrates data using pictures and symbols.

place value The value given to the place in which a digit appears in a number.

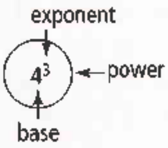
In the number 2345, 2 is in the thousands place, 3 is in the hundreds place, 4 is in the tens place, and 5 is in the ones place.

polygon A two-dimensional closed figure whose sides are line segments.

polyhedron A three-dimensional figure with faces that are polygons.

positive integer One of the numbers +1, +2, +3, ….

power A number in exponential form. Includes a base and an exponent.



primary data Data you collect yourself.

Data from a survey are primary data.

prism A three-dimensional object with two parallel, congruent polygonal bases. A prism is named by the shape of its bases, for example, rectangular prism, triangular prism.

probability The chance that something will happen.

product A number resulting from multiplication.

proper fraction A fraction in which the denominator is greater than the numerator, such as $\frac{5}{8}$.

pyramid A polyhedron with one base and the same number of triangular faces as there are sides on the base.

Q

quadrilateral A four-sided polygon.

quotient A number resulting from division.

R

random A type of choice or pick in which each outcome is equally likely.

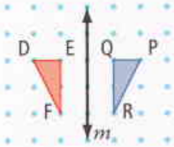
ray A part of a line with one endpoint.

rectangle A quadrilateral with two pairs of equal opposite sides and four right angles.

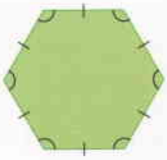
rectangular prism A prism whose bases are congruent rectangles.

rectangular pyramid A pyramid with a rectangular base.

reflection A flip over a mirror line.



regular polygon A polygon with all sides equal and all angles equal.



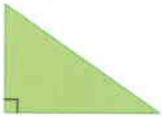
relationship A pattern formed between two sets of numbers. Can often be seen by plotting ordered pairs on a coordinate grid.

repeating decimal A decimal with a digit or group of digits that repeats forever. Write the repeating digits with a bar: $0.333... = 0.\overline{3}$.

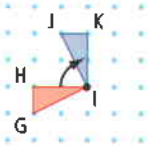
rhombus A quadrilateral in which the lengths of all four sides are equal.

right angle An angle that measures 90°.

right triangle A triangle containing a 90° angle.

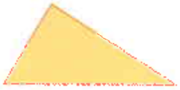


rotation A turn about a fixed point.



S

scalene triangle A triangle with no sides equal.



secondary data Data obtained from someone else.

An encyclopedia is an example of secondary data.

similar figures Figures that have the same shape but different size.

△RST and △UVW are similar.

simulation A probability experiment used to model a real situation.

solution A number that makes an equation true.

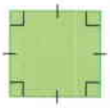
solving by inspection A method of solving equations using mental math.

solving by systematic trial A method of solving equations by substituting values for the variable until the correct answer is obtained.

sphere A round ball-shaped object. All points on its surface are the same distance from a fixed point called the centre.

spreadsheet A software tool for organizing and displaying numeric data.

square A rectangle in which the lengths of all four sides are equal.



square-based pyramid A pyramid with a square base.

square (number) The product of two equal factors. Represents the area of a square.

$3 \times 3 = 3^2$

square root (of a number) A factor that multiplies by itself to give that number.

Since 8 × 8 = 64, the square root of 64 is 8.

statistic A value calculated from a set of data.

stem-and-leaf plot A way of organizing numerical data by representing part of each number as a stem and the other part of the number as a leaf.

sum A number resulting from addition.

surface area The number of square units needed to cover the outside of an object.

survey A sampling of information. Can be conducted by asking people questions or interviewing them.

symmetry A balanced arrangement on either side of a centre line. This line is called a line of symmetry.

T

table of values A table listing two sets of numbers that may be related.

tally chart A table used to record experimental results or data. Tally marks are used to count the data.

tessellation A pattern that covers a plane without overlapping or leaving gaps. Also called a tiling pattern.

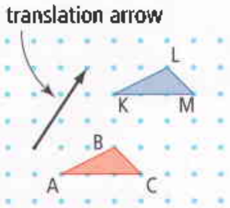
tetrahedron A polyhedron with four triangular faces.

tiling pattern A pattern that covers a plane without overlapping or leaving gaps. Also called a tessellation.

tiling the plane Using repeated congruent shapes to cover a region completely.

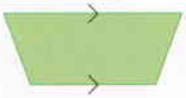
transformation A change in a figure that results in a different position, orientation, or size. Three types of transformations are translations, rotations, and reflections.

translation A slide along a straight line.



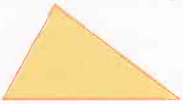
translation arrow An arrow that shows the distance and direction of a translation.

trapezoid A four-sided figure with exactly one pair of opposite sides parallel.



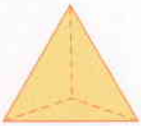
tree diagram A diagram that shows outcomes as sets of branches. Useful for organizing combined outcomes.

triangle A closed, three-sided figure.



triangular prism A prism whose bases are congruent triangles.

triangular pyramid A pyramid with a triangular base.



turn centre A fixed point about which a figure rotates.

V

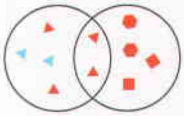
variable A letter that represents an unknown number.

In $2x + 4$, the letter x is a variable.

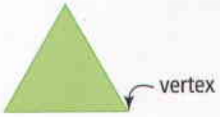
variable expression An expression that contains variables and operations with numbers.

$2x + 4$ is a variable expression.

Venn diagram A diagram that uses nested and/or overlapping shapes to show relationships.



vertex A point at which two sides of a figure meet.



volume The number of cubic units contained in a space.

X

***x*-axis** The horizontal number line on a coordinate grid.

***x*-coordinate** The first number in the ordered pair describing a point on a coordinate grid.

The point P(2, 5) has x-coordinate 2.

Y

***y*-axis** The vertical number line on a coordinate grid.

***y*-coordinate** The second number in the ordered pair describing a point on a coordinate grid.

The point P(2, 5) has y-coordinate 5.

Z

zero principle The principle that opposite integers cancel each other out. The sum of a pair of opposite integers is zero.

For example, $(+1) + (-1) = 0$.

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