CHAPTER 5

Fractions, Decimals, and Percents

In 2002, Canada scored a remarkable double. Both the women's and men's hockey teams won gold at the Olympics.

Action and skill play a big part in sports. But the difference between a very good athlete and a great one often comes down to numbers.

By the end of this chapter, you will have a better understanding of the numbers used to report on athletic performances. You will be able to discuss and compare sports statistics using fractions, percents, and decimals.

Chapter Problem

A hockey goalie's performance can be measured in two ways:
- the goals-against average
- the save percentage

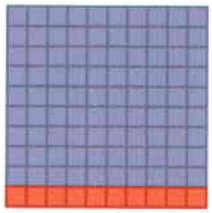
What does each statistic (or stat) tell about the goalie? How might such information help you compare two goalies?

Get Ready

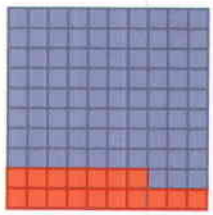
Write Fractions as Decimals

You can use base 10 blocks to represent fractions and decimals. For example, let a hundreds flat represent 1.

The fractions $\frac{1}{10}$ and $\frac{17}{100}$ can be represented as in the diagram.



The fraction $\frac{1}{10}$ is the same as the decimal 0.1.



$\frac{17}{100}$ is the same as 0.17.

1. Use base 10 blocks or grid paper to represent each fraction. Then, write each fraction as a decimal.

a) $\frac{3}{10}$ **b)** $\frac{7}{10}$ **c)** $\frac{1}{100}$ **d)** $\frac{23}{100}$

2. Write each fraction as a decimal.

a) $\frac{6}{10}$ **b)** $\frac{32}{100}$ **c)** $\frac{97}{100}$ **d)** $\frac{5}{10}$

Compare and Order Decimals

To order decimals, look at place values. For example, to order 0.74, 0.7, and 0.732, you can use a place value chart.

Ones	Tenths	Hundredths	Thousandths
0	7	4	
0	7	0	
0	7	3	2

I added a zero placeholder to 0.7 to compare the hundredths place.

All three numbers have the same tenths digit. So, look at the hundredths digit. 0.74 is the greatest. 0.70 = 0.7 is the least.

The order of the decimals, from least to greatest, is 0.7, 0.732, 0.74.

This can also be written as $0.7 < 0.732 < 0.74$.

3. Order each set of decimals from least to greatest.

a) 0.25, 0.225, 0.2

b) 1.34, 1.334, 1.43

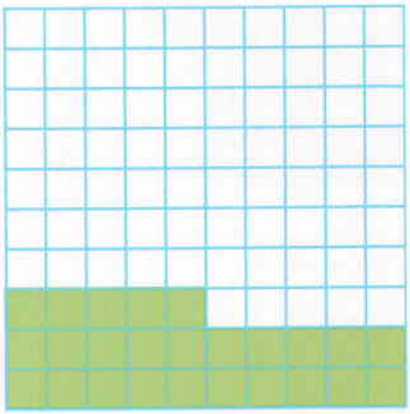
4. Order each set of decimals from greatest to least. Use the symbol ">" to stand for "is greater than."

a) 0.082, 0.0802, 0.08

b) 5.45, 5.545, 5.454

Representing Percents

You can use a hundred grid to show a **percent**. For example, 25% means 25 out of 100. To show 25%, colour 25 squares on a hundred chart.



5. Show each percent on a hundred grid.

a) 50% **b)** 75%

6. Show each percent on a hundred grid.

a) 100% **b)** 60%

Decimals and Percents

To write a decimal as a percent, multiply the decimal by 100%. Remember to add the % sign. For example:

$0.71 = 0.71 \times 100\%$
$= 71\%$

The decimal point moves two places to the right.

$0.2 = 20\%$

Add a zero because you move two decimal places.

To write a percent as a decimal, express the percent as a fraction with denominator 100 and drop the % sign. For example:

$25\% = \frac{25}{100}$
$= 25 \div 100$
$= 0.25$

The decimal point moves two places to the left.

$5\% = \frac{5}{100}$
$= 0.05$

Add a zero placeholder because there are two decimal places.

7. Write each decimal as a percent.

a) 0.43 **b)** 0.6
c) 0.05 **d)** 0.02

8. Write each percent as a decimal.

a) 75% **b)** 90%
c) 3% **d)** 45%

5.1 Fractions and Decimals

Focus on...
- comparing and ordering fractions and decimals
- writing fractions as decimals
- writing decimals as fractions

Suppose your hockey team won 38 out of 50 games last year. How could you show this statistic as a fraction? a decimal?

What other sports statistics use fractions? Which use decimals? How are they used?

Discover the Math

Materials
- grid paper

or
- counters

How can you relate fractions and decimals?

1. Look at the first three columns in this table. Who do you think is having the best season? Explain why you think so.

Goalie	Total Goals Against	Games Played	Goals-Against Average
Akina	36	12	
Julie	60	15	
Katrien	35	14	

statistic
- a value calculated from a set of data

2. Goalies can be rated by their goals-against average. This **statistic** compares the number of goals scored against a goalie to the number of games played.

$$\text{Goals-against average} = \frac{\text{total goals against}}{\text{total games played}}$$

a) Copy and complete the table. Write the goals-against average for each goalie as a fraction and as a decimal.

b) Do you want to change your answer from step 1? Explain.

Strategies Choose a formula

3. a) How does the goals-against average help you predict a goalie's performance?

b) Is it better for a goalie to have a lower goals-against average or a higher one? Explain why.

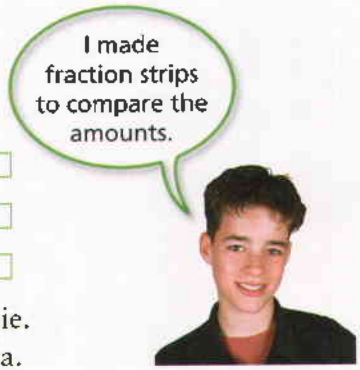
4. **Reflect** The goals-against average is usually stated as a decimal.
   a) How is the decimal form different from the fraction form?
   b) How are they related?
   c) Which form is more useful? Justify your choice.

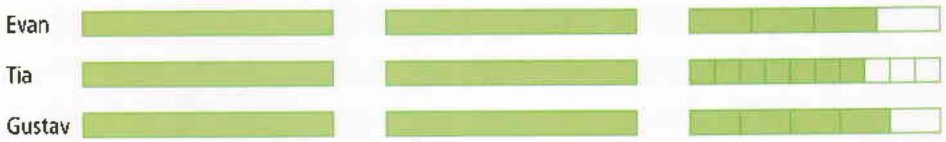
Example 1: Compare and Order Fractions

In a pie-eating contest, Evan ate $2\frac{3}{4}$ pies, Tia ate $2\frac{7}{10}$ pies, and Gustav ate $2\frac{4}{5}$ pies. The person who ate the most pies won the contest. Order the contestants from first place to third place.

Solution

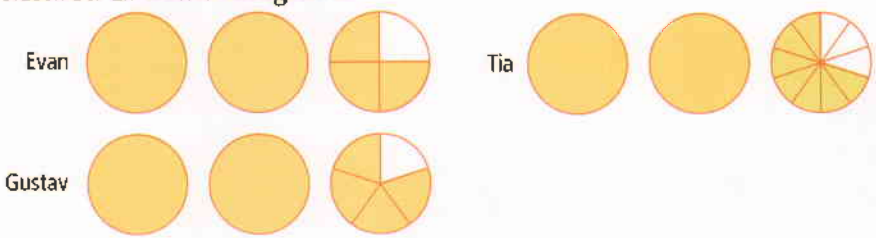
Method 1: Model the Amounts





The fraction strips show that Gustav ate the most pie. Tia ate the least pie.
The contestants, from first place to third place, are Gustav, Evan, and Tia.

Method 2: Draw Diagrams



Strategies
Make a picture or diagram

The diagrams show that Gustav ate the most pie. Tia ate the least pie.
The contestants, from first place to third place, are Gustav, Evan, and Tia.

Method 3: Convert to Decimals

Evan:

$$2\frac{3}{4} = 2 + 3 \div 4$$
$$= 2 + 0.75$$
$$= 2.75$$

Tia:

$$2\frac{7}{10} = 2 + 7 \div 10$$
$$= 2 + 0.7$$
$$= 2.7$$

Gustav:

$$2\frac{4}{5} = 2 + 4 \div 5$$
$$= 2 + 0.8$$
$$= 2.8$$

$2.8 > 2.75 > 2.7$
The contestants, from first place to third place, are Gustav, Evan, and Tia.

Strategies
What other strategy could you use to compare and order these fractions?

Example 2: Repeating Decimals

Use a calculator to write each fraction as a **repeating decimal**. Then, order the fractions from least to greatest.

a) $\frac{1}{3}$ **b)** $\frac{2}{3}$ **c)** $\frac{1}{6}$

repeating decimal

- a decimal with a digit or group of digits that repeats forever
- write the repeating digits with a bar: $0.333... = 0.\overline{3}$

Solution

a) $\frac{1}{3} = 1 \div 3$

$= 0.333...$

$= 0.\overline{3}$ **Place a bar above the repeating digit.**

b) $\frac{2}{3} = 2 \div 3$

$= 0.666...$ **The calculator displays the final 7 because it rounds up.**

$= 0.\overline{6}$ **It would show more 6's if it had a larger display.**

c) $\frac{1}{6} = 1 \div 6$

$= 0.1666...$

$= 0.1\overline{6}$ **The digit 1 does not repeat. Place the bar above the 6.**

$0.1\overline{6} < 0.\overline{3} < 0.\overline{6}$

So, $\frac{1}{6} < \frac{1}{3} < \frac{2}{3}$.

Literacy Connections

Reading Repeating Decimals

$0.\overline{3}$ means that the 3 repeats. Another way of showing this is 0.333....

$0.1\overline{6}$ means that the 6 repeats. Another way of showing this is 0.1666....

Example 3: Convert Decimals to Fractions

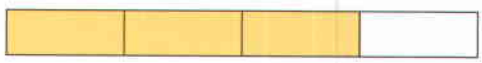
a) What fraction of a dollar is \$0.75?

b) What fraction of a dollar is \$0.44?

Solution

a) $0.75 = \frac{75}{100}$

\$0.75 is $\frac{75}{100}$ of a dollar.



Another way of saying this is $\frac{3}{4}$ of a dollar.

b) $0.44 = \frac{44}{100}$

\$0.44 is $\frac{44}{100}$ of a dollar.

Another way of saying this is $\frac{11}{25}$ of a dollar.

Making Connections

If you need help with equivalent fractions, go to Chapter 3, page 85.

Key Ideas

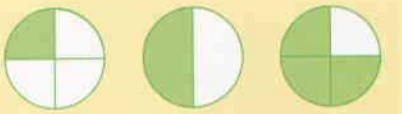
- To write a fraction as a decimal, divide the numerator by the denominator. For example,

$$\frac{1}{2} = 1 \div 2$$
$$= 0.5$$

- To express a decimal as a fraction, think of place value.

For example, $0.7 = \frac{7}{10}$ and $0.37 = \frac{37}{100}$.

- Compare and order fractions using diagrams, manipulatives, number lines, or common denominators, or by writing them as decimals.



$\frac{1}{4} = 0.25$ $\frac{1}{2} = 0.5$ $\frac{3}{4} = 0.75$

- Compare and order decimals using place value.
- Some fractions produce repeating decimals. For example, $\frac{5}{6} = 0.8333\ldots$. Another way of writing this is $0.8\overline{3}$.

Communicate the Ideas

1. Write three fractions. Describe two methods you can use to order these fractions.

2. Write each repeating decimal correctly. Explain what error has been made.

a) 0.444444444 $0.444\ldots = 0.4$

b) 2.166666667 $2.1666\ldots = 2.1\overline{6}$

3. a) How could you compare the numbers in a mixed list of fractions and decimals?

b) What other strategy might be useful? Explain.

c) Use one strategy to compare $\frac{4}{5}$, 0.6, 0.9, and $\frac{3}{4}$. Order the numbers from greatest to least.

4. a) Express 0.7 and 0.51 as fractions.

b) How many numerals are after the decimal of each number? How many zeros are in the denominator of each fraction? Explain how the answers to these questions are related.

Check Your Understanding

Practise

5. Show each fraction or mixed number using a diagram.

a) $\frac{1}{2}$ **b)** $\frac{3}{4}$

c) $1\frac{2}{5}$ **d)** $3\frac{3}{8}$

6. Write each fraction or mixed number in question 5 as a decimal.

7. Order these decimals from greatest to least.

a) 1.3, 1.301, 0.34, 0.3

b) 0.29, 0.489, 0.06, 0.2

For help with questions 8 and 9, refer to Example 1.

8. In a pie-eating contest, Robert ate $2\frac{3}{4}$ pies, Dora ate $2\frac{3}{10}$ pies, and Gianetta ate $2\frac{2}{5}$ pies.

a) Who won?

b) Order the contestants from first place to third place.

9. Order these fractions from least to greatest.

a) $\frac{4}{9}, \frac{1}{2}, \frac{1}{3}$ **b)** $1\frac{2}{3}, 1\frac{5}{6}, 1\frac{3}{4}, 1\frac{3}{8}$

For help with questions 10 and 11, refer to Example 2.

10. Write these decimals using repeating notation.

a) 0.3333333333 **b)** 0.5555555556

c) 2.166666667 **d)** 0.183333333

11. Use a calculator to write each fraction or mixed number as a repeating decimal.

a) $\frac{1}{6}$ **b)** $1\frac{1}{3}$

c) $3\frac{2}{3}$ **d)** $2\frac{5}{6}$

For help with questions 12 and 13, refer to Example 3.

12. What fraction of a dollar is \$0.50?

13. What fraction of a kilometre is 250 m?

14. Express each decimal as a fraction.

a) 0.2 **b)** 0.3

c) 0.6 **d)** 0.4

e) 0.25 **f)** 0.85

g) 0.64 **h)** 0.78

Apply

15. a) Order these numbers from least to greatest.

$\frac{17}{20}$ 1.04 $\frac{21}{27}$

0.87 $\frac{23}{25}$ $\frac{6}{5}$

b) Order these numbers from greatest to least. Use two different methods.

12.84 $12\frac{5}{6}$ $\frac{319}{25}$

Chapter Problem

16. Several teams used different methods to show goals-against averages. You are the coach of an all-star team. You want to choose the best goalie to start your first game. Which one would you choose?

Andrea: $\frac{44}{20}$ Mustafa: $\frac{32}{8}$

Delilah: $\frac{27}{8}$ Thomas: 2.7

a) What strategies might you use to answer this question?

b) Choose one strategy and solve the problem.

17. Carlo's last three quiz scores in math class were $\frac{8}{8}$, $\frac{9}{12}$, and $\frac{14}{16}$.

a) Arrange these quiz scores from greatest to least.

b) Explain your method.

18. Trina's three cats all love Meow Munch cat food. One day, the supermarket ran out of Meow Munch, and Trina had to buy Kitty Kibble instead. That evening,

- Fuzzball ate $\frac{2}{3}$ of his dinner
- Princess ate $\frac{3}{4}$ of her meal
- Pounce ate $\frac{3}{5}$ of his supper

a) Show visually the fraction of food that each cat ate.

b) Who liked Kitty Kibble the most? Who disliked Kitty Kibble the most? Explain.

c) Can you be sure of your answers in part b)? Explain.

19. Some of these gymnastics scores out of 10 were given as mixed numbers, and some as decimals.

$6.7, 6\frac{2}{5}, 6\frac{1}{3}, 6\frac{3}{8}, 6.05, 6\frac{9}{20}$

a) Order the scores from greatest to least.

b) What strategy did you use to solve this question? What other strategies might you use? Describe one.

20. Create your own problem involving fractions and decimals. Trade problems with a partner and solve your partner's problem.

21. Express each decimal as a fraction. Simply the fraction.

a) 0.75 **b)** 0.8

c) 0.50 **d)** 0.65

22. A Student Council gives a "School Spirit Award" to the class with the best attendance at a dance. Four classes were surveyed.

Teacher	Number of Students	Number to Attend
Mr. Tamaki	30	21
Mrs. Galante	32	24
Ms. Serafini	26	19
Mr. Hetfield	28	20

a) Without calculating, predict which class should win. Explain your thinking.

b) Which class actually wins the School Spirit Award? Justify your decision.

Making Connections

In Chapter 10, you will learn more about conducting and analysing surveys.

Extend

23. Use a calculator to explore $\frac{4}{7}$ as a decimal.

a) Do you think this fraction produces a repeating decimal? Why or why not?

b) What additional information would help you decide?

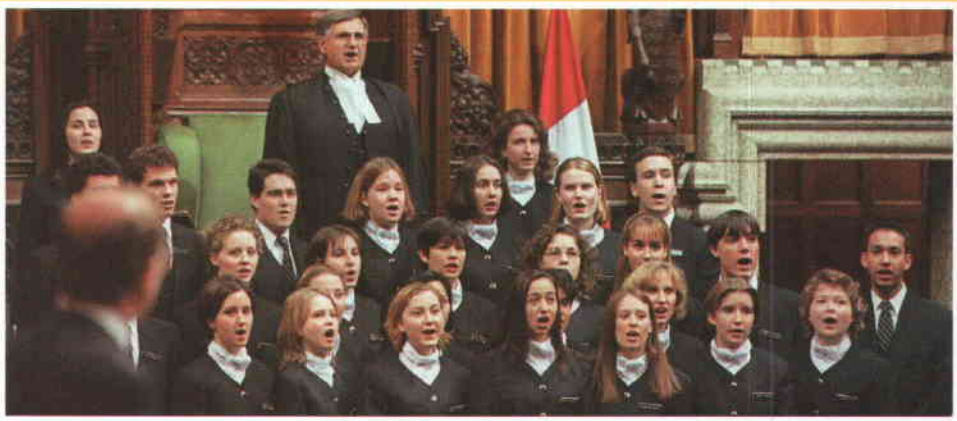
c) Try to find fractions with similar decimal forms to $\frac{4}{7}$. Write a brief report on what you discover.

24. The table shows United Nations estimates for Earth's population. What is the current growth rate of Earth's population? Use fractions and/or decimals to answer this question in at least two different ways.

Year	Earth's Population
2000	6.06 billion
2001	6.14 billion
2002	6.22 billion
2003	6.30 billion

5.2 Calculate Percents

Focus on...
- writing fractions and decimals as percents
- representing percents



Each year in Ontario, a few grade 7 and grade 8 students are chosen to assist Members of Provincial Parliament (MPPs). These students are called pages. They run errands and deliver messages.

To qualify as a page, you need an overall average mark of at least 80%. How could you check whether you qualify?

Literacy Connections

What is a page?
Page has more than one meaning. Right now, you are reading the page of a book. "Page" can also mean a young person who runs errands and carries messages in a hotel, theatre, or parliament.

Discover the Math

How can you calculate percents from fractions or decimals?

Materials
- grid paper

or
- base 10 blocks (hundreds, tens, and ones)

1. Maia wants to become a page. On her last four math quizzes, she got $\frac{19}{25}$, $\frac{13}{20}$, $\frac{18}{20}$, and $\frac{8}{10}$. She needs an average of 80%.
- **a)** Look at Maia's quiz scores. Find a common denominator and write Maia's scores as equivalent fractions.
- **b)** Show each equivalent fraction on a hundred grid or using base 10 blocks.
- **c)** Explain why using equivalent fractions makes it easy to compare the scores.

2. **a)** Write each of Maia's quiz scores as a **percent**.
- **b)** Find the mean of Maia's percent scores.
- **c)** Does Maia need to improve her scores? Explain.

3. **Reflect** Describe how you wrote Maia's scores as percents. What other method could you have used? Describe it.

percent
- out of 100
- 50% means $\frac{50}{100}$ or 0.5

Making Connections
If you need help with calculating means, go to Chapter 4, page 114.

Example: Write Fractions and Decimals as Percents

On a Student Council, it takes 60% of the votes to make a major decision. The students on the Council have voted on whether to hold a dance. The results are shown in the table.

Grade Level	Votes in Favour	Number on Council
Grade 7	4	6
Grade 8	7	9

a) Did the Student Council decide to hold the dance?
b) What is the minimum number of votes needed to make a decision?

Solution

a) There were $4 + 7 = 11$ votes in favour. The total number of votes was $6 + 9 = 15$.

$$\frac{11}{15} = 0.7333\ldots$$
$$= 0.7\overline{3}$$
$$= 0.7\overline{3} \times 100\%$$
$$\doteq 73\%$$

First, I divided 11 by 15 to write this fraction as a decimal.

Then, I multiplied by 100 to find the percent.

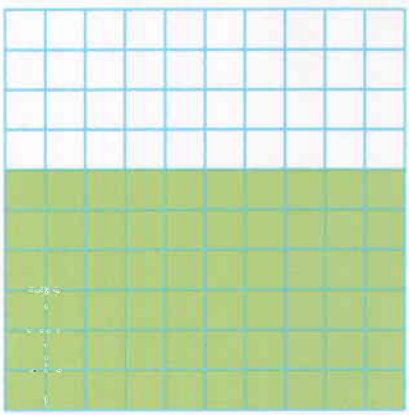
73% is greater than 60%. The council decided to hold the dance.

b) Try 10 votes out of 15:

$$\frac{10}{15} = 0.\overline{6}$$
$$= 0.\overline{6} \times 100\%$$
$$\doteq 67\%$$

So, 10 is more than 60%.

Half of 15 is 7.5. 60% is more than half. I'll start by trying 10.



Try 9 votes out of 15:

$$\frac{9}{15} = 0.6$$
$$= 0.6 \times 100\%$$
$$= 60\%$$

The minimum number of votes to make a decision is 9 out of 15.

Key Ideas

- To write a fraction as a percent, write the fraction as a decimal. Then, multiply by 100%. For example:

$$\frac{5}{8} = 0.625$$
$$= 0.625 \times 100\%$$
$$= 62.5\%$$

- You can represent percents visually.

$\frac{1}{4} = 0.25$ or 25%

Communicate the Ideas

1. Describe two methods to write $\frac{16}{25}$ as a percent.
2. Represent 75% visually.
3. What percent is green? Explain.

Check Your Understanding

Practise

For help with questions 4 to 9, refer to the Example.

4. Write each fraction as a decimal.
   a) $\frac{7}{10}$ b) $\frac{17}{20}$ c) $\frac{4}{5}$
5. Show each fraction in question 4 as a percent.
6. Write each fraction as a decimal, and then as a percent.
   a) $\frac{3}{20}$ b) $\frac{27}{50}$ c) $\frac{19}{25}$
7. Represent each percent in question 5 visually.

Apply

8. On a Student Council, it takes 55% of the votes to make a major decision. The results on a decision to set up a study support group are shown in the table.

Grade Level	Votes in Favour	Number on Council
Grade 7	3	8
Grade 8	7	12

   a) Did the Student Council decide to set up the study support group?
   b) What is the minimum number of votes needed to make a decision?

9. A Student Council is made up as follows:
   - Grade 6 4 students
   - Grade 7 6 students
   - Grade 8 6 students

   A majority of 60% is needed to make major decisions. What is the minimum number of votes needed?

10. Leah got 32 out of 45 on her geography test. What was her score as a percent? Round your answer to the nearest percent.

11. a) David got 45 out of 60 on a test. What percent did he get?
    b) What other method could you use to solve this question? Explain.

12. In a recent taste test, 177 out of 250 people said they preferred the taste of Fizzo to Splash.
    a) What percent of the people surveyed preferred Fizzo?
    b) What percent preferred Splash?
    c) The makers of Fizzo announce that "7 out of 10 people prefer Fizzo to Splash." Is this fair? Explain.

13. Amir got 65% on his science quiz. Amir said to a friend, "I got 65% out of 100." Did Amir tell his friend the correct score? Explain.

14. Rocco and Biff are koala bears. They are trying to eat eucalyptus leaves that are almost out of reach.

- Rocco successfully reaches his target leaf on 12 out of 15 tries.
- Biff succeeds on 16 of 20 tries.

a) Predict which is the more acrobatic bear. Explain why you think so.
b) Write each bear's score as a fraction.
c) Write each bear's score as a percent.
d) Can you determine which is the more acrobatic bear? Explain.

15. Suppose a friend borrowed your notes for this lesson. Later, the friend sent you an e-mail.

> Have I got this right? To change a decimal to a percent, I move the decimal point 2 places to the left and add the % sign?

a) Does your friend understand the procedure? Explain your thinking.
b) Write an e-mail reply to your friend. Make sure your friend understands why your explanation makes sense.

Try This!

16. On her next four math quizzes, Maia got $\frac{16}{25}$, $\frac{14}{20}$, $\frac{18}{20}$, and $\frac{8}{10}$. She wants to be a page in the Provincial Legislature. To be accepted, she needs an average of 80%.

a) Write Maia's scores as percents. What is her mean score?
b) Maia's next quiz is out of 25. What minimum score should she try to get on the quiz?
c) Use pictures, numbers, and words to explain how you answered part b).

Extend

17. When no party wins more than 50% of the seats in an election, a minority government is formed.

a) There are 300 seats in Canada's federal Parliament. Give an example of how a minority government could be formed in a federal election. Show the number of seats for each of these parties: Liberal, Conservative, NDP, Bloc Québecois.
b) Who would likely be the governing party in your example? What percent of the seats do they hold?
c) What is the total percent held by non-governing parties?
d) Explain why a minority government has less power than a majority government.

Did You Know?

Would you like to learn more about how our government works? You can, by becoming a legislature page. Find out how to do this. Go to **www.mcgrawhill.ca/links/math7** and follow the links.

Internet Connect

18. Approximately 70% of Earth's surface is covered by water. The total surface area of Earth is about 510 000 km^2.

a) What total area of Earth is covered by water?
b) What is the total land area on Earth?
c) What effect do you think global warming could have on these values? Explain your answer. Hint: Research any information you need.

5.3 Fractions, Decimals, and Percents

Focus on...
- presenting numerical data
- converting between fractions, decimals, and percents
- estimating

Why are election results often reported as percents?

Discover the Math

Materials
- pencil crayons

How do percents make data easier to understand?

1. At Sherwood Junior High School, three students ran for president of the Student Council. Each student picked a different colour for signs and buttons. The results of the election were as follows. Who won the election? How can you tell?

Candidate	Colour	Percent of Votes
Justine	red	21%
Bram	green	27%
Lucia	blue	52%

2. Design a bar graph for the school newspaper to represent the election results.

3. Write a paragraph about the results of the election. Report the results as fractions. Choose approximate fractions with small denominators. Hint: Use your estimation skills.

4. **Reflect** What are the advantages of reporting the results as percents? as approximate fractions? What other reporting methods could be useful? Explain.

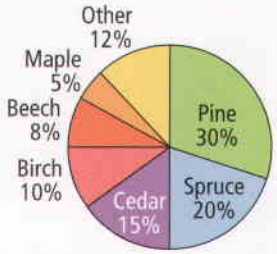
Example: Write Percents as Fractions and Decimals

Ari is studying tree species in the Canadian Shield. Ari's research in one woodlot is shown in the circle graph.

a) What fraction of the forest is made up of each tree species?

b) To plan for harvesting, Ari needs the decimal equivalent of each percent. Convert each percent to decimal form.

c) The forest has approximately 2000 trees. Approximately how many are pine?



Solution

a)

Type of Tree	Percent	Fraction
Pine	30%	$\frac{30}{100}$
Spruce	20%	$\frac{20}{100}$
Cedar	15%	$\frac{15}{100}$
Birch	10%	$\frac{10}{100}$
Beech	8%	$\frac{8}{100}$
Maple	5%	$\frac{5}{100}$
Other	12%	$\frac{12}{100}$

I put 100 as the denominator and dropped the % sign.

I can simplify this fraction:

$$\frac{30}{100} = \frac{30 \div 10}{100 \div 10} = \frac{3}{10}$$

b)

Type of Tree	Percent	Decimal
Pine	30%	$30 \div 100 = 0.3$
Spruce	20%	$20 \div 100 = 0.2$
Cedar	15%	$15 \div 100 = 0.15$
Birch	10%	$10 \div 100 = 0.1$
Beech	8%	$8 \div 100 = 0.08$
Maple	5%	$5 \div 100 = 0.05$
Other	12%	$12 \div 100 = 0.12$

c) Number of pine trees $= 0.3 \times$ total number of trees

$= 0.3 \times 2000$

$= 600$

Approximately 600 of the trees are pine.

Key Ideas

- Fractions, decimals, and percents are useful forms of data for different types of presentations.
- To express a percent as a fraction, write the value over 100 and drop the % sign.

  For example, $31\% = \frac{31}{100}$.

Lucia got just over $\frac{1}{2}$ the vote.

Communicate the Ideas

1. What fraction is yellow? Explain how to write the fraction as a decimal and as a percent.

2. What approximate fraction is 24%? Explain.

Check Your Understanding

Practise

For help with questions 3 to 5, refer to the Example.

3. Write each percent as a fraction. Simplify the fraction.

a) 25% **b)** 50% **c)** 10%
d) 20% **e)** 75% **f)** 100%

4. Write each percent as a fraction. Can you simplify the fraction?

a) 40% **b)** 55% **c)** 72%
d) 4% **e)** 5%

5. Write each percent as a decimal.

a) 32% **b)** 64% **c)** 70%
d) 83% **e)** 5%

6. Write each percent as a fraction.

a) 53% **b)** 77%
c) 9% **d)** 26%

7. Choose approximate fractions with small denominators to show the fractions from question 6 in a different way.

8. Write each fraction as a decimal. Then, write the decimal as a percent.

a) $\frac{13}{20}$ **b)** $\frac{14}{25}$ **c)** $\frac{3}{8}$
d) $\frac{9}{16}$ **e)** $\frac{2}{15}$

9. Copy and complete the table.

Fraction	Percent	Decimal
		0.15
	65%	
$\frac{1}{50}$		

Apply

10. Dilip and Françoise tried a quiz with 20 multiple-choice questions. Their scores were

Dilip 95% Françoise 80%

a) To skip the homework on this topic, students had to get at least 17 out of 20 correct answers. Who had to do the homework?

b) What strategy did you use to solve this problem? What other strategy might you use?

11. A 25-question multiple-choice quiz has these passing grades:

A at least 80%
B at least 68%
C at least 56%

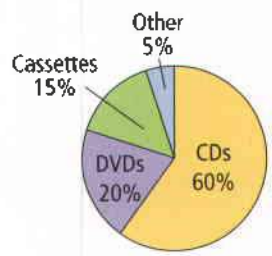
How many correct questions do you need for each passing grade?

12. What's wrong? Clara wrote 0.6 as a fraction this way.

$$0.6 = 6\%$$
$$= \frac{6}{100}$$
$$= \frac{3}{50}$$

Correct Clara's mistake. Explain your answer.

13. A music store had these sales for the month of January.

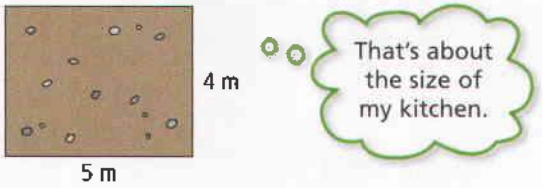


Write the percent sales

a) as fractions (for stock-taking)

b) as decimals (for tax calculations)

14. Wally bought 8 tomato plants, 3 cucumber plants, and 5 pepper plants for his vegetable garden. He wants to allow equal space for each plant.



a) What fraction of Wally's garden should he use for each type of plant?
b) What percent of the garden does each type of plant take?
c) Sketch a diagram to show how Wally could organize his plants.
d) What area will each type of plant need?

15. A newspaper report claims that "63% of children in the developing world do not have a safe water supply."

a) Rewrite this statement, using an approximate fraction.
b) You want to check how accurate the percent figure is. What information would you need to check? What relationship should you look for in the data? Explain.

16. A sales breakdown for the Ontario Beverage Company shows 22% cans, 67% bottles, and 9% party packs. Report the sales breakdown using approximate fractions with small denominators.

17. The Barkerville Dog Pound adopts out dogs and cats. One year, they found homes for 800 animals. 53% of these were cats.

a) Write the percent of cats as a decimal.
b) How many cats were adopted out?
c) How many dogs were adopted out?

18. Marie competes in chess tournaments. In her first year of competition, she won 9 games, lost 12, and tied 4.

a) Find the percents of wins, losses, and ties.
b) Marie plans to play 25 games a year in the next three years of competition. How many of these games do you expect Marie to win? Explain your answer.
c) Think about your answer to part b). What factors might change your answer? How could you investigate these factors?

Extend

19. Refer to question 18.

a) Create a single statistic that describes Marie's performance. Give your statistic a name. Calculate Marie's statistic for her first-year record.
b) Suppose Marie's record in her second year is 10 wins, 7 losses, and 8 ties. How will her statistic change?

20. Earth's mass is 0.0031 times the mass of Jupiter. This can be written as Earth = 0.0031 Jupiters. Similarly,

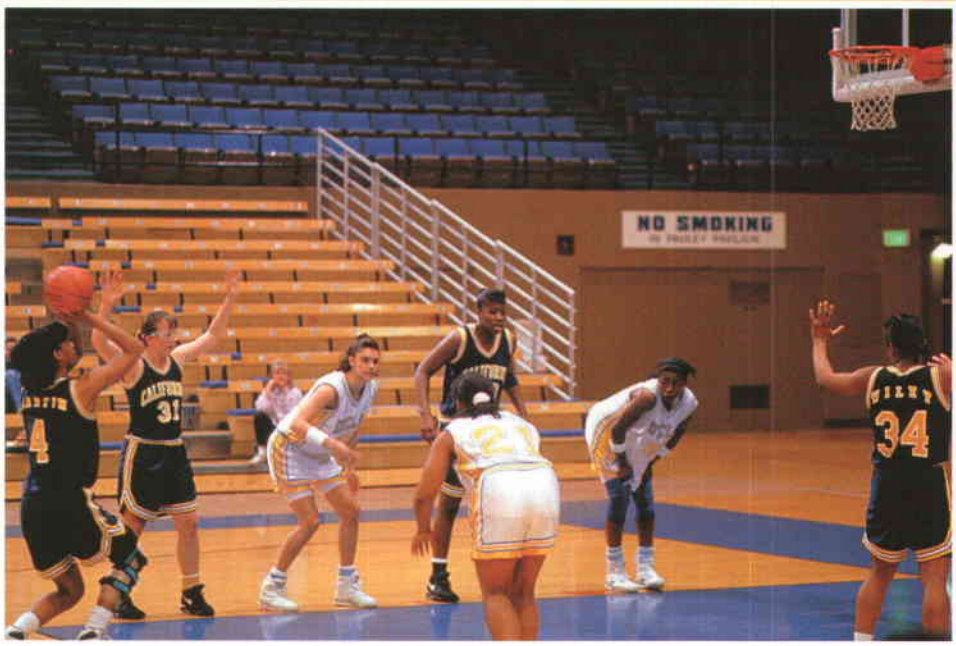
Mercury = 0.0002 Jupiters
Venus = 0.0026 Jupiters
Mars = 0.0003 Jupiters
Saturn = 0.2989 Jupiters
Uranus = 0.0457 Jupiters
Neptune = 0.0539 Jupiters
Pluto = 0.000 007 Jupiters

a) What is Earth's mass as a percent of Jupiter's mass?
b) Write the other planets' masses as percents of Jupiter's.
c) Determine each planet's mass, as a percent of the *total* planetary mass in our solar system. Illustrate your answer visually.

5.4 Apply Fractions, Decimals, and Percents

Focus on...

- fraction, decimal, and percent problems
- multi-step problems
- estimation



Your basketball team is in a tight playoff game. You are awarded a free throw. Scoring on this free throw could make the difference between winning and losing.

How can you use statistics to help you decide which player to send on the court?

Discover the Math

What strategies might you use to solve problems like these?

Example 1: Compare Sports Data

Pemba, Heather, and Roberto are the best free-throw players on your basketball team.

Player	Free-Throw Attempts	Good/No Good
Pemba	25	✓✓✓XX✓✓XXX✓✓X✓XXXX✓✓XX✓✓X
Heather	25	X✓X✓X✓XXX✓✓✓✓✓XXX✓✓✓✓XX✓✓
Roberto	20	XXX✓✓XX✓✓XXXX✓✓X✓✓✓X

a) Based on these records, who should you send on the court?
b) What other factors might you consider in making your decision?

Solution

Understand

a) Assume the player with the best record is the person most likely to make a free throw. You need to identify the best free-throw record.

Strategies
Make an assumption

Plan

1. Show each player's record as a fraction: $\frac{\text{successful free throws}}{\text{number of attempts}}$
2. Express the fractions as decimals or percents.
3. Compare to decide who has the best chance of scoring on a free throw.

Do It!

1. Show each player's record as a fraction.

Pemba: $\frac{\text{successful free throws}}{\text{number of attempts}} = \frac{12}{25}$

Heather: $\frac{14}{25}$ Roberto: $\frac{9}{20}$

2. Express the fractions in a different way.

Method 1: Write Fractions as Decimals

Pemba's record: $\frac{12}{25} = 12 \div 25$
$= 0.48$

Heather's record: $\frac{14}{25} = 14 \div 25$
$= 0.56$

Roberto's record: $\frac{9}{20} = 9 \div 20$
$= 0.45$

Method 2: Write Fractions as Percents

Pemba's record: $\frac{12}{25} = 12 \div 25$
$= 0.48 \times 100\%$
$= 48\%$

Heather's record: $\frac{14}{25} = 14 \div 25$
$= 0.56 \times 100\%$
$= 56\%$

Roberto's record: $\frac{9}{20} = 9 \div 20$
$= 0.45 \times 100\%$
$= 45\%$

3. Compare the free-throw records.

Method 1: As Decimals
$0.56 > 0.48 > 0.45$

Method 2: As Percents
$56\% > 48\% > 45\%$

Look Back

Heather scored on more than half of her attempts. Less than half of Pemba's and Roberto's attempts were successful. Heather should take the free throw.

b) Other factors to consider are
- current injuries
- most recent performances
- the ability to perform under pressure

Did You Know?

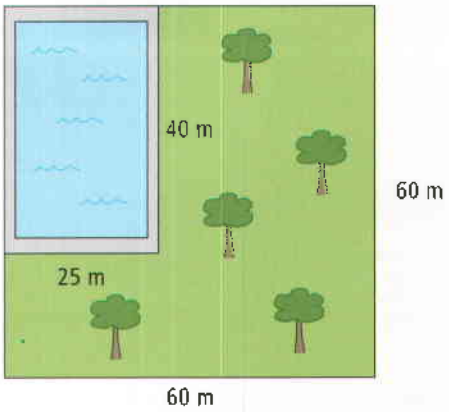
The statistic in Example 1 is called the free-throw percentage. It is used in several leagues, including the National Basketball Association (NBA).

Example 2: Calculate Percent of Area

A community park has a swimming pool.

a) Estimate the percent of the area of the park taken up by the pool.

b) Calculate the percent of the area taken up by the pool. Round your answer to the nearest percent.



Solution

a) The pool appears to take up approximately a quarter of the park. This is approximately 25%.

b) Calculate the areas.

	Shape	Area Formula	Area (m^2)
Swimming Pool	Rectangle	$A = l \times w$	$A = l \times w$ $A = 40 \times 25$ $A = 1000$
Park	Square	$A = s^2$	$A = s^2$ $A = 60 \times 60$ $A = 3600$

$$\frac{\text{area of pool}}{\text{area of park}} = \frac{1000}{3600}$$
$$= 0.2\overline{7}$$

Write this decimal as a percent.

$$0.2\overline{7} = 0.2\overline{7} \times 100\%$$
$$= 27.\overline{7}\%$$
$$\doteq 28\% \qquad \text{Round to the nearest percent.}$$

The pool takes up approximately 28% of the park.

Strategies

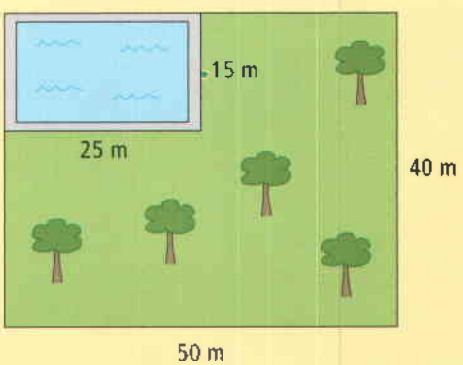
What strategy is used here? What other strategy might be used?

Making Connections

If you need help with calculating areas, go to Chapter 1, page 11.

Key Ideas

- In problems involving fractions, decimals, and percents, choose which form of the number to work with.
- Estimation is a good tool to check your conversion work. For example, the pool in this park is less than $\frac{1}{4}$ of the area of the park.

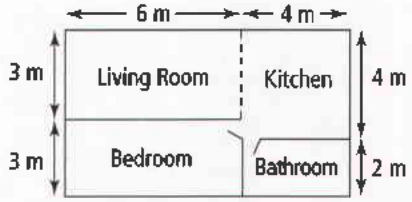


Communicate the Ideas

1. Pemba attempted 25 free throws. She made 12 of them and missed 13. Show her free-throw results as a fraction. Explain why a fraction is a useful way to show these statistics.

2. What other methods might you use to show Pemba's free-throw results? Describe how to do this.

3. Jasmine's apartment has a living room, a bedroom, a kitchen, and a bathroom. How can Jasmine find the percent of her apartment that is used for each room? Describe different strategies.



Check Your Understanding

Practise

4. Write each fraction as a decimal.

   a) $\frac{3}{16}$ b) $\frac{13}{20}$

   c) $\frac{5}{6}$ d) $\frac{17}{25}$

5. Write each fraction in question 4 as a percent.

6. Write each percent as a fraction. Simplify the fraction.

   a) 40% b) 55%

   c) 28% d) 66%

For help with questions 7 and 8, refer to Example 1.

7. Tamika was selling chocolate bars for a youth group. Two out of every five people she asked bought a chocolate bar. Show this result as

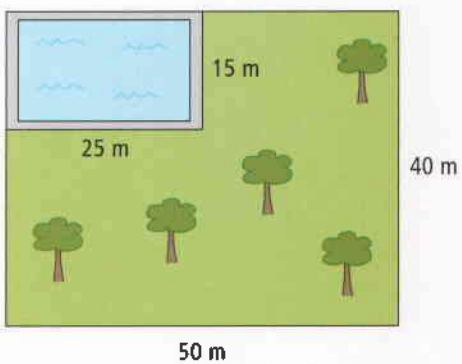
   a) a fraction
   b) a decimal
   c) a percent

8. In a survey, 7 out of every 20 people preferred hot chocolate to any flavour of juice. Show this result as a fraction, a decimal, and a percent.

Apply

For help with question 9, refer to Example 2.

9. A community park has a swimming pool.



   a) Estimate the percent of the area taken up by the pool.
   b) Calculate the percent of the area taken up by the pool. Round your answer to the nearest percent.

10. During hockey penalty shots, Paco can score a goal 9 times out of 10. Frederica can score a goal 13 times out of 15.

a) Show each player's statistic as a fraction, a decimal, and a percent. Use a table or some other method to organize your answer.

b) Who is the better scorer? Explain why.

11. During a snowy January, Snowbelt City buses were on schedule 60% of the time. What fraction of the time were the buses *not* on schedule?

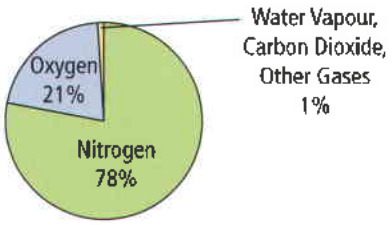
12. a) Rico played only 18 out of 32 hockey games last season due to an ankle injury. What percent of the season did Rico miss?

b) What strategy did you use to solve part a)? What other strategy might be useful?

13. The make-up of Earth's atmosphere is shown in the circle graph. What approximate fraction of air is made up of

a) nitrogen? **b)** oxygen?

c) other gases?



14. Kendra's CD collection has 12 dance CDs, 15 alternative CDs, and 18 rock CDs.

a) How large is Kendra's collection?

b) Estimate the percent of each type of CD in Kendra's collection.

c) Calculate the percent of each type of CD.

d) What fraction of the collection does each type of CD represent?

e) Describe another method you could have used in part d).

15. The Ontario Beverage Company makes seven different flavours of pop. Sales for the past two weeks are shown in the table.

Drink Flavour	Sales (Thousands)
Koala Cola	13
Lizard Lime	5
Lemur Lemon	8
Gorilla Grape	3
Roary Root Beer	9
Oliphaunt Orange	10
Jumping Ginger	2

Write a brief report on drink sales. Include

- total sales of drinks
- a visual showing percent sales of each flavour

Literacy Connections

Visuals in Reports

Bar graphs and circle graphs are good ways to show sales data. Visuals and few words make reports easy to read.

Try This!

16. Santino got $\frac{26}{40}$ on a test. He needs to get 80% to be on the honour roll.

a) Will he make it? Explain.

b) If not, how many marks does he need to get on the make-up test?

Extend

17. The producers of a 90-min action film decide that between 50% and 65% of the film should be action scenes. The film editor has 70 min of good action footage. Should the director ask the editor to cut any of this footage? If so, how much? Use pictures, numbers, and words to explain your answer.

18. In a board game, you are trapped in "detention." You have two chances to escape, based on rolling a pair of number cubes:

- roll doubles (the same number on both cubes)

or

- roll a total of at least 9

There is a catch: You have to decide in advance to try for one way or the other. Should you go for doubles, or a total of 9 or more? Justify your decision, using calculations.

19. A jacket is marked down 30% from the regular price of $150.

a) Find the sale price.

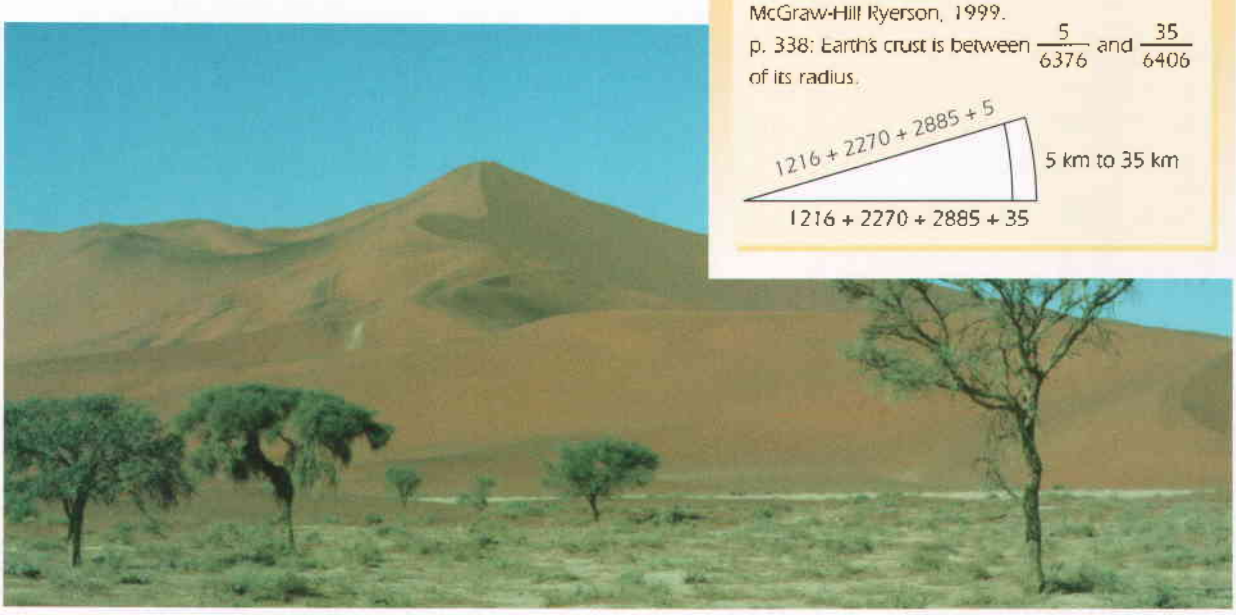
b) The sale price is not the final price! On most clothing in Ontario, there is a provincial sales tax (PST) of 8% and a Goods and Services Tax (GST) of 7%. Find the final price of the jacket.

Making Connections

Planet Earth in Percents

Good atlases and almanacs contain interesting data. Sometimes you have to work with data to create percents.

Research some information about planet Earth. Use fractions and percents to report your findings.



Literacy Connections

Keeping Track of Sources

File cards can help you research. Use one file card for each source.

Galbraith, Don. Sciencepower 7. McGraw-Hill Ryerson, 1999.
p. 338: Earth's crust is between $\frac{5}{6376}$ and $\frac{35}{6406}$ of its radius.

$$\frac{1216 + 2270 + 2885 + 5}{1216 + 2270 + 2885 + 35}$$

5 km to 35 km

CHAPTER 5

Review

Key Terms

For questions 1 to 3, copy the statement and fill in the blanks. Use some of these words:

fraction *percent*
repeating decimal *statistic*

1. A [redacted] is a value calculated from a set of data.
2. You can write a [redacted] decimal using a bar or a dot.
3. Another way to say "out of a hundred" is "[redacted]."
4. Unscramble the circled letters in questions 1 to 3 to answer this question:

   Who called the MPP to the committee meeting?

5.1 Fractions and Decimals, pages 152–157

5. Show each fraction using a diagram.
   a) $\frac{3}{5}$ b) $\frac{2}{3}$ c) $\frac{5}{8}$
6. A car has just come into Otto's Auto Body for emergency repairs. Otto checks to see which mechanic is closest to finishing his or her current job.

   Nico: "I'm $\frac{1}{3}$ of the way finished, boss."

   Jacques: "I'm about halfway done, Otto."

   Leah: "I still have about $\frac{3}{4}$ of the job to do, boss."

   a) Use a fraction to show how much of his or her work each mechanic has done.
   b) Who should be assigned the new job? Justify your answer.
7. Write each fraction as a decimal.
   a) $\frac{7}{12}$ b) $\frac{5}{9}$ c) $\frac{17}{31}$
8. Order the fractions in question 7 from least to greatest.

5.2 Calculate Percents, pages 158–161

9. Write each decimal as a percent.
   a) 0.65 b) 0.4
   c) 0.237 d) 0.008
10. Write each percent as a decimal.
    a) 26% b) 94.3%
    c) 7% d) 2%
11. Write each fraction as a decimal.
    a) $\frac{33}{100}$ b) $\frac{3}{50}$
    c) $\frac{7}{9}$ d) $\frac{11}{90}$
12. Sheila decides to put her best test into her portfolio. Her three top test scores are $\frac{28}{35}$, $\frac{33}{39}$, and $\frac{31}{37}$.
    a) Write each test score as a percent.
    b) Which test should Sheila put into her portfolio? Explain your decision.
13. On average, Chantal gets a hit 3 out of every 10 times at bat.
    a) What percent of the time does Chantal get a hit?
    b) Express this batting average as a decimal.
    c) How many hits would you expect Chantal to get in 180 at-bats? Explain how you found your answer.

5.3 Fractions, Decimals, and Percents, pages 162–165

14. Write each percent as a fraction. Simplify the fraction.

a) 20% **b)** 72%

c) 8% **d)** 98%

15. Copy and complete the table.

Percent	Decimal	Fraction
	0.88	
36%		
		$\frac{1}{5}$
42%		
4%		

16. A lasagna recipe calls for three types of cheese.

Cheese	Amount (Makes 8 Servings)
Mozzarella	400 g
Ricotta	350 g
Parmesan	250 g
Total	1000 g

a) Show each cheese type as a fraction of the total.

b) You want to calculate the amounts of cheese for three servings. Which form should you use for the amount of each cheese: fraction, decimal, or percent? Explain your choice.

c) Find the amount of each cheese in three servings of lasagna.

5.4 Apply Fractions, Decimals, and Percents, pages 166–171

17. Professor Bish has a collection of reference books. He estimates that he has these percents of each type of book.

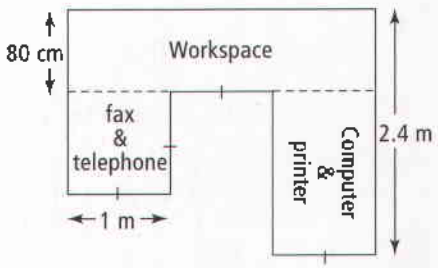
Type of Book	Percent
Engineering	35%
Mathematics	30%
Science	25%
Other	10%

Professor Bish has three equal-sized shelves that can easily fit all of his books.

a) How do you think Professor Bish should organize his books? Illustrate with a diagram.

b) Professor Bish has 180 books in total. How many would be placed on each shelf? Discuss any assumptions you must make.

18. Professor Bish organizes his office desk as shown.



Professor Bish feels that he needs at least 50% of his desk for workspace. Should he ask the university for a larger desk? Support your answer with calculations.

CHAPTER 5

Practice Test

Strand	NSN	MEA	GSS	PA	DMP
Questions	1–11	11	11		

Multiple Choice

For questions 1 to 6, choose the best answer.

1. In a bag of coloured candy, 25% of the candies are green. If the bag contains 48 candies, how many are green?

A 12 **B** 15
C 24 **D** 25

2. The decimal equivalent of $1\frac{1}{6}$ is

A 1.16 **B** $0.1\overline{6}$
C $1.1\overline{6}$ **D** $1.\overline{6}$

3. The fraction equivalent of 4% is

A $\frac{1}{4}$ **B** $\frac{2}{5}$
C $\frac{1}{25}$ **D** $\frac{1}{400}$

4. Vincent scored 17 out of 20 on his geography test. What percent did he achieve?

A 75% **B** 85%
C 90% **D** 95%

5. In a school survey, 75% of students were wearing blue jeans. What fraction of students surveyed does this represent?

A $\frac{3}{4}$ **B** $\frac{3}{5}$
C $\frac{75}{10}$ **D** $\frac{4}{5}$

6. The number 3.75 is greater than which fraction or mixed number?

A $\frac{19}{5}$ **B** $3\frac{2}{3}$
C $\frac{16}{3}$ **D** $3\frac{4}{5}$

Short Answer

7. Copy and complete the table.

Fraction	Decimal	Percent
		28%
$\frac{3}{8}$		
	0.45	
		5%
$\frac{24}{240}$		

8. Three types of recordable CD-ROMs are checked for faults.

Brand	Number Tested	Number Passed
Electro-Zip	20	19
Ultraback	10	7
A-Retrieve	30	23

a) What fraction of each brand passed the test?
b) Which brand of CD-ROM seems the most reliable? the least reliable? Justify your answers.

9. The table shows Hoshi's performance during a unit in science class.

Evaluation Item	Result
Lab	55%
Quiz	half of multiple-choice questions answered correctly
Test	29 out of 50

a) Arrange Hoshi's evaluation items from strongest performance to weakest.
b) Describe the method you used.

Extended Response

10. The table shows estimated percents for various tree species in a forest.

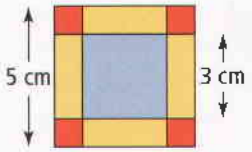
Type of Tree	Percent
Maple	30%
Oak	25%
Pine	20%
Birch	15%
Other	10%

a) What fraction of the forest is made up of each tree species?

b) Write each percent as a decimal.

c) The forest has approximately 1500 trees. Would you expect more or fewer than 500 maples? Explain.

11. At a fairground game, you can throw a dart at the target to win a prize.



a) Which do you think is easiest to hit: red, yellow, or blue? Explain your choice.

b) Every dart that lands in one of the coloured regions is a winner. What fraction of the total area is each colour?

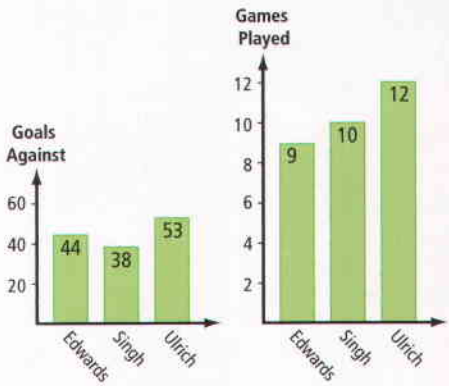
c) Rank the colours in order from greatest to least. Which colour should get the best prize, and why?

Chapter Problem Wrap-Up

A hockey coach is choosing a new goalie for her team. She has 20 games left in the season and only three players to choose from.

1. Use the information in the table to recommend which player the coach should choose. Provide as many reasons as you can for your recommendation.

2. What can the coach hope for from her goalie in the next 20 games? Explain.



Goalie	Save Percentage
Edwards	0.88
Singh	0.96
Ulrich	0.92

Patterning and Algebra

- Find and describe patterns in sequences of numbers and shapes.
- Extend a pattern, complete a table, and write words to explain the pattern.
- Use patterns to make predictions.
- Use patterning strategies in problem solving.
- Interpret a variable as a symbol that stands for a number.
- Evaluate simple algebraic expressions.

Data Management and Probability

- Organize, analyse, interpret, and present data mathematically and on graphs.

Key Words

variable natural numbers

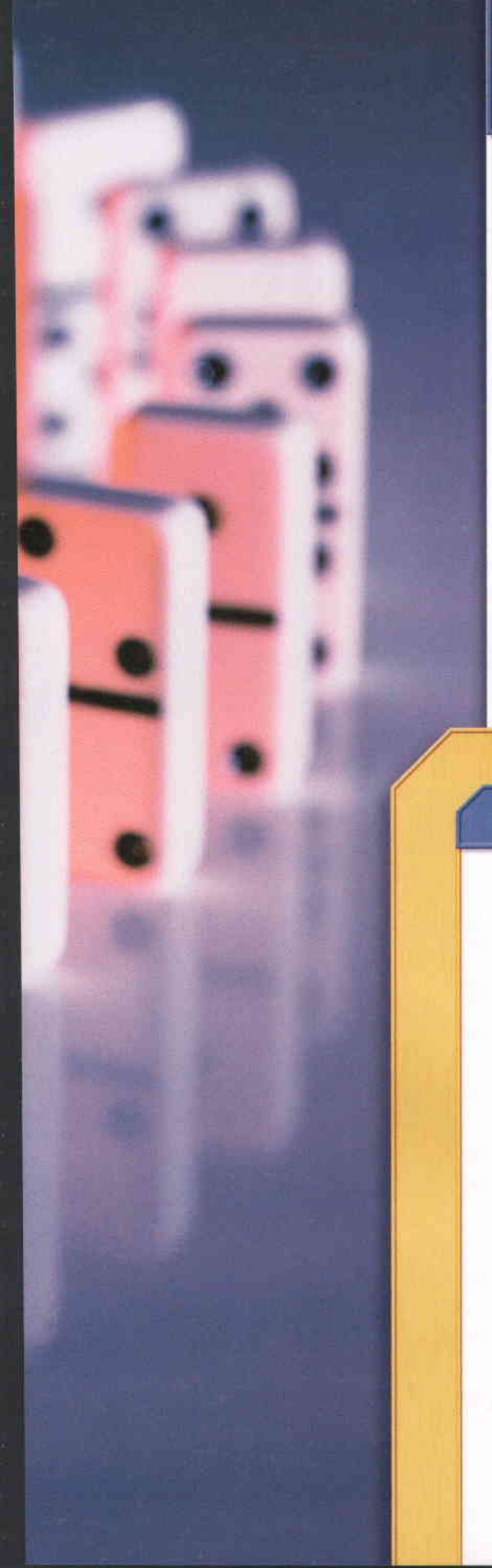
variable expression

pattern rule

table of values

relationship

algebraic equation



CHAPTER 6

Patterning

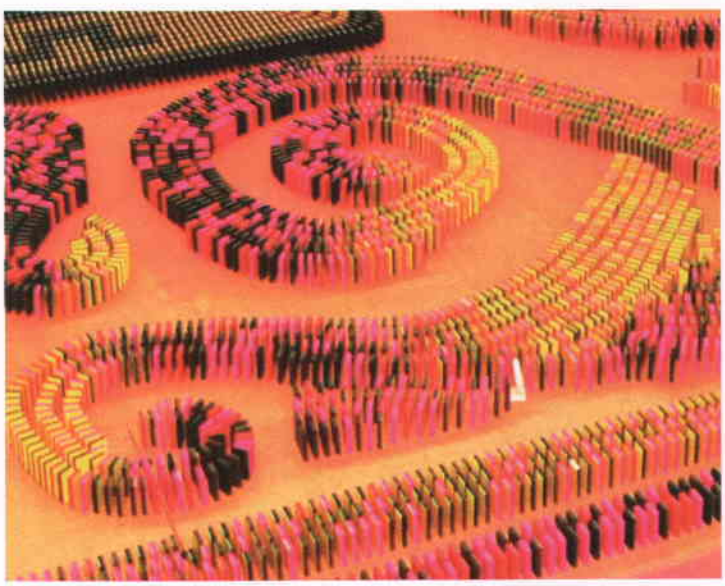
Dominoes can be set up to fall in a chain reaction called a domino topple. The world record for the most dominoes toppled in one attempt is 3 847 295. For record-breaking domino topples, the dominoes are often set up in unusual and interesting designs.

To prevent accidental toppling of the entire chain during set-up, stoppers are placed between the dominoes at regular intervals. The stoppers are removed just before the first domino is toppled.

How can math help in the topple-safe design of a large domino chain?

Chapter Problem

Describe all the patterns you can see in this domino layout.

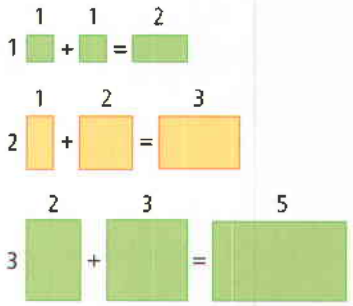


Get Ready

Explore Patterns

In math, many patterns can be predicted with complete accuracy. For example, to extend this pattern of rectangles,

- turn each rectangle through 90°
- add a square to the right
- the square's sides are the length of the rectangle
- put the rectangle and the square together to form a new rectangle
- repeat



This instruction predicts the entire pattern.

Examine the sequence of rectangle widths: 1, 1, 2, 3, 5, 8, 13, ... Each number in the sequence is the sum of the two previous numbers. For example,

$1 + 1 = 2$
$1 + 2 = 3$
$2 + 3 = 5$

By following this rule, you can predict the next two numbers in this sequence.

$8 + 13 = 21$
$13 + 21 = 34$

Did You Know?

The sequence 1, 1, 2, 3, 5, 8, 13, ... is called a Fibonacci sequence. Leonardo Fibonacci discovered it over 800 years ago, while investigating how quickly rabbits breed. Fibonacci sequences describe many patterns in nature. For example, the spiral of blades on a pine cone follows a Fibonacci sequence.

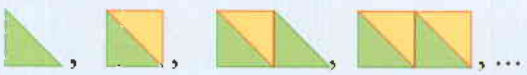
1. Describe the pattern in each sequence. Then, continue each pattern for three more items.

a) 

b) 2, 4, 6, 8, ...

c) 100, 95, 90, 85, ...

d) 3, 6, 12, 24, ...

e) 

f) $\frac{1}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$

g) ab, abc, abcd, ab, abc, abcd, ...

h) 😐, 😐😐, 😐😐😐, ...

2. Extend each number pattern.

a) Start with 8. Repeatedly add 7. List the next six numbers.

b) Start with 2. Repeatedly add 15. List the next three numbers.

c) Start with 100. Repeatedly subtract 12. List the next four numbers.

d) Start with 4. Repeatedly multiply by 3. List the next five numbers.

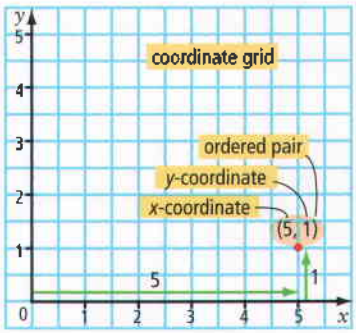
e) Start with 128. Repeatedly divide by 2. List the next three numbers.

f) Start with 2, 5. Repeatedly add the previous two numbers. List the next five numbers.

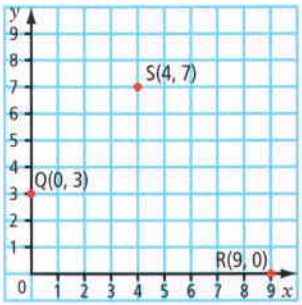
Use Graphing Skills

(5, 1) is an **ordered pair**. To plot (5, 1) on a **coordinate grid**,

- put your pencil at (0, 0)
- move 5 units to the right for the ***x*-coordinate**
- then, move up 1 unit for the ***y*-coordinate**
- mark a dot and label the coordinates



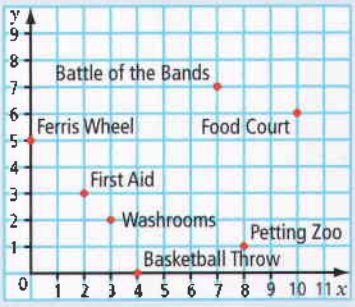
These points are plotted on the coordinate grid to the right.
Q(0, 3), R(9, 0), S(4, 7)



3. **a)** Draw a coordinate grid with numbers from 0 to 6 on each axis.
b) Plot these ordered pairs on your coordinate grid.
A(1, 0), B(2, 1), C(0, 2), D(2, 2), E(3, 3), F(4, 2), G(6, 2), H(4, 1), I(5, 0), J(3, 1)
c) Join the points in alphabetical order. Finish by joining J back to A.
d) Identify the shape.

4. For each ordered pair, copy and complete this statement: "Move ■ units to the right. Then, move ■ units up."
a) (4, 2) **b)** (7, 3) **c)** (3, 1) **d)** (1, 3)

5. The organizer of a fair plots the fair's layout on a coordinate grid. Write an ordered pair for each location.



6.1 Investigate and Describe Patterns

Focus on...
- patterns with numbers
- patterns with shapes
- describing patterns

The numbers on a calendar make some interesting mathematical patterns. If the last day of March is a Monday, what are the dates of the Fridays in April? How did you find these dates? What other patterns do you see?

Discover the Math

Materials

Optional:
- BLM 6.1A Calendar Patterns
- calendar page showing one month

What patterns can you find and describe in a calendar?

Look at this calendar page.

1. a) Can you find any patterns in the red square $\begin{array}{|c|c|}\hline 5 & 6 \\ \hline 12 & 13 \\ \hline \end{array}$ of the calendar?

b) Shift the square one position to the right. See if you get the same results.

c) Describe in words the pattern or patterns that you found.

JANUARY

S	M	T	W	T	F	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
19	20	21	22	23	24	25
26	27	28	29	30	31	

2. Find other patterns in the calendar.

3. **Reflect** Describe in words all the patterns that you found in step 2. Are some patterns similar to others? Try grouping your patterns.

Example 1: Describe the Number Pattern

a) Describe the pattern 1, 4, 9, 16, 25, ... in as many ways as you can. Use pictures, words, and numbers.

b) Find the next three numbers in the pattern.

Solution

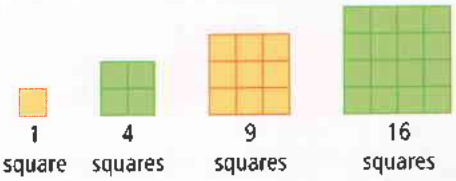
a) *Method 1: Use Words and Multiplication*

$1 \times 1 = 1$ $2 \times 2 = 4$ $3 \times 3 = 9$

$4 \times 4 = 16$ $5 \times 5 = 25$ ...

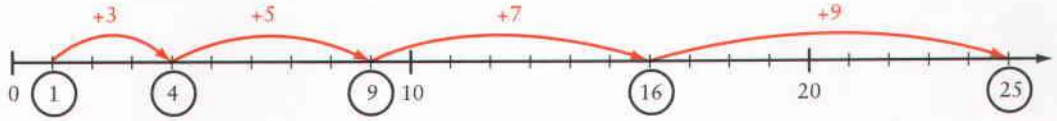
Multiply 1 by itself. Then, multiply 2 by itself, 3 by itself, and so on. The answers form the pattern.

Method 2: Use Pictures



The total number of squares at each stage creates the pattern 1, 4, 9, 16, 25, ...

Method 3: Draw a Number Line



The pattern jumps to the right in steps of ordered odd numbers.

b) $6 \times 6 = 36$ $7 \times 7 = 49$ $8 \times 8 = 64$

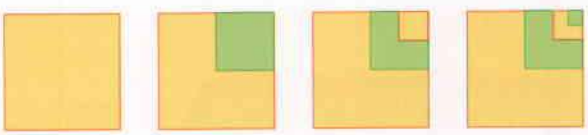
The next three numbers are 36, 49, and 64.

Strategies

What other methods can you use to find the next three numbers?

Example 2: Describe the Geometric Pattern

Study these figures. What geometric pattern do they show?



Solution

Method 1: Describe How to Create the Pattern

- Begin with a square.
- For the next square, halve the side length.
- Draw this smaller square in the upper-right corner of the previous square.
- Repeat this pattern over and over.

Method 2: Describe What the Pattern Looks Like

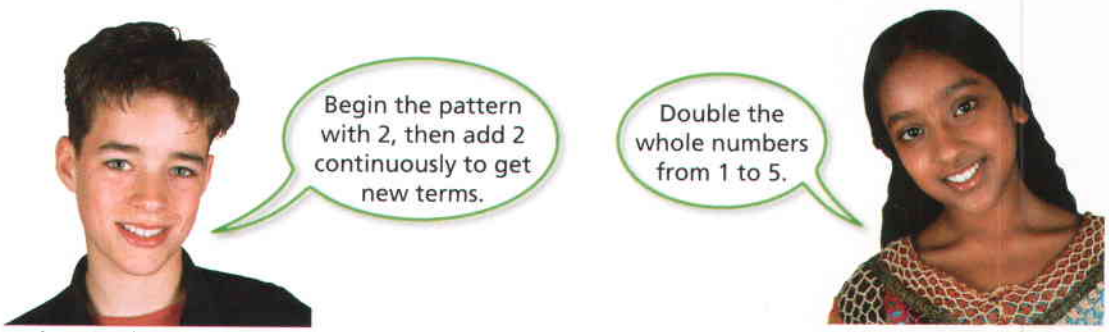
This is a pattern of squares within squares. Each new square has $\frac{1}{4}$ of the area of the previous square, and is in the upper-right corner.

Key Ideas

- In a pattern, you can predict what comes next.
- Some patterns are based on number operations. Other patterns are based on geometric shapes.
- To describe a pattern, identify the first item. Then, describe how the numbers or shapes that follow are generated. Relate each new item to previous items, or to the counting numbers 1, 2, 3, ...

Communicate the Ideas

1. Michel and Fareeha described the pattern 2, 4, 6, 8, 10, ...



Decide whether Michel, Fareeha, or both are correct. Explain.

2. Create your own pattern. Is there more than one way to describe the pattern? Explain.

Check Your Understanding

Practise

For help with questions 3 to 8, refer to Example 1.

3. a) Describe this pattern. Use pictures, words, and numbers.
5, 10, 15, 20, 25, ...
b) Find the next three numbers in the pattern.

4. a) Describe this pattern in as many ways as you can. Use pictures, words, and numbers.
1, 7, 13, 19, 25, ...
b) What are the next three numbers in the pattern?

5. a) Describe the pattern 0, 6, 14, 24, 36, 50, ... using numbers.
b) Predict the next two numbers in the pattern.

6. Describe each pattern. State the next three items.

a) 1, 2, 2, 3, 3, 3, ...
b) 2, 4, 6, 8, 10, ...
c) 100, 95, 90, 85, ...
d) abc, bcd, cde, ...
e) 256, 128, 64, 32, ...
f) vxz, uwy, tvx, ...

7. Make a number pattern and a letter pattern. Trade with a friend. Solve each other's patterns.

8. a) Describe this pattern using numbers.

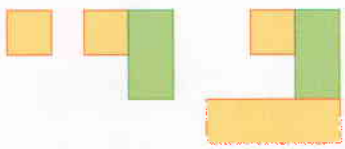
b) List the next three numbers of dots.

For help with questions 9 to 12, refer to Example 2.

9. Study these figures. Describe the geometric pattern.

10. Study these figures. Describe the geometric pattern.

11. Describe the geometric pattern.



12. Make a geometric pattern. Describe your pattern.

Apply

13. Copy and complete each pattern by replacing each ■ with the appropriate value.

a) 5, ■, 15, ■, ... (addition pattern)
b) 3, ■, 12, ■, ... (multiplication pattern)
c) ■, 16, ■, 12, ... (subtraction pattern)
d) ■, 200, ■, 50, ... (division pattern)

14. The old Chinese calendar used three 10-day weeks.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30

a) Choose any four numbers that form a square.
b) Multiply each pair of numbers in the diagonals.
c) Repeat this with another square of four numbers.
d) Describe what you notice.

Use this calendar page to answer questions 15 and 16.

JANUARY

S	M	T	W	T	F	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

15. a) Add the date of the first Wednesday to the date of the second Wednesday. Repeat for the first and second Thursdays, Fridays, and Saturdays.
b) Describe the pattern.
c) Use your description to find the next three numbers in the pattern.
d) Predict whether or not similar patterns will occur in later weeks. Explain your reasoning.

16. a) Multiply the date of the first Wednesday by the date of the second Wednesday. Repeat for the first and second Thursdays, Fridays, and Saturdays.
b) Describe the pattern.
c) Use your description to find the next three numbers in the pattern.

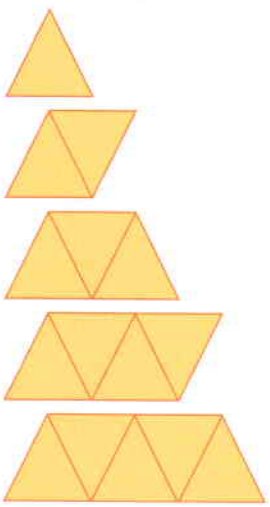
17. Create a pattern using simple geometric figures. Explain your pattern.

18. A pattern begins with the fraction $\frac{1}{2}$.
Then, 1 is added to both the numerator and the denominator.
a) State the next five fractions in the pattern.
b) As the pattern continues, do the fractions get bigger or smaller? Explain your reasoning.

Making Connections

In question 18, use what you learned about fractions in Chapter 3 to develop and describe a pattern.

19. Study this growing pattern.



a) Describe the pattern.
b) Why do you think it is called a growing pattern?
c) Draw the next two stages in the pattern.

20. a) Use toothpicks to make your own growing pattern. Describe your pattern in words.
b) Ask a partner to draw the next two stages in your pattern.

21. Study this shrinking pattern.

a) Describe the pattern.
b) Draw the next two stages in the pattern.

Try This!

22. a) Use toothpicks to make a shrinking pattern. Describe your pattern in words.
b) Ask a partner to draw and describe your pattern.

Extend

23. Alex gave Sasha the pattern 2, 4, ... , and asked him to write the next three numbers. Does Alex's question have a single answer? Explain your thinking.

24. Regularized Islamic years have 6 months with 30 days, alternating with 6 months of 29 days.
a) How long is an Islamic year?
b) How many Islamic years will it take to "gain" one year, compared to the Western calendar? Use a pattern to explain your answer.

Did You Know?

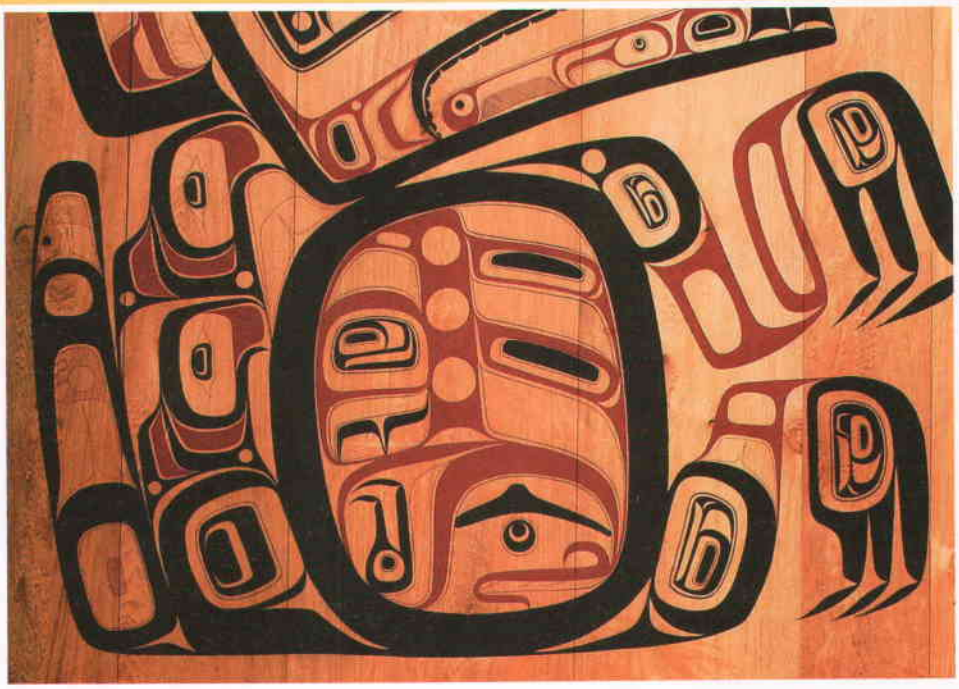
In the original Islamic calendar, each month begins when the first, thin crescent of the new moon is seen.

Internet Connect Research the history of calendars. Go to www.mcgrawhill.ca/links/math7 and follow the links to get started.

6.2 Organize, Extend, and Make Predictions

Focus on...

- describing patterns using numbers
- describing patterns using variables
- extending patterns
- making predictions



This is a house screen painting from Port Simpson, British Columbia. Identify the patterns on this painting. How many times does each pattern repeat?

Discover the Math

Materials

- toothpicks

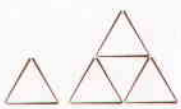
or

- plain paper or triangle dot paper

Optional

- BLM 6.2A Patterns With Toothpicks

How can you organize and predict patterns?



1. Use toothpicks to build the first four stages of this triangle pattern. Record all the patterns you can see.

2. **a)** How many toothpicks would you need for a triangle diagram with a base of 5 toothpicks?
   b) Predict the number of toothpicks in a triangle diagram with a base of 6 toothpicks.
   c) Check your predictions by modelling with toothpicks. How close were your predictions to your models? Explain.

3. **Reflect** How could you organize each stage of the pattern? How would this help you to extend the pattern?

Example 1: Organize and Extend a Pattern

a) Determine the perimeter (the number of toothpicks) of each of the first four triangles.

b) Show how each perimeter relates to the set of **natural numbers**.

c) Find the next three perimeters in the pattern.

natural numbers

- the numbers 1, 2, 3, ...
- also called positive integers

Solution

a) The first four perimeters use 3, 6, 9, and 12 toothpicks.

b)

Natural Number (Base Length)	Perimeter	Operation
1	3	3×1
2	6	3×2
3	9	3×3
4	12	3×4

Strategies
How could you show this pattern using addition?

c) The next three perimeters are

$3 \times 5 = 15$ $3 \times 6 = 18$ $3 \times 7 = 21$

Example 2: Describe and Extend a Pattern Using Variables

a) Show the relationship between the number of cubes in each stack and the number of vertical faces.

b) Use a **variable** to write a **variable expression** for the pattern.

c) Determine the number of vertical faces in a stack of 100 cubes.

variable

- a letter that can stand for any number

variable expression

- numbers and variables, combined by operations

Strategies
What other strategy could you use to show this relationship?

Solution

a) *Method 1: Use a Table*

Number of Cubes	Number of Vertical Faces	Expression
1	4	4×1
2	8	4×2
3	12	4×3

Method 2: Use Equations

$4 \times 1 = 4$

$4 \times 2 = 8$

$4 \times 3 = 12$

b) Use the variable n to stand for the number of cubes in a stack. Multiply the number of cubes, n, by 4. The variable expression for the number of vertical faces is $4 \times n$, or $4n$ for short.

c) $4n = 4 \times 100$ **Substitute 100 for n.**

$= 400$

There will be 400 vertical faces in a stack of 100 cubes.

Literacy Connections

Reading Expressions
A number beside a variable means you multiply the variable by the number. So, $4n = 4 \times n$.

Key Ideas

- Any letter can be used as a variable to represent a number.
- A variable can be used in an expression that shows how a pattern works. For example, $3 \times n$ or $3n$ shows how the perimeters of toothpick triangles grow.

Communicate the Ideas

1. Look at these growing patterns:
2, 4, 6, 8, ... 2, 4, 8, 16, ...
Explain how to find the next number in the growing patterns. What different techniques can you use?

2. You tell two friends a story.
- These friends each tell another two friends and then they each tell another two friends.
- Assume no two friends are the same.

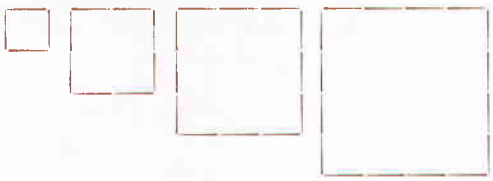
Explain how you can find out how many different people know the story after 5 rounds of telling it. Use pictures and words.

Check Your Understanding

Practise

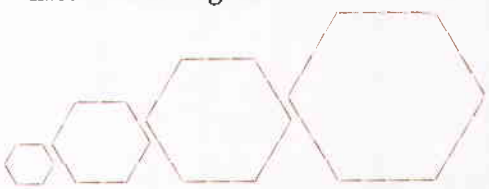
For help with questions 3 and 4, refer to Example 1.

3. a) Determine the perimeter of each of the first four squares.



b) Show how each perimeter relates to the set of natural numbers.

4. a) Determine the perimeter of each of the first four hexagons.



b) Describe how each perimeter relates to the natural numbers.

5. Describe each number pattern. Relate each item to the set of natural numbers.

a) 5, 10, 15, 20, ... **b)** 1, 4, 9, 16, 25, ...

c) 1, 3, 5, 7, ... **d)** $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$

6. For each pattern in question 5, predict the next three numbers.

For help with questions 7 to 9, refer to Example 2.

7. a) Show the relationship between the number of cubes in each block and the number of horizontal faces.

b) Write a variable expression for the pattern.

c) Use your variable expression to determine the number of horizontal faces in a 10×10 block.

8. In a room with 1 person, there are 8 large joints: 2 knees, 2 hips, 2 elbows, and 2 shoulders. How many large joints are there in a room with

a) 2 people? **b)** 3 people?
c) 4 people? **d)** x people?
e) 73 people?

9. Sam always eats 3 more candies than his friend. How many candies will Sam eat if his friend eats

a) 5 candies? **b)** 12 candies?
c) 100 candies? **d)** k candies?

Apply

10. One trillium has 3 petals.

a) How many petals do 2, 3, and 4 trilliums have?

b) How many petals do 100 trilliums have?

c) If there are 3000 petals, how many trilliums are there?

d) Explain the strategy you used to solve these questions.

11. a) Study these three toothpick shapes. Describe the pattern of the number of toothpicks in each shape.

b) Predict how many toothpicks would be in the fourth diagram. Give a reason for your prediction.

c) Draw the fourth diagram. If you need to change your prediction, explain why.

d) Predict how many toothpicks would be in the fifth diagram.

e) Consider the method you used to make pattern predictions in parts b) and d). Describe another method for predicting growing patterns.

12. A pattern of zigzags is made from line segments.

a) Draw the next two stages in the pattern.

b) Describe the relationship between the number of line segments and the number of acute angles.

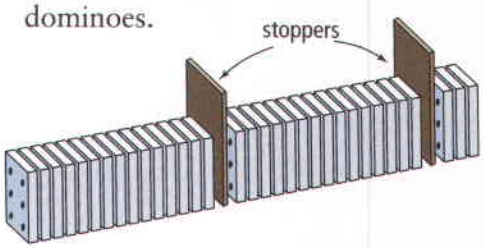
c) Choose a variable to represent either line segments or angles. Develop an expression to show the relationship.

Making Connections

You identified acute angles in Chapter 2.

Chapter Problem

13. a) Describe the pattern in this chain of dominoes.



b) If the pattern continues, how many stoppers are needed for a 40-domino chain? a 100-domino chain?

14. Try this magic math game. Copy the table to record your answers.

Directions	How the Numbers Change			Expression
Think of a number.				n
Add 7.				
Multiply by 3.				
Subtract 8.				
Subtract 3 × your original number.				

a) Think of a number. Add 7. Triple your answer. Subtract 8. Now subtract three times your original number.

b) Try this two more times with different numbers.

c) Using n as a variable, write an expression for each step.

d) Explain what you learned about this expression.

Try This!

15. Create your own magic math question. Fill in a table to show how it works.

Did You Know?

Internet Connect

Are you good at brain teasers? You might like to find out about Mensa, an international society that encourages and supports good thinkers and problem solvers. Go to www.mcgrawhill.ca/links/math7 and follow the links.

Extend

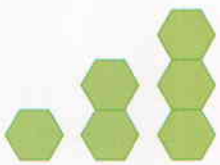
16. A solitaire game, sometimes called *The Tower of Hanoi*, uses different-sized coins or counters. For example, you could use a toonie, a loonie, a quarter, a nickel, a penny, and a dime.

- You have three tower bases.
- To begin with, the tower is on the left-hand base.
- You must rebuild the tower on another base.
- You can only move one counter at a time to any other base.
- You cannot place a counter on top of a smaller counter.

a) Try the game, with different tower heights. Discover and explain any patterns used in a successful game.

b) Investigate the minimum number of moves for each tower height to complete the game.

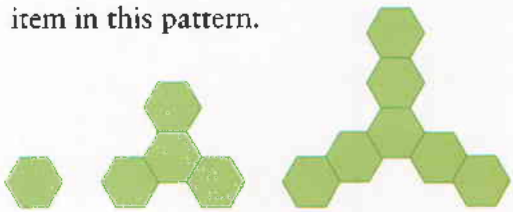
17. Sanjay is making hexagon trails. The table shows two different methods to find the perimeter for each trail length.



Steps	Perimeter	Pattern A	Pattern B
1	6	2 + 4	6
2	10	2 + 4 + 4	6 + 4
3	14	2 + 4 + 4 + 4	6 + 4 + 4

a) Choose either pattern A or Pattern B. Use your chosen pattern to find the perimeter of a trail with 150 hexagons.

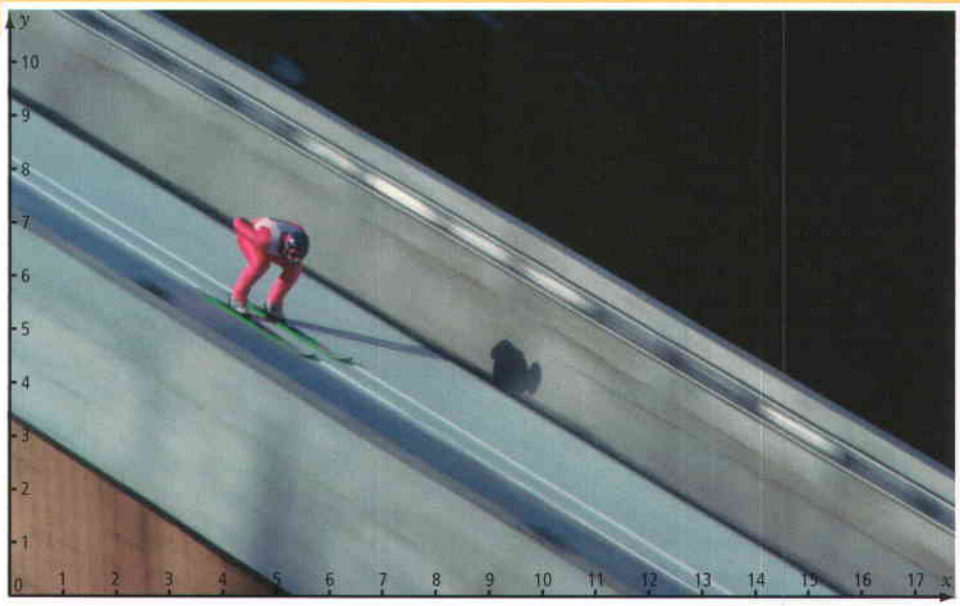
b) Determine the perimeter of the 10th item in this pattern.



6.3 Explore Patterns on a Grid or in a Table of Values

Focus on...

- relationships between numbers and variables
- describing patterns on a grid



Engineers can design long, steep ski jumps. They use mathematics to make the smooth curve by plotting ordered pairs on a grid.

How can you use a grid to represent where the skier is right now?

Materials

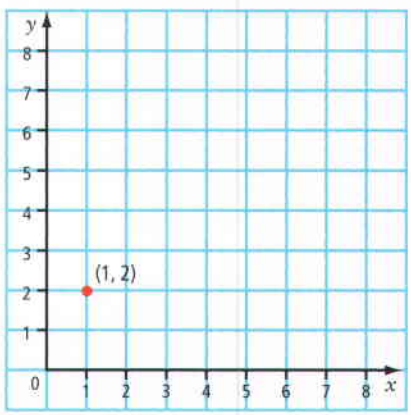
- grid paper

Discover the Math

How can you explore patterns on a grid?

1. Copy the coordinate grid below onto a piece of grid paper. Plot the ordered pair (1, 2) as shown.

2. **a)** From the point (1, 2), move 1 space right and 2 spaces up. Plot a second point.
   b) Repeat two more times, moving 1 space right and 2 spaces up each time.
   c) Describe the resulting pattern of points.



3. Suppose the x-coordinate is 25. Predict what the y-coordinate will be.

4. **a)** State a **pattern rule** for determining the y-coordinate when you are given the x-coordinate.
   b) How would your pattern rule change if the initial point were (3, 1)? Explain.

5. **Reflect** How does displaying a pattern on a grid help you to describe and extend the pattern?

pattern rule
- a simple statement that tells how to form or continue a pattern

Example 1: Plot Points and Describe Patterns

a) Plot the points given in the **table of values**.

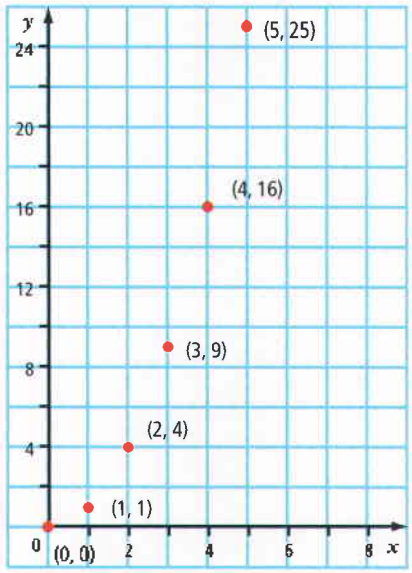
x	0	1	2	3	4	5
y	0	1	4	9	16	25

b) Describe the pattern of points.

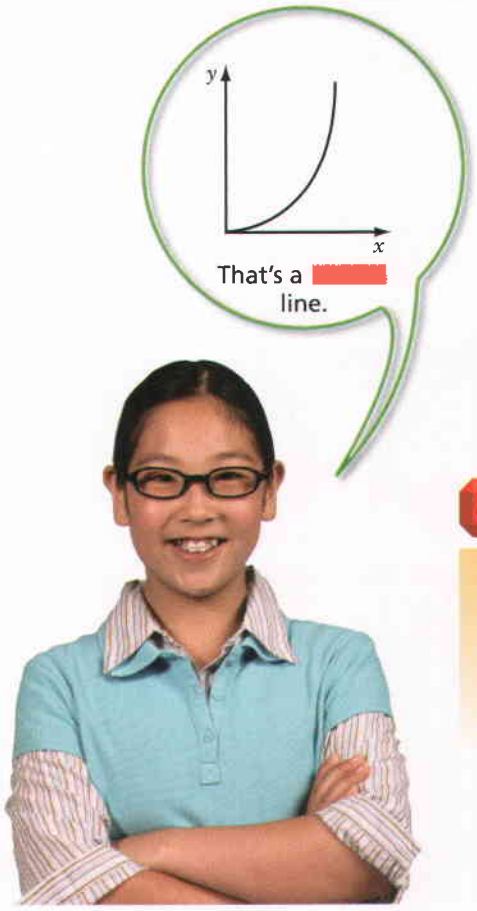
table of values
- table listing two sets of numbers that may be related

Solution

a)



b) The pattern of points curves upward. The y-values grow very quickly.

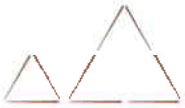


Literacy Connections

Look for a Pattern
Use your finger to trace a line joining the dots. What kind of line have you traced?

Example 2: Plot Points and Examine Relationships

Describe the **relationship** between the base and the perimeter of the toothpick triangles.



relationship

- pattern formed between two sets of numbers
- often seen in a table of values
- can be plotted on a coordinate grid

Solution

Understand

The relationship connects base length and perimeter.

Plan

1. Make a table of values comparing the base and perimeter.
2. Describe the relationship seen in the table.

Do It!

1. Table of values:

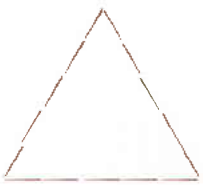
Base, b	Perimeter, P	(b, P)
1	3	(1, 3)
2	6	(2, 6)
3	9	(3, 9)
4	12	(4, 12)

2. As the base increases by 1, the perimeter increases by 3.

$1 \times 3 = 3$: That's (1, 3).
$2 \times 3 = 6$: That's (2, 6).

Look Back

The fifth diagram should have 15 toothpicks.



Key Ideas

- Patterns can be shown by listing ordered pairs in a table of values.
- A pattern rule is a description, in words, of a pattern. It is often used to predict the pattern. For example, in the pattern shown in the table, you multiply the base by 3 to get the perimeter.

Base, b	Perimeter, P	(b, P)
1	3	(1, 3)
2	6	(2, 6)
3	9	(3, 9)
4	12	(4, 12)

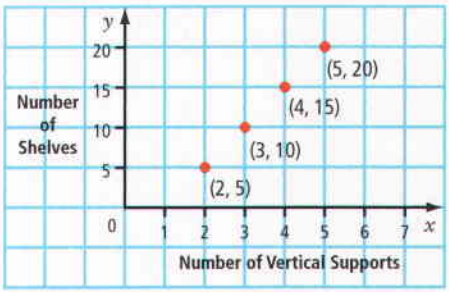
- A pattern between two sets of numbers is called a relationship.
- Plotting ordered pairs on a grid can help identify relationships.

Communicate the Ideas

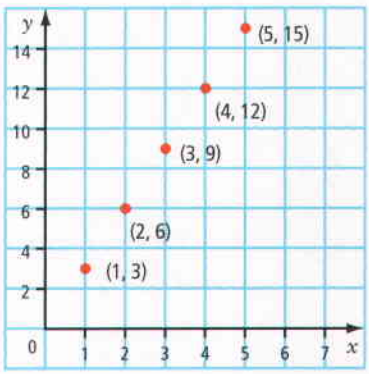
1. Andrea used a table of values to show the number of shelves in a library shelving unit. Carlos used a coordinate grid to show the same information.

Number of Vertical Supports	2	3	4	5
Number of Shelves	5	10	15	20

Which method do you find easier to use? Explain why.



2. Make a table of values for the ordered pairs shown on the grid. Then, create a question that leads to this pattern.



Check Your Understanding

Practise

For help with questions 3 to 5, refer to Example 1.

3. Plot the points given in each table of values. Then, describe the pattern of points.

a)

x	0	1	2	3	4	5
y	0	1	2	3	4	5

b)

x	0	1	2	3	4	5
y	15	12	9	6	3	0

4. Plot the points given in each table of values. Then, describe the pattern of points.

a)

x	3	4	5	6	7	8	9
y	1	2	3	4	5	6	7

b)

x	0	1	2	3	4	5	6
y	10	9	8	7	6	5	4

5. a) Plot the points given in the table of values.

x	0	1	2	3	4	5	6
y	0	7	12	15	16	15	12

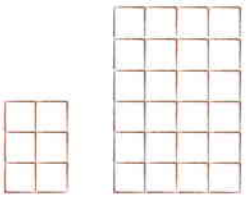
b) Describe the pattern of points. How is this pattern different from the patterns in questions 3 and 4?

For help with questions 6 and 7, refer to Example 2.

6. Describe the relationship between the base and the perimeter of these toothpick squares.

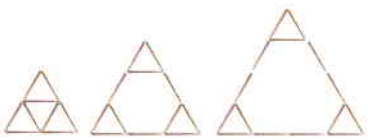
7. Describe the relationship between the width and the length of these toothpick rectangles.

8. Describe the relationship between the width and the length of these toothpick rectangles.



Apply

9. Give a pattern rule for the number of toothpicks in each stage of this pattern.



10. a) Find, and describe in words, a relationship between the width and the area of the squares in question 6.
b) Repeat part a) for question 7. Hint: Use the shorter sides.
c) Repeat part a) for question 8.

11. In the game of Battleship, ships sit on a 12 by 12 grid. Battleships can cover 4 points in a row, vertically, horizontally, or diagonally. A battleship sits with one endpoint at (5, 7). List all the points for each possible set of ordered pairs for this battleship. Hint: Use a picture or diagram.

12. When plotting these points on a grid, Ginny reversed the x- and y-coordinates. What happened to the resulting pattern?

x	0	1	2	3	4	5
y	0	1	4	9	16	25

Try This!

13. a) Make a table of values for the expression $4x + 1$. Use x-values from 0 to 5.
b) Plot the points on a coordinate grid.
c) Describe the pattern.

Making Connections

In question 13, you gave the variable x six values: 0, 1, 2, 3, 4, and 5. Learning how to use variables is a useful skill. This knowledge can help you program computers. You will work with variables in Chapter 12.

Extend

14. Asumi threw a ball into the air. The table of values gives

- the horizontal distance, d, in metres, from Asumi
- the height, h, in metres, of the ball at the same instant

d (horizontal distance)	h (height)
0	1
1	16
2	21
3	16
4	1

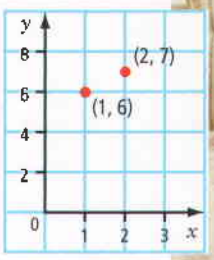
a) Plot the points on a grid.
b) Describe the shape of the path.
c) Explain the shape of the path at its highest point.

15. a) A pattern rule says that the y-coordinate is 5 more than the x-coordinate. Rewrite the rule to find the x-coordinate from the y-coordinate.
b) Create another pattern rule in the form "y-coordinate = one operation on x-coordinate." Rewrite your pattern to reverse it.
c) Create a method for reversing any simple pattern rule of the type in parts a) and b).

6.4 Express Simple Relationships

Focus on...
- relationships between numbers and variables
- solving problems with patterns
- evaluating algebraic expressions

Yukio charges \$6 per hour for babysitting the first child, plus \$1 per hour for each extra child. What is the relationship between hourly rate and the number of children?



Discover the Math

How can you express mathematical relationships?

Example 1: Describe an Algebraic Equation

The table of values shows the cost of a taxi ride from 0 km to 5 km in length.

a) Copy and complete the table.

b) Describe an **algebraic equation** for the cost of the taxi ride.

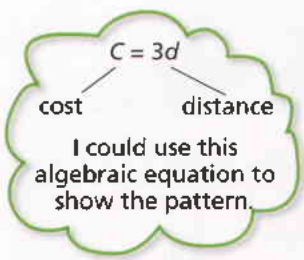
Distance, d (km)	Cost, C (\$)	(d, C)
0	0	(0, 0)
1	3	
2	6	
3	9	
4	12	
5	15	

algebraic equation
- an equation or formula that describes a relationship
- uses numbers and variables

Solution

a)

Distance, d (km)	Cost, C (\$)	(d, C)
0	0	(0, 0)
1	3	(1, 3)
2	6	(2, 6)
3	9	(3, 9)
4	12	(4, 14)
5	15	(5, 15)



b) The cost, in dollars, is 3 times the distance, in kilometres.
The formula for the cost, in dollars, of a taxi ride is $C = 3d$.

Example 2: Describe a Relationship

The equation $y = 4x$ shows the relationship between the number of squares and the number of sides.

a) Create a table of values for $y = 4x$. Use four consecutive values for x.

b) Describe how the number of sides changes relative to the number of squares.

Solution

a) Choose four x-values, for example, 2, 3, 4, and 5 squares.

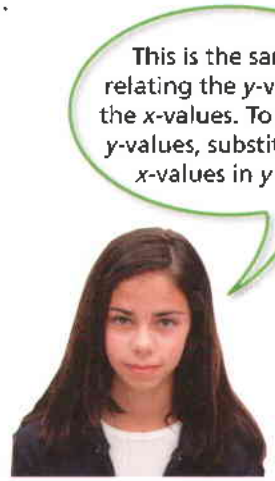
For $x = 2$, $y = 4(2)$, $y = 8$

For $x = 3$, $y = 4(3)$, $y = 12$

For $x = 4$, $y = 4(4)$, $y = 16$

For $x = 5$, $y = 4(5)$, $y = 20$

x	y	(x, y)
2	8	(2, 8)
3	12	(3, 12)
4	16	(4, 16)
5	20	(5, 20)



b) As x, the number of squares, increases by 1, y, the number of sides, increases by 4.

You can also say, There are four times as many sides as there are squares.

Did You Know?

Emmy Noether was a great mathematician and teacher. She worked closely with her university students. Together they made many breakthroughs in algebra. In the 1930s, she fled to the United States to escape Nazi persecution.

Key Ideas

- You can use an algebraic equation to express a relationship.

$$C = 3d$$

C: cost of taxi ride; d: distance of taxi ride

- Ordered pairs of data can be analysed to identify a relationship.

(2, 8): number of squares, number of sides

Communicate the Ideas

1. Develop an equation for the table of values to the right. Explain how you know you are correct.

Number of Wings, w	Number of Birds, b
2	1
4	2
6	3

2. Sara developed this table of values. She described the relationship: "My pay is eight times the number of hours I work." Then, she wrote this as

$h = 8p$

hours pay

Hours	Pay
1	8
2	16
3	24

Is Sara correct? Explain.

Check Your Understanding

Practise

For help with questions 3 to 6, refer to Example 1.

3. List the set of ordered pairs for each table of values. State the pattern rule.

a)

x	y
1	5
2	10
3	15
4	20

b)

x	y
0	4
1	5
2	6
3	7

4. State the pattern rule for each relationship.

a)

x	y
5	1
6	2
7	3
8	4

b)

x	y
0	6
1	4
2	2
3	0

5. The table of values shows the cost of a bus ride across 5 zones.

Zone, z	Cost, C ($)	(z, C)
1	2	(1, 2)
2	3	
3	4	
4	5	
5	6	

a) Copy the table. Complete the third column.

b) Describe the pattern.

6. The table shows Yukio's babysitting rates.

Number Children, c	Hourly Rate, R ($)	(c, R)
1	6	(1, 6)
2	7	(2, 7)
3	8	(3, 8)

a) Copy the table, and add two extra rows for 4 and 5 children. Complete the second and third columns.

b) Describe the pattern.

For help with questions 7 and 8, refer to Example 1.

7. **a)** Create a table of values for $y = 2x$. Use any four consecutive values for x.
b) Describe how the y-values change relative to the x-values.

8. For each equation, create a table of values. Describe the relationship.
a) $y = 3x$
b) $y = x + 2$
c) $y = 10 - x$
d) $y = 3x - 1$

Apply

9. **a)** Use the data in the table of values to write and solve a pattern problem involving money.
b) Trade problems with a friend. Use different strategies to solve each other's problems.

x	y
1	15
2	30
3	45
4	60
5	75

10. For the equation $y = x + 1$:
a) Complete a table of values for these values of x: 1, 2, 3, 4, and 5.
b) Plot the ordered pairs on a grid.
c) Explore how the y-values change as the x-values change. Explain what you see.

11. A box of Valentine candies is priced at $10.
a) Complete a table of values for selling from 1 to 10 boxes of candy.
b) Graph the information.
c) Write a formula for the cost, C, to purchase n boxes of Valentine candies.
d) Use your formula, or patterning, to predict the cost of 15 boxes of Valentine candies. Use your graph, or a diagram, to check your results.

Chapter Problem

12. A domino topple is being set up. Stoppers are used every 15 dominoes to prevent toppling during set-up.
a) How many stoppers are needed for an n-domino chain? Explain your solution.
b) How many stoppers are needed for a 1000-domino chain?
c) An individual world record for domino toppling was set in 2003 by Ma Lihua, in Singapore. She used 303 621 dominoes. How many stoppers would she have needed?

13. For each set of ordered pairs (x, y), describe in words how y relates to x. Write an equation using x and y.
a) (5, 4), (6, 5), (7, 6), (8, 7)
b) (0, 0), (1, 6), (2, 12), (3, 18)
c) (10, 6), (11, 7), (12, 8), (13, 9)
d) (3, 0), (5, 2), (7, 4), (9, 6)

Try This!

14. A catering company prepares meals for parties. They use the equation $M = p + 5$ to calculate the number of meals to make for a party. M stands for the number of meals. p stands for the number of people planning to go.
a) Copy and complete the table of values for the values of p shown.

p	M = p + 5	(p, M)
30	$M = p + 5$ $M = 30 + 5$ $M = 35$	
40		
50		
60		
70		

b) Describe how the values for M change relative to changes in p.
c) Why would the catering company include "+ 5" in the equation?

Extend

15. **a)** Develop a table of values for the area of squares with sides 1 cm to 6 cm long.

b) Explain the relationship between the side length and the area.

16. For the last day of school, special T-shirts with the name of every student in Mr. Vaish's class were ordered. It costs \$50 to set up for the printing and \$5 for every shirt printed.

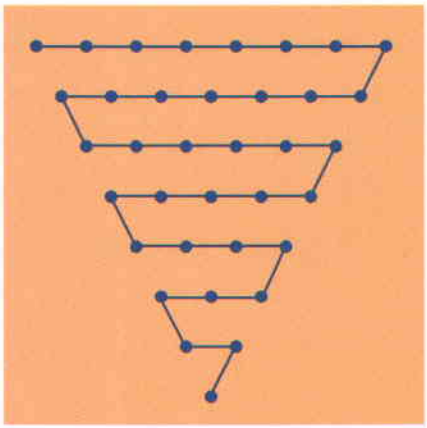
a) Make a table of values and a graph for this situation. Write an equation to represent it.

b) There are 23 students in the class. Which is the most efficient way, the graph or the equation, to predict the cost of T-shirts for the whole class? Justify your choice.

Making Connections

The Game of Bounce

The game of *Bounce* is for two or three players. Each player has two counters of the same colour.

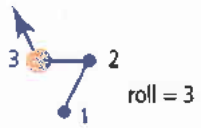


Materials

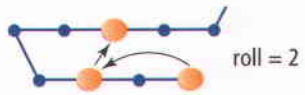
- BLM 6.4A Bounce Game Board
- two counters of the same colour per player
- number cube

1. Players take turns rolling the number cube and moving one of their counters forward by the number shown on the cube.

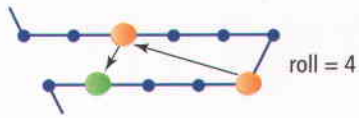
For example: This player started with a roll of 3.



2. Suppose you move a counter onto a dot that already has one of your own counters on it. You can "bounce" one of your counters forward one line, as shown.



3. Suppose you move a counter onto a dot that has an opponent's counter on it. You can "bounce" the other counter *back* one line, as shown.



4. You must roll the exact number needed to land on the final dot. You can then take that counter off.

5. The winner is the first player to take both of her or his counters off.

CHAPTER 6

Review

Key Words

For questions 1 to 4, choose the letter representing the term that best matches each statement.

1. all the counting numbers (1, 2, 3, ...)
2. a table used to record x- and y-coordinates
3. a pattern between two sets of numbers
4. a description of a pattern

a) relationship
b) natural numbers
c) pattern rule
d) variable
e) table of values

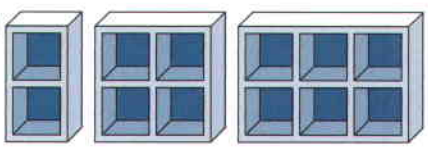
5. Write each sentence in your notebook. Fill in each blank using appropriate words from the list.

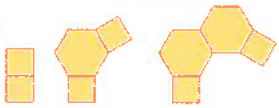
variable — algebraic equation
ordered pair — variable expression

a) $4x$ is an example of a/an _____.
b) (2, 7) is an example of a/an _____.
c) x is a/an _____.
d) $y = 4x$ is a/an _____.

6.1 Investigate and Describe Patterns, pages 180–184

6. Describe each pattern in words. Predict the next two items.

a) 

b) 7, 9, 11, 13
c) 243, 81, 27
d) a, cc, eee, gggg
e) 
f) 3, 7, 10, 17, 27, ...

7. Patterns that get smaller as they repeat forever are called fractals. Study this fractal pattern of triangles.

a) Describe the rule used to create this pattern.
b) Draw the next item in the pattern.

8. Invent a calendar for one month of your choice. Use your calendar to answer these questions.

a) Choose any four numbers that form a square.
b) Multiply each pair of numbers in the diagonals.
c) Choose another four numbers that form a square. Repeat step b).
d) Identify and describe any patterns that you notice.

6.2 Organize, Extend, and Make Predictions, pages 185–189

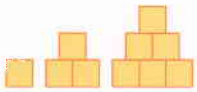
9. In a science experiment, Zena measured the growth of a bean plant.

Days After Germination	Height (cm)
1	1
2	3
3	5
4	7

If this pattern continues, what will the height of the plant be

a) after 5 days?
b) after 9 days?
c) after n days?
d) after 18 days?

10. Study this sequence of stacked 1-cm squares.



a) Draw the next two stacks.

b) Calculate the perimeter of each diagram. Organize the data. Hint: Use a table.

c) How does the perimeter relate to the previous perimeter?

d) How does the perimeter relate to the diagram number? Write an expression to show this.

6.3 Explore Patterns on a Grid or in a Table of Values, pages 190–194

11. Plot each set of points. Describe the pattern of points.

a)

x	y
1	0
2	3
3	6
4	9
5	12
6	15

b) (0, 21), (1, 13), (2, 9), (3, 7), (4, 6)

12. a) Describe the relationship between the width and the length of these toothpick rectangles.

b) Find, and describe in words, a relationship between the shorter side length and the area of the rectangles.

6.4 Express Simple Relationships, pages 195–199

13. a) List the ordered pairs in the table.

b) Describe the pattern in words.

x	y
1	3
2	6
3	9
4	12

14. a) Copy this table of values. Use the equation $y = x + 3$ to complete the table for the values of x shown.

x	y	(x, y)
1		
2		
3		
4		

b) List the ordered pairs.

c) Describe how the y-values change relative to changes in x.

15. A new computer business predicted its sales as shown.

Month	Number of Computers Sold
1	13
2	26
3	39
4	52
5	65
6	78

a) Show the information as ordered pairs.

b) Describe the pattern rule.

c) Using the pattern rule, predict the number of computers to be sold in the seventh month.

CHAPTER 6

Practice Test

Strand	NSN	MSN	GSS	PA	DMP
Questions				1–13	4, 5, 6, 13

Multiple Choice

For questions 1 to 6, select the best answer.

1. The next two terms in the sequence 4, 7, 10, 13, ... are

A 16, 19 **B** 17, 22
C 23, 36 **D** none of these

2. The next two terms in the sequence 7, 14, 28, 56, ... are

A 70, 84 **B** 84, 106
C 84, 119 **D** 112, 224

3. To find the next term in the sequence 1, 2, 4, 7, 11, ... ,

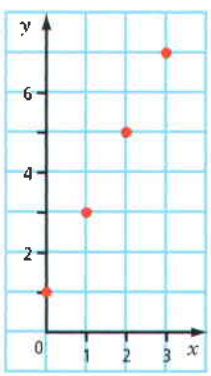
A add 8 **B** add 5
C add 11 **D** add 18

4. Match the table of values to the correct formula.

Number of Bees, b	Number of Legs, l
1	6
2	12
3	18

A $b = 6l$ **B** $b = l - 6$
C $l = 6b$ **D** $l = b \div 6$

5. The ordered pairs on the coordinate grid are



A (0, 0), (1, 3), (2, 6), (3, 9)
B (0, 1), (1, 3), (2, 5), (3, 7)
C (1, 0), (3, 1), (5, 2), (7, 3)
D (0, 1), (1, 2), (2, 4), (3, 6)

6. Which relationship describes the ordered pairs (0, 0), (2, 6), (4, 12), (6, 18)?

A $y = 2x$ **B** $6y = x$
C $y = 3x$ **D** $y = 6x$

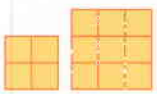
Short Answer

7. **a)** Plot the data as ordered pairs on a coordinate grid.
b) Describe the pattern in words.
c) Write an algebraic equation for y in terms of x.
d) Use your equation to find the value of y if $x = 15$.

x	y
2	4
3	5
4	6
5	7
6	8

8. Given the formula $y = 3x$, do the following:
a) Create a table of values for $x = 0, 1, 2, 3$, and 4.
b) List the ordered pairs.
c) Describe how the y-values relate to the x-values.

For questions 9 to 11, use this pattern of squares.



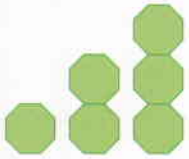
9. For a 2 by 2 square:
a) How many 1 by 1 squares are there?
b) How many 2 by 2 squares are there?
c) What is the total number of squares of various sizes?

10. For a 3 by 3 square:
a) How many 1 by 1 squares are there?
b) How many 2 by 2 squares are there?
c) How many 3 by 3 squares are there?
d) What is the total number of squares of various sizes?

11. **a)** What pattern can you see in the answers to questions 9 and 10? Use your pattern to predict the total number of squares in a 4 by 4 square.

b) Draw a 4 by 4 square and check your prediction.

12. Kara is planning a garden path with this pattern. The table shows two different patterns to find the path's perimeter.



Steps	Perimeter	Pattern A	Pattern B
1	8	2 + 6	8
2	14	2 + 6 + 6	8 + 6
3	20	2 + 6 + 6 + 6	8 + 6 + 6
4	26	2 + 6 + 6 + 6 + 6	8 + 6 + 6 + 6

Choose either Pattern A or Pattern B. Use your chosen pattern to find the perimeter of a path with 8 steps.

Extended Response

13. After a "Reduce, Reuse, Recycle" campaign, the Environment Club measured the mass of paper collected each week for a month.

Week Number, n	Total Mass, M, of Paper (kg)
1	175
2	350
3	525
4	700

a) Plot the data as ordered pairs on a coordinate grid.

b) Describe the pattern in words.

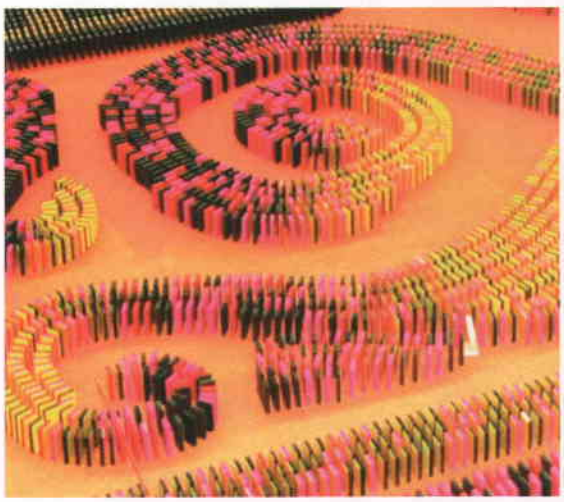
c) Predict how much paper will be recycled in one school year. Hint: How many weeks are there in a school year? Will exam weeks generate more recycling than usual?

Chapter Problem Wrap-Up

A domino chain begins with 100 dominoes, then splits off into three rows of dominoes. One row has 200 dominoes, the second has 300 dominoes, and the third has 400 dominoes.

1. Stoppers are placed every 30 dominoes. How many stoppers are needed? Where would you place your stoppers?

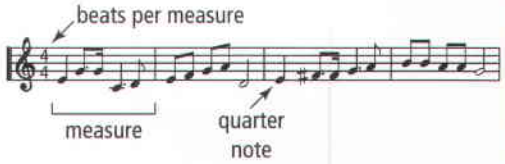
2. Are there other ways to place the stoppers? Do they use the same number of stoppers? Explain.



Making Connections

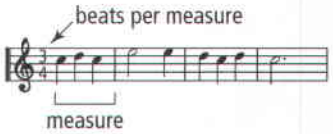
What do patterns have to do with music?

In $\frac{4}{4}$ time, one measure of music has 4 beats.



1. a) Calculate the total number of beats in 2, 3, and 4 measures.
b) What is the total number of beats in 12 measures?
c) What is the total number of beats in n measures?

In $\frac{3}{4}$ time, one measure of music has 3 beats. The nursery tune *Rock-a-Bye Baby* is in $\frac{3}{4}$ time.



2. a) Calculate the total number of beats in 4 measures of $\frac{3}{4}$ time.
b) How many beats are there in n measures of $\frac{3}{4}$ time?

Making Connections

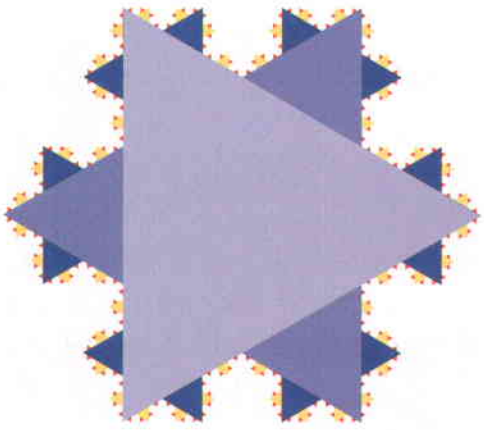
What do you get when you mix mathematics and art?

Fractals!

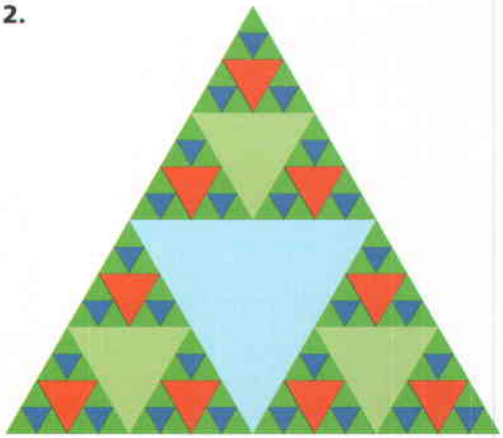
Fractals are often used to model natural forms such as jagged shorelines and peaked mountains.

What patterns can you see in each fractal?

1.



2.



Fold Fractals

TASK

Fractals are mathematical patterns made by repeating the same patterns over and over again, at smaller and smaller sizes. Follow these steps to make a fractal sheet.

Materials

- cardboard or construction paper
- ruler
- scissors
- pencil

Optional:

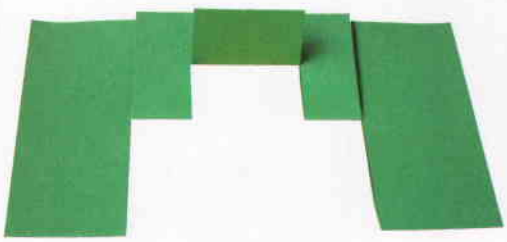
- pencil crayons

1. a) Fold a piece of paper exactly in half, with the two short sides together.

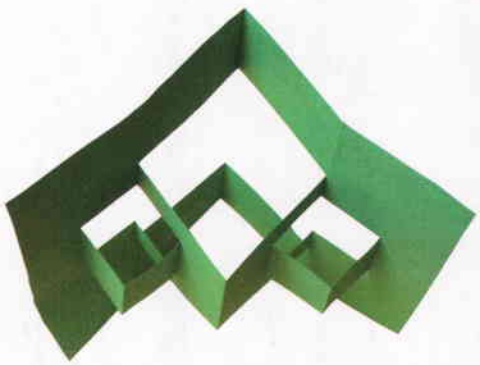
b) Find and mark the $\frac{1}{4}$ and $\frac{3}{4}$ positions along the fold.

c) At both the $\frac{1}{4}$ and $\frac{3}{4}$ marks, make a vertical cut from the fold to halfway up the paper. Fold the midsection up to form an upside-down U-shape.

2. Repeat steps 1b) and c) using the smaller, double-layer rectangle formed by the cuts and folds. You should now have two sets of cuts.



3. Unfold the paper. Pop out the fractals.



4. Investigate the shapes and patterns in your fractal sheet. Use your knowledge of shapes, patterns, fractions, and percents to report on what you see. What happens if you extend the pattern?

Number Sense and Numeration

- Compare, order, and represent decimals, multiples, factors, and square roots.
- Represent exponents as repeated multiplication.
- Represent perfect squares and their square roots in a variety of ways.
- Explain the process of problem solving using appropriate mathematical language.

Patterning and Algebra

- Identify and use a pattern.
- Explain and extend a pattern.

Key Words

square
cube
exponential form
base
exponent
square root
perfect square
power

CHAPTER 7

Exponents

A healthy lake or pond is home to many living things, including birds, fish, and plants. Small plants called algae live on the surface of the water. As they grow, they add oxygen to the water. They also provide food for fish and other animals.

Pollutants, such as fertilizers and sewage, can make the algae grow too quickly. This can make a large body of water look like green soup. When the huge numbers of algae die and decay, they use up much of the oxygen in the water. Then the water may not support life.

Large numbers like the number of algae in a polluted pond can be expressed using exponents. In this chapter, you will learn how exponents can be used to represent numbers.

Chapter Problem

How long will it take for algae to completely cover a polluted pond?

By the end of this chapter, you will be able to answer this question.

Get Ready

Factors and Multiples

The **factors** of 6 are 1, 2, 3, and 6.
The following pairs of these factors multiply to make 6.
$1 \times 6 = 6$ $2 \times 3 = 6$ $3 \times 2 = 6$ $6 \times 1 = 6$

The first four **multiples** of 5 are 5, 10, 15, and 20.
Each multiple is the product of 5 and a natural number.
$5 \times 1 = 5$ $5 \times 2 = 10$ $5 \times 3 = 15$ $5 \times 4 = 20$

1. List the factors of each number.
a) 8 **b)** 17 **c)** 24

2. Which of the following numbers have 2 as a factor? How can you tell?
100 301 456 294 279 193

3. List the first four multiples of each number.
a) 4 **b)** 8 **c)** 6

4. List the first three multiples of each number.
a) 10 **b)** 12 **c)** 20

True Statements

The symbol > means "is greater than." So, $3 > 2$ is a true statement.
The symbol < means "is less than." So, $2 < 3$ is a true statement.
The symbol = means "is equal to." So, $3 = 3$ is a true statement.

5. Rewrite each statement. Replace ■ with >, <, or = to make a true statement.
a) 13 ■ 14 **b)** 13.6 ■ 13.5
c) 8×3 ■ 2×12

6. Find the missing number that makes a true statement.
a) ■ + 7 = 12 **b)** 9 − ■ = 3
c) 5 × ■ = 20 **d)** ■ ÷ 6 = 4

Unit Conversions

Here are some relationships between metric units of length.
Measurements are often converted from one unit to another.

$3 \text{ m} = 3 \times 100 \text{ cm}$
$= 300 \text{ cm}$

I am converting from metres to centimetres. I am converting to a smaller unit, so multiply.

$5000 \text{ m} = \frac{5000}{1000} \text{ km}$
$= 5 \text{ km}$

I am converting to a larger unit, so divide.

10 mm = 1 cm
1000 mm = 1 m
100 cm = 1 m
1000 m = 1 km

7. Copy and complete each unit conversion.

a) 2 km = ■ m
b) 2000 mm = ■ m
c) 30 cm = ■ mm

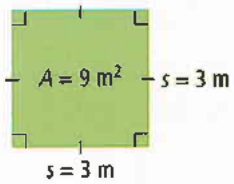
8. Convert.

a) 200 m to kilometres
b) 550 cm to metres
c) 0.5 m to millimetres

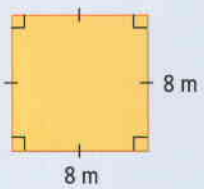
Area of a Square

You can find the area of a square by multiplying the side length by itself.

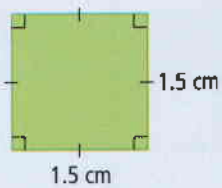
$A = s \times s$
$A = 3 \times 3$
$A = 9$
The area is 9 m^2.



9. Find the area of the square.



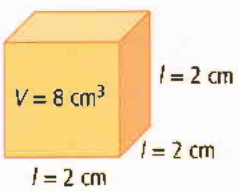
10. Find the area of the square.



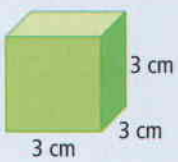
Volume of a Cube

You can find the volume of a cube by multiplying three edge lengths.

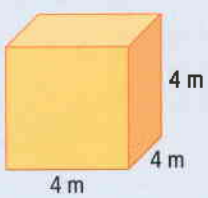
$V = l \times l \times l$
$V = 2 \times 2 \times 2$
$V = 8$
The volume is 8 cm^3.



11. Find the volume of the cube.



12. Find the volume of the cube.



7.1 Understand Exponents

Focus on...
- squares
- cubes
- exponents
- exponential form

How are the designs of these square flags related?


Switzerland


International Red Cross

Discover the Math

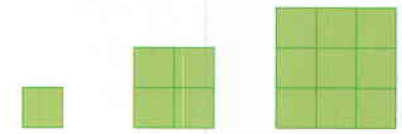
How can you represent squares and cubes?

Materials
- ruler
- centimetre grid paper or square tiles
- centimetre cubes

1. The diagrams represent the first three square numbers on a grid.

a) Use centimetre grid paper or square tiles to continue the pattern.

b) How is each diagram related to its number sentence?

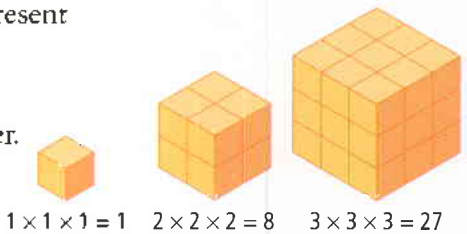


square number
- the product of two equal factors, for example, $3 \times 3 = 9$
- represents the area of a square

2. The diagrams show models that represent the first three cubic numbers.

a) Use centimetre cubes or diagrams to represent the next cubic number.

b) How is each diagram related to its number sentence?



cubic number
- the product of three equal factors, for example, $2 \times 2 \times 2 = 8$
- represents the volume of a cube

3. Which part of a model of a cubic number represents a square number? Explain.

4. **Reflect** Describe a square number and a cubic number.

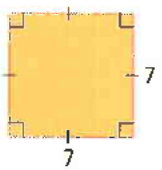
Example 1: Evaluate Squares

Find the area of a square with each side length.

a) 7 **b)** 3.1

Solution

a) $A = s^2$
$A = 7^2$
$A = 7 \times 7$
$A = 49$
The area is 49 square units.



Literacy Connections

Reading Squares
You can write expressions like 3×3 as squares.
$3 \times 3 = 3^2$
You can read 3^2 as "three squared."

b) Use a calculator.

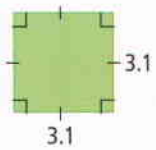
$A = s^2$

$A = 3.1^2$

$A = 9.61$

C 3.1 x^2 9.61 or C 3.1 × 3.1 = 9.61

The area is 9.61 square units.



Estimate: $3^2 = 9$

Example 2: Evaluate Cubes

Find the volume of a cube with each edge length.

a) 6 **b)** 15

Solution

a) $V = l^3$

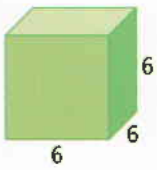
$V = 6^3$

$V = 6 \times 6 \times 6$

$V = 36 \times 6$

$V = 216$

The volume is 216 cubic units.



b) Use a calculator.

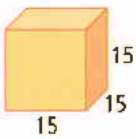
$V = l^3$

$V = 15^3$

$V = 3375$

C 15 x^y 3 = 3375. or C 15 × 15 × 15 = 3375.

The volume is 3375 cubic units.

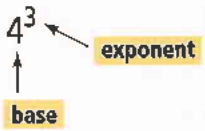


Estimate: $15 \times 15 \doteq 10 \times 20$
$= 200$
$200 \times 15 = 3000$

A number in the form 4^3 is in **exponential form**.

4^3 is the product of three 4s.

$4^3 = \underbrace{4 \times 4 \times 4}_{3 \text{ factors}}$



Example 3: Use Exponential Form

Write each repeated multiplication in exponential form.

a) 18×18 **b)** $11 \times 11 \times 11$

Solution

a) $18 \times 18 = 18^2$

There are two 18s, so the exponent is 2.

b) $11 \times 11 \times 11 = 11^3$

There are three 11s, so the exponent is 3.

Literacy Connections

Reading Cubes

You can write expressions like $4 \times 4 \times 4$ as cubes.

$4 \times 4 \times 4 = 4^3$

You can read 4^3 as "four cubed."

exponential form

- a shorter method for writing numbers expressed as repeated multiplication

$4 \times 4 \times 4 = 4^3$

repeated multiplication — exponential form

base

- the factor you multiply

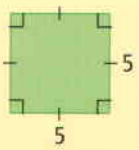
exponent

- the number of factors you multiply

Key Ideas

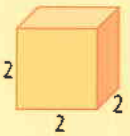
- Exponents represent repeated multiplication. For example, $7 \times 7 = 7^2$. The exponent is 2.
- A square is the product of two equal factors.

$A = 5 \times 5$
$A = 5^2$
$A = 25$



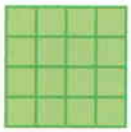
- A cube is the product of three equal factors.

$V = 2 \times 2 \times 2$
$V = 2^3$
$V = 8$



Communicate the Ideas

1. Exponents can be thought of as a form of shorthand. Explain why.
2. Use exponents to represent the area of this square.



3. Explain why 2^3 can be named "two cubed."

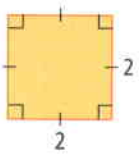
Check Your Understanding

Practise

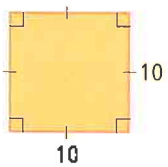
For help with questions 4 and 5, refer to Example 1.

4. Find the area of each square.

a)



b)

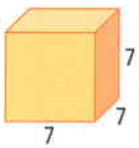


5. Find the area of a square with each side length.
   - **a)** 6
   - **b)** 12
   - **c)** 11

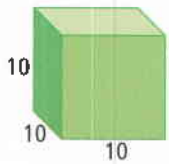
For help with questions 6 and 7, refer to Example 2.

6. Find the volume of each cube.

a)



b)



7. Find the volume of a cube with each edge length.
   - **a)** 5
   - **b)** 12
   - **c)** 20

For help with questions 8 and 9, refer to Example 3.

8. Write each repeated multiplication in exponential form.

a) 13×13 **b)** $25 \times 25 \times 25$

9. Write as a repeated multiplication.

a) 9^2 **b)** 7^3 **c)** 12^3

Apply

10. Rewrite each statement. Replace ■ with >, <, or = to make a true statement.

a) 2^3 ■ 3^3 **b)** 4^3 ■ 4^2

c) 5^3 ■ 10^2 **d)** 1^2 ■ 1^3

11. Evaluate using a calculator.

a) 1.3^2 **b)** 2.4^2

c) 4.1^2 **d)** 1.2^3

e) 3.2^3 **f)** 2.5^3

12. Does $2^3 = 2 \times 3$? Explain.

13. Write these expressions in order from the greatest value to the least value.

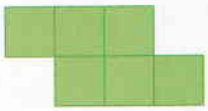
$10 \times 10 \times 10$ 20^2 8^3 25×25

14. The pyramid of Menkaure in Egypt has a square base with a side length of 105 m. What is the area of the base?

15. A square with a side length of 1 cm has an area of 1 cm^2. Explain why the unit of area includes the exponent 2.

16. The total length of all the edges on a cube is 36 cm. What is the volume of the cube?

17. The perimeter of this figure is 48 cm. What is its area?



Try This!

18. a) Use diagrams to show why 16 and 4 are square numbers.

b) The product of 8×2 is 16. The quotient of $8 \div 2$ is 4. Find three other pairs of numbers whose product and quotient are both square numbers.

c) What is the relationship between the numbers in each pair?

Extend

19. Rama used his calculator to explore some patterns using exponents. Copy and complete the first three rows of each pattern. Then, write the next three rows. Describe each pattern in words.

a) $1^3 = ■^2$

$1^3 + 2^3 = ■^2$

$1^3 + 2^3 + 3^3 = ■^2$

b) $3^2 - 1^2 = ■^3$

$6^2 - 3^2 = ■^3$

$10^2 - 6^2 = ■^3$

20. The diagrams represent the first four triangular numbers.

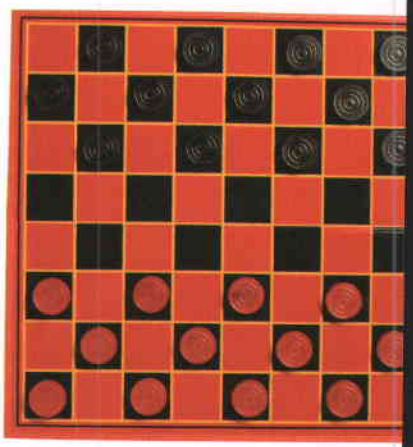
a) Describe a relationship between triangular numbers and square numbers.

b) Use diagrams to explain the relationship.

7.2 Represent and Evaluate Square Roots

Focus on...
- square roots
- perfect squares

A checkerboard is a square. The playing surface is divided into smaller squares. How many small squares are along each side? How many small squares are there altogether?



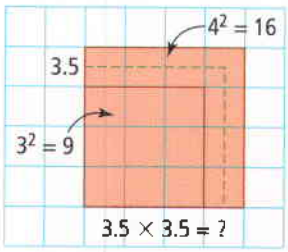
Notice that 8 is the side length of a square with an area of 64.
$8 \times 8 = 64$
The two factors, 8 and 8, are equal.
8 is the **square root** of 64.

square root (of a number)
- a factor that multiplies by itself to give that number

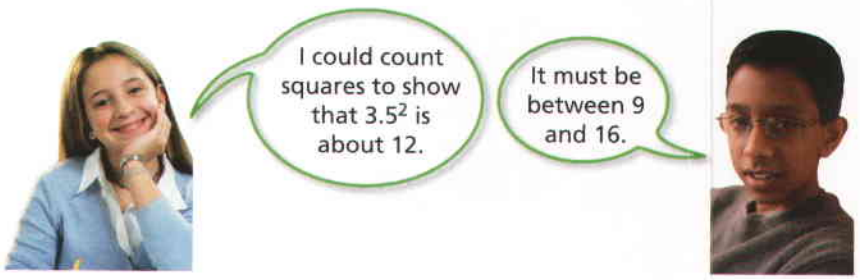
Discover the Math

How can you represent and evaluate square roots?

Materials
- ruler
- grid paper



1. Cheryl and Amar used grid paper to estimate the value of 3.5^2.
- **a)** Describe what each of them did.
- **b)** How does the diagram show that the square root of 12 is between 3 and 4?



2. The diagram gave Amar an idea. He said, "I can use grid paper to show that the square root of 30 is between 5 and 6." Describe what he might have done.

3. **Reflect** Describe how you could use grid paper to
- **a)** estimate 4.5^2
- **b)** show that the square root of 42 is between 6 and 7

Example 1: Evaluate Square Roots

Find the side length of a square with the given area.

a) 81 cm^2 **b)** 11.56 m^2

Solution

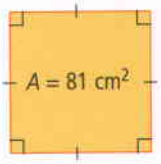
The side length is the square root of the area.

a) Find a factor that multiplies by itself to give 81.

$9 \times 9 = 81$

So, $\sqrt{81} = 9$

The side length is 9 cm.



b) Find a factor that multiplies by itself to give 11.56.

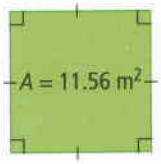
$3 \times 3 = 9$ **Too low.**

$4 \times 4 = 16$ **Too high.**

$\sqrt{11.56}$ must be between 3 and 4.

Use a calculator. C 11.56 √ 3.4

The side length is 3.4 m.



Example 2: Identify Perfect Squares

Decide if each number is a **perfect square**.

a) 121 **b)** 18

Solution

a) Try the natural number 10.

$10 \times 10 = 100$ **Too low.**

Try 11.

$11 \times 11 = 121$ **Correct!**

So, $\sqrt{121} = 11$

The square root of 121 is a natural number, 11.

So, 121 is a perfect square.

Strategies
Use systematic trial

b) Try the natural number 4.

$4 \times 4 = 16$ **Too low.**

Try 5.

$5 \times 5 = 25$ **Too high.**

Since 18 is between 16 and 25, the square root of 18 is between 4 and 5.

The square root of 18 is not a natural number.

So, 18 is not a perfect square.

Check using a calculator.

C 18 √ 4.242640687

Literacy Connections

Reading Square Roots

The $\sqrt{\ }$ symbol indicates the square root of a number.

Read $\sqrt{81} = 9$ as "the square root of 81 equals 9."

Technology Tip

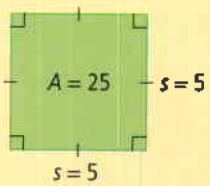
- Key sequences vary. On some calculators, you need to enter C 11.56 √. On other calculators, you need to enter C √ 11.56 =.

perfect square

- a number whose square root is a natural number
- example is 4

Key Ideas

- The side length of a square represents the square root of a number.
- A perfect square is a number whose square root is a natural number.
- The $\sqrt{\ }$ symbol indicates the square root of a number.



Communicate the Ideas

1. How does the diagram show the square root of 16?
2. How could you use grid paper, tiles, or blocks to show that $\sqrt{36} = 6$?
3. Decide if 49 is a perfect square. Show how you know.
4. Is it possible to find $\sqrt{6.25}$? Explain and justify your answer.

Check Your Understanding

Practise

5. State the side length of each square.

a) 

b) 

For help with questions 6 and 7, refer to Example 1.

6. Find the side length of a square with the given area.
   a) 25 m^2
   b) 49 cm^2
   c) 100 km^2
   d) 9 m^2

7. Use a calculator to find the side length of a square with the given area.
   a) 1.69 cm^2
   b) 12.25 m^2
   c) 0.04 mm^2
   d) 1.96 cm^2

For help with question 8, refer to Example 2.

8. Decide if each number is a perfect square. Show how you know.
   a) 16
   b) 24
   c) 58
   d) 225

9. Evaluate.
   a) $\sqrt{64}$
   b) $\sqrt{144}$
   c) $\sqrt{400}$

10. Use a calculator to evaluate.
    a) $\sqrt{625}$
    b) $\sqrt{441}$
    c) $\sqrt{10\,000}$

11. Evaluate.
    a) $\sqrt{1.44}$
    b) $\sqrt{2.25}$
    c) $\sqrt{5.76}$
    d) $\sqrt{0.25}$

Apply

12. How does the game board show that $\sqrt{100} = 10$?

13. **a)** The square root of 81 is 9. How can you use your calculator to find this? Write your steps.

b) Ask a classmate to check your steps.

14. On a baseball diamond, the first-base bag is a square with an area of about 1444 cm^2. What is the length of one side of the bag?

15. Find the perimeter of a square with each area.

a) 9 m^2

b) 144 cm^2

16. Each face of a cube has an area of 36 cm^2. What is the volume of the cube?

17. Which of the following square roots are between 6 and 7? Explain how you know.

$\sqrt{52}$ $\sqrt{41}$ $\sqrt{35}$ $\sqrt{38}$ $\sqrt{45}$

18. The playing surface on a checkerboard has an area of 576 cm^2. What is the side length of each small square on the board?

19. A square picture has an area of 100 cm^2. It is centred on a square mat with an area of 324 cm^2.

a) Draw a diagram to show the dimensions of the picture and mat.

b) Identify one item that is approximately the size of the picture and one that is the size of the mat.

c) How wide is the border around the picture? Justify your response.

Extend

20. Tamar is building a fence around her deck. The deck is a square with an area of 25 m^2. It costs $120 to build each metre of wood fencing. How much will Tamar spend?

21. Since $2^3 = 8$, 2 is known as the "cube root" of 8. Find the cube root of each of these numbers. Explain how you found your answers.

a) 27 **b)** 125 **c)** 1 000 000

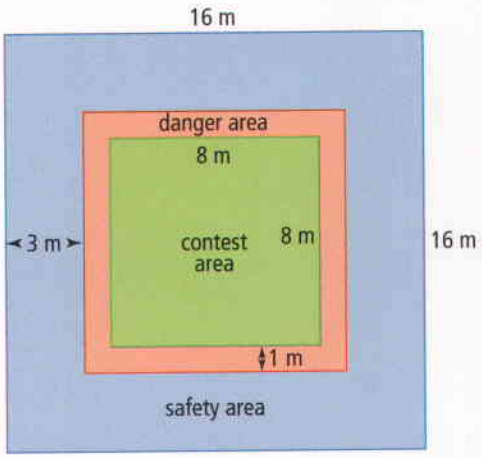
Making Connections

What does math have to do with recreation?

Some sports, such as judo, use squares on their playing surfaces. The diagram shows the square shapes on a judo mat.

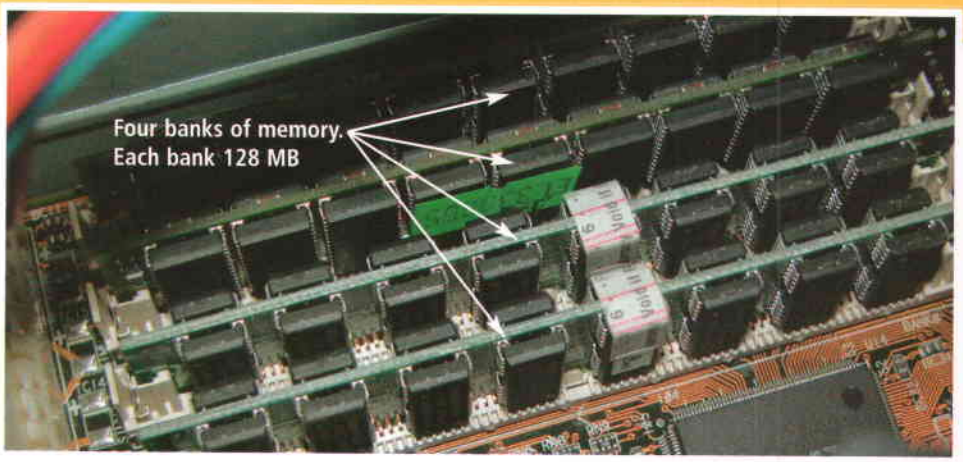
Use your personal experiences or your research skills to find another sport that uses a square on its playing surface. Find the side length and the area of this square. Use the rules of the sport to describe the purpose of the square.

List other recreational activities that involve square surfaces. When might you need to know the perimeter or the area of each square?



7.3 Understand the Use of Exponents

Focus on...
- exponential form
- powers



Computer memory can be measured in megabytes. The photograph shows some memory chips in a computer. How many megabytes of memory do the chips contain?

A megabyte can be defined as either a million bytes or as 2^{20} bytes. Do you think these two values are the same? To find out, you need to learn more about exponents.

Discover the Math

Materials
- scientific calculator
- sheet of paper

Optional
- BLM 7.3A Paper Folding Layers Worksheet

How do you write and evaluate numbers in exponential form?

1. Fold a sheet of paper in half. Then, fold it in half again. Count the layers of paper. Copy the table. Complete it as you continue to fold the paper. Notice that each number of layers is written in three ways in the table.

	Number of Layers		
Number of Folds	Standard Form	Repeated Multiplication	Exponential Form
2	4	2×2	2^2
3			
4			
5			
6			

2. a) Describe the pattern you see in the table.
b) Predict the next two rows.

3. Use your calculator to evaluate 2^6. Describe the method you used.

4. Use your calculator to evaluate 2^{20}.

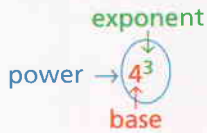
5. Compare the value of 2^{20} with 1 000 000.

6. Do you agree or disagree that 2^{20} bytes should be called a megabyte? Explain and justify your answer.

7. Reflect In step 1, you looked at three ways to represent the number of layers. Which way do you think is easiest to use and why? Are there times when either of the other two ways would have advantages? Explain.

power

- a number in exponential form
- includes a base and an exponent



Example 1: Evaluate Powers

Evaluate the power 3^4.

Solution

Method 1: Use Paper and Pencil

$3^4 = 3 \times 3 \times 3 \times 3$
$= 9 \times 3 \times 3$
$= 27 \times 3$
$= 81$

I see two sets of 3×3.
$3^4 = (3 \times 3) \times (3 \times 3)$
$= 9 \times 9$
$= 81$

Method 2: Use a Calculator

Calculators vary.
Try these methods on your calculator.

C 3 y^x 4 = 81.

or

C 3 × = = = 81.

or

C 3 × 3 × 3 × 3 = 81.

Literacy Connections

Reading Powers

You can name powers in words by describing the base and the exponent. These are some common ways to name 3^4:

- three to the fourth
- three to the exponent four
- the fourth power of three

Technology Tip

- On some calculators, the power key will appear as y^x. On others, the power key may appear as a^x or x^y.

Example 2: Write Powers

Write each expression as a power. Then, name it in words.

a) $6 \times 6 \times 6 \times 6 \times 6$

b) $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

Solution

a) $6 \times 6 \times 6 \times 6 \times 6 = 6^5$

The power is six to the fifth.

You could also say six to the exponent five or the fifth power of six.

b) $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^7$

The power is two to the seventh.

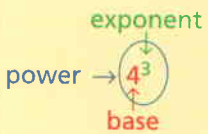
You could also say two to the exponent seven or the seventh power of two.

Powers can be described according to the value of the base.
For example, 2^2, 2^3, and 2^4 are powers of two.
They represent the repeated multiplication of 2 by itself.

Similarly, 3^2, 3^5, and 3^9 are powers of three.
They represent the repeated multiplication of 3 by itself.

Key Ideas

- Repeated multiplication can be represented using exponents.
  $7 \times 7 \times 7 \times 7 = 7^4$
- A number written in exponential form is called a power.
- A power includes a base and an exponent.



Communicate the Ideas

1. Describe how you would evaluate 6^4
   a) using a calculator
   b) using paper and pencil
2. Rachel said that 10^3 equals 30. How could you help Rachel correct her error?
3. Show the method you would use to write 625 as a power of 5.
4. David said that 3^4 is a power of 4. Is David correct? Explain.

Check Your Understanding

Practise

5. Identify the base and the exponent of each power.

a) 2^4 **b)** 1^6 **c)** 4^3

For help with questions 6 and 7, refer to Example 1.

6. Evaluate.

a) 4^1 **b)** 3^3 **c)** 5^4

7. Evaluate.

a) 2^5 **b)** 1^7 **c)** 6^3

For help with questions 8 and 9, refer to Example 2.

8. Write each expression as a power.

a) $5 \times 5 \times 5 \times 5 \times 5$

b) $11 \times 11 \times 11 \times 11 \times 11 \times 11 \times 11$

c) $100 \times 100 \times 100 \times 100 \times 100 \times 100$

9. Write each expression as a power.

a) $4 \times 4 \times 4 \times 4 \times 4 \times 4$

b) $9 \times 9 \times 9$

c) $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

10. Name each power in words.

a) 4^5 **b)** 8^3 **c)** 2^8

11. Express as a power.

a) three to the sixth

b) five to the exponent four

c) seventh power of nine

12. Rewrite each statement. Replace ■ with >, <, or = to make a true statement.

a) 3^3 ■ 2^5

b) 6^4 ■ 10^3

c) 8^1 ■ 2^3

d) 5^4 ■ 4^5

13. Use a calculator to evaluate.

a) 0.2^5

b) 1.4^4

c) 0.3^6

Apply

14. The metric system of measurement is based on powers of 10.

a) The metric prefix *kilo-* means 1000. Write this number as a power of 10.

b) The metric prefix *mega-* means 1 000 000. Write this number as a power of 10.

Making Connections

In grade 8, you will learn more about powers of 10 and about their importance in math and science.

15. Express as a power of 4.

a) 64

b) 256

c) 4

16. Find the unknown number in each equation.

a) $3^5 = ■$

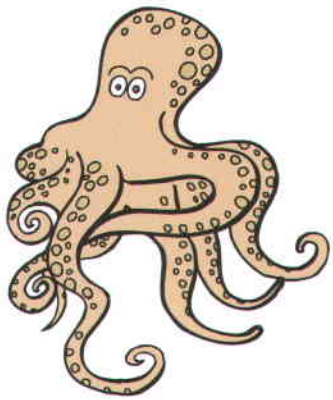
b) $2^■ = 64$

c) $■^4 = 4096$

17. Write the following expressions in order from the greatest value to the least value.

2^4 17 19.5 $\sqrt{225}$ 1^{18} 18^1

18. Express the total number of arms on 16 octopi in exponential form. (Note: Octopi is the plural of octopus.)



19. A *googol* is a very large number defined as 10^{100}. Scientists think that this is greater than the number of atoms in the universe.

a) Describe how you would find the number of zeros needed to write a googol in standard form.

b) How many zeros are needed?

20. Different values of the megabyte are used in the computer industry. Go to www.mcgrawhill.ca/links/math7 and follow the links to some sites with information on megabytes. Describe how different values of the megabyte are used in the computer industry.

Internet Connect

Did You Know?

Mers Kutt invented the world's first personal computer in Ontario in 1973. It was called the MCM-70 Microcomputer. It had only 2 to 8 kilobytes of random access memory (RAM) and 14 kilobytes of read-only memory (ROM).

21. Find two whole numbers between 10 and 1000 that can be written as a power of 3 and as a power of 9.

Chapter Problem

22. A pond is polluted with fertilizer. Algae are growing on the water. The area of the algae on the pond is 1 m^2. The area of the algae doubles every week.

a) What will the area of the algae be after 1 week, 2 weeks, and 3 weeks? Express each area as a power of 2.

b) The pond has an area of 32 m^2. How long will it take for the pond to be completely covered in algae?

Try This!

23. a) Express 16 as a power of 2.

b) Express 16 as a power of 4.

c) Express 16 as a power of 16.

d) Find a whole number greater than 16 that can be expressed as a power of 2, a power of 4, and a power of 16. Write the three powers.

Extend

24. Large numbers are sometimes estimated to the nearest power of 10. For example, 8500 is about 10 000, or 10^4. Estimate each of the following to the nearest power of 10.

a) the number of people in the world
b) the height of the CN Tower, in centimetres
c) the length of the Trans-Canada Highway, in metres
d) your age, in minutes

Did You Know?

- The world's population was about 6.4 billion people at the end of 2003. What is it now?
- The CN Tower is 553 m high.
- The Trans-Canada Highway is the world's longest national highway. Its length is 7821 km.

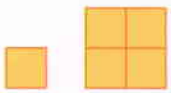
Making Connections

Modelling Changes in Powers

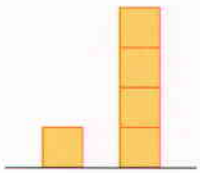
Materials

- square blocks or tiles

1. a) Use tiles or blocks to model 1^2, 2^2, 3^2, 4^2, and 5^2.



Rearrange the tiles for each number into a straight line. Put the five lines of tiles next to each other. The diagram shows how to place the first two lines.

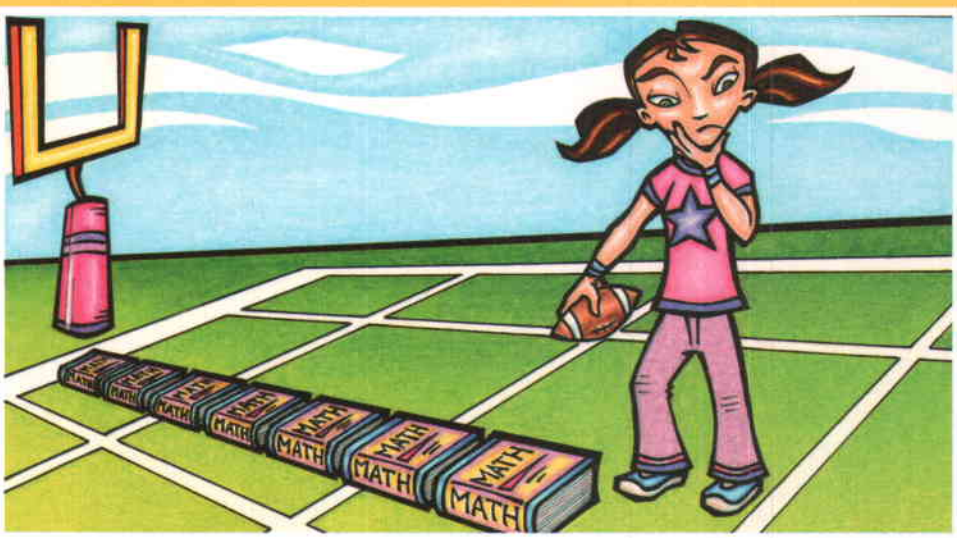


b) Describe the way that changing the base changes the size of the power.
c) Suppose you changed the exponent from 2 to 1. Would you see the same kind of change in the size of the power? Try it and describe what you see.

2. a) Use lines of tiles to model 2^1, 2^2, 2^3, 2^4, and 2^5. Put the lines of tiles next to each other.
b) Describe the way that changing the exponent changes the size of the power.
c) Suppose you changed the base from 2 to 1. Would you see the same kind of change in the size of the power? Try it and describe what you see.

7.4 Fermi Problems

Focus on...
- problem solving
- estimation



How many math textbooks would cover a football field?
How many tennis balls would fill your classroom?

Estimation problems like these are known as Fermi problems. They are named after a famous scientist, Enrico Fermi (1901–1954). He liked to ask his students to solve them.

Fermi problems may seem impossible at first. Different people may estimate different answers. As long as the answer makes sense, the process you follow is more important than your answer.

Discover the Math

How can you solve Fermi problems?

Example: Toonies

How many toonies are needed to cover the floor of a classroom?

Solution

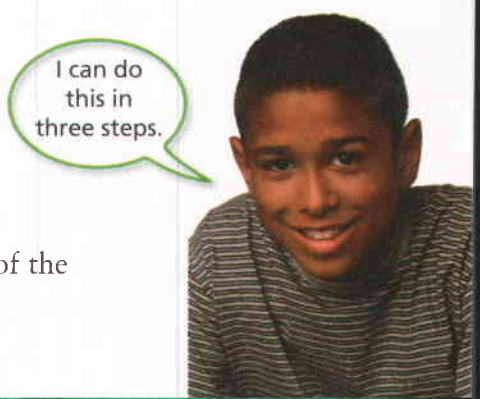
Method 1: Use the Areas

Understand

Find out how many small objects will cover a large area.

Plan

1. Estimate the area of the toonie.
2. Find the area of the classroom floor.
3. Divide the area of the toonie into the area of the classroom floor.



Do It!

Strategies
Find needed information

1. Estimate the area of the toonie.

A toonie is about 2.8 cm across.

Strategies
Make an assumption

2.8 cm

2.8 cm

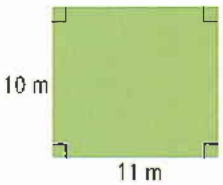
A toonie roughly covers a square with a side length of 2.8 cm.

Literacy Connections

Reading ≐
The symbol ≐ means "is approximately equal to."

Approximate area of one toonie = 2.8 cm × 2.8 cm
≐ 8 cm^2

2. Find the area of the classroom floor.



I measured the dimensions of my classroom floor. It is an 11 m by 10 m rectangle.

Convert the side lengths of the classroom floor to centimetres.

11 m = 11 × 100 cm
= 1100 cm

10 m = 10 × 100 cm
= 1000 cm

The toonie is measured in centimetres. The classroom floor is measured in metres. To divide the areas, the units must be the same.

Area of floor = 1100 cm × 1000 cm
= 1 100 000 cm^2

3. Divide the area of the toonie into the area of the classroom floor.

$$\text{Number of toonies needed} = \frac{1\ 100\ 000}{8}$$

$$= 137\ 500$$

C 1100000 ÷ 8 = 137500.

About 137 500 toonies are needed to cover the floor of this classroom.

Look Back

Can you think of another way to solve the problem?
After you think about this, look at Method 2 on the next page.

Strategies
Solve a simpler problem

Method 2: Solve a Simpler Problem

I am going to estimate the number of toonies that would cover a 1 m by 1 m square.

The side length of the square is 1 m.
The diameter of a toonie is about 2.8 cm.

My measurements need to have the same units.

Change the side length of the square to centimetres.
1 m = 100 cm

Find the number of toonies that will fit along each side of the square.
$100 \div 2.8 \doteq 36$

The side length of the square is 100 cm. The diameter of the toonie is 2.8 cm. Now I can divide to find the number of toonies that will fit along one side of the square.

Find the total number of toonies inside the square.
$36 \times 36 = 1296$
$\doteq 1300$

I need to multiply the number of toonies along two sides of the square

The area of the classroom floor is
$11 \text{ m} \times 10 \text{ m} = 110 \text{ m}^2$

This area is 110 times the area of the 1 m by 1 m square.

Find the number of toonies needed to cover the classroom floor.
$110 \times 1300 = 143\,000$

About 1300 toonies cover 1 m^2. The classroom floor covers about 110 m^2. I can multiply to find out how many toonies can fit on the classroom floor.

About 143 000 toonies are needed to cover the floor of this classroom.

Did You Know?

The value of the toonies on the floor of this classroom would be about $280 000!

The answers to Method 1 and Method 2 are both very large. They differ by only 5500. Both numbers round to 140 000, so both answers seem reasonable.

Key Ideas

- Fermi problems are estimation problems that may involve large numbers.
- To solve Fermi problems, you need to research missing information and make reasonable assumptions.
- A Fermi problem can have several solutions.

Communicate the Ideas

1. Fermi problems can have many "right answers." Explain why.
2. Suppose two students get two different answers to a Fermi problem. How might the students decide if one answer is more reasonable than the other?

Check Your Understanding

Apply

In each question, explain and justify the process that you used to get the result.

For help with question 3, refer to the Example.

3. How many loonies are needed to cover the floor of *your* classroom?

4. Estimate the number of table tennis balls that would fill a suitcase. Hint: Assume that the volume of the ball approximates the volume of a cube.

5. How many words are in this textbook?

6. Estimate the number of CD cases needed to cover the walls of your classroom.

7. How many math textbooks would it take to cover a football field?

8. How many tennis balls would fill your classroom?

9. How many bananas would fill a garbage bag?

Try This!

10. How many of your footprints would cover the floor of your classroom? Solve this problem in three different ways.
    a) Assume that your foot approximates a rectangle. Calculate the area it covers. Complete the solution.
    b) Use a different method to find the area covered by your foot. Solve the problem again. Compare your solution with the one from part a). Is one solution better than the other? Explain.
    c) Use a method that does not involve calculating the area of your foot.

Extend

11. a) Write a Fermi problem of your own. Make sure that you can solve it.
    b) Have a classmate solve your problem.
    c) Compare the two solutions. If they are different, decide which method you prefer. Explain why. Which method do you think is more accurate?

CHAPTER 7

Review

Key Words

Unscramble the letters for each puzzle. Use the clues to help you solve the puzzles.

1. E P R O W
a number in exponential form

2. F S R Q R E U C E A T E P
a number whose square root is a natural number (2 words)

3. A S E B
the factor you multiply

4. P O X N T E N E
the number of factors you multiply

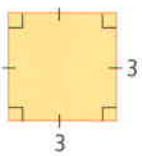
5. R O E S U T Q R O A
a factor that multiplies by itself to give a number (2 words)

6. E O A P E X R M N O N T L F I
a shorter method for writing numbers expressed as repeated multiplication (2 words)

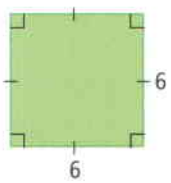
7.1 Understand Exponents, pages 210–213

7. Find the area of each square.

a)

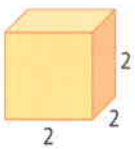


b)

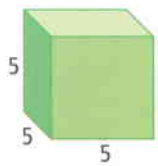


8. Find the volume of each cube.

a)



b)



9. Use a calculator to evaluate.

a) 16^2 **b)** 1.3^2
c) 8^3 **d)** 11^3

10. Write each number in exponential form.

a) 29×29
b) $14 \times 14 \times 14$

11. Decide which number is greater in each pair.

a) 3^3 or 5^2
b) 14^2 or 6^3
c) 3.2^2 or 2.2^3

12. a) Evaluate 11^2, 111^2, and 1111^2.
b) Predict the values of $11\ 111^2$ and $111\ 111^2$.

13. A square pizza with a side length of 30 cm is cut into 9 equal square pieces.

a) Draw a diagram to show this situation.
b) What is the area of the top of each piece?

14. Suppose that 1 mL of paint covers an area of 10 cm^2. How much paint would you need to cover the outside of a wooden cube with an edge length of 5 cm?

7.2 Represent and Evaluate Square Roots, pages 214–217

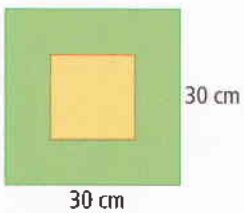
15. Explain why 9 is a perfect square.

16. Evaluate.

a) $\sqrt{16}$
b) $\sqrt{169}$
c) $\sqrt{6.25}$

17. The playing surface of a Snakes and Ladders game board has 100 small squares on it. Each small square has an area of 9 cm^2. What is the perimeter of the playing surface?

18. A square picture is glued to a square piece of construction paper. The construction paper is 4 times the area of the square picture mounted on it. The construction paper has a side length of 30 cm. What is the side length of the picture?



19. The first four diagrams in a pattern are shown. Each square has an area of 4 cm^2. Find the perimeter of the 30th diagram in the pattern.

7.3 Understand the Use of Exponents, pages 218–223

20. Write each number as a power. Do not evaluate.

a) $8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8$

b) $3.6 \times 3.6 \times 3.6 \times 3.6$

21. Write each power.

a) 625 as a power of 5

b) a billion as a power of 10

c) 243 as a power of 3

22. Write the total number of legs on 27 tripods in exponential form.

23. Evaluate.

a) 4^5 **b)** 1^{12} **c)** 0.1^4

24. Write the following expressions in order from the least value to the greatest value.

$\sqrt{900}$ 30.5 3^3 29 $2 \times 2 \times 2 \times 2 \times 2$

25. Suppose that a bacterium takes 1 h to split into two bacteria. If there is 1 bacterium at the start, how many bacteria are there after 8 h?

26. Is there a number between 100 and 1000 that can be written as a power of 6 and as a power of 36? Explain.

7.4 Fermi Problems, pages 224–227

In each solution, explain and justify the process that you used to get the result.

27. How many telephone books would cover a basketball court?

28. Estimate the number of sheets of paper in a stack with a height of 1 km.

29. a) Estimate the number of coins you receive in change in a year.

b) Estimate the volume of all those coins.

CHAPTER 7

Practice Test

Strand	NSN	MEA	GSS	PA	DMP
Questions	1–19			5, 13, 18	

Multiple Choice

For questions 1 to 5, choose the best answer.

1. The number 5^2 is expressed

A in standard form
B as a repeated multiplication
C in factored form
D in exponential form

2. The number 6^3 means

A 6×3
B $3 \times 3 \times 3 \times 3 \times 3 \times 3$
C $6 \times 6 \times 6$
D both B and C

3. A square has an area of 10 cm^2. Its side length is

A 100 cm
B between 3 cm and 4 cm
C over 1 cm
D between 2 cm and 5 cm

4. Which number is not a perfect square?

A 25
B 169
C 114
D 400

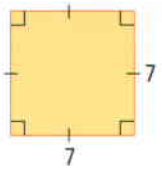
5. The first four numbers in a pattern are 1, 8, 27, and 64. The next number in the pattern is

A 125
B 128
C 216
D 192

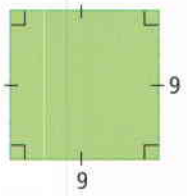
Short Answer

6. Find the area of each square.

a)

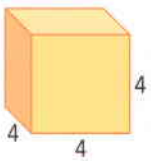


b)

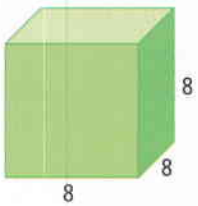


7. Find the volume of each cube.

a)



b)



8. Evaluate.

a) 5^2 **b)** 10^2
c) 6^3 **d)** 7^3

9. Evaluate.

a) $\sqrt{64}$ **b)** $\sqrt{400}$
c) $\sqrt{1.44}$ **d)** $\sqrt{2.25}$

10. Decide if each number is a perfect square. Show how you know.

a) 121 **b)** 47

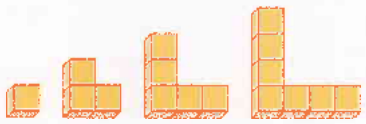
11. Write as a repeated multiplication. Then, evaluate.

a) 2^9 **b)** 3^6 **c)** 4^5

12. Write the following expressions in order from the least value to the greatest value. Show your reasoning.

$3 \times 3 \times 3 \times 3$ 10^2 $\sqrt{6400}$ 2^6 79.5

13. The diagrams show the first four shapes in a pattern. Each cube has an edge length of 2 cm. What is the volume of the 5th shape?



14. A square photograph has an area of 361 cm^2. Will it fit inside a 30 cm by 18 cm rectangular frame? Explain.

15. a) Norah said, "2^6 is greater than 2^5, because 6 is greater than 5." Do you agree? Explain.

b) Paul said, "1^6 is greater than 1^5, because 6 is greater than 5." Do you agree? Explain.

16. Express the number of pennies in $100 as a power of 100.

17. A 4 m by 3 m rectangular floor is covered by 300 square floor tiles. What is the side length of each tile?

18. a) Evaluate 9^2, 99^2, 999^2, and 9999^2.

b) Predict the values of $99\,999^2$ and $999\,999^2$.

Extended Response

19. Describe how you would solve one of the following problems. In your description, list the missing information you would need and the assumptions you would make.

a) How many years would it take to walk the distance from Earth to the Moon?

b) How many cell phones would fill a backpack?

c) How many litres of soft drinks do the students in your school drink in a year?

Chapter Problem Wrap-Up

In question 22, on page 222, you solved a problem involving algae on a pond. The following questions refer to three different bodies of water. Each one is polluted. In each case, the area of the algae doubles every week.

1. The area of the algae on a lake is 5 m^2. What will the area of the algae be after 4 weeks?

2. Algae covered an area of 2 m^2 in a square water-storage tank. Three weeks later the tank was completely covered. What was the side length of the tank?

3. It took 6 weeks for a pond to be completely covered by algae. What percent of the pond was covered after 4 weeks? Explain and justify your solution.

Geometry and Spatial Sense

- Identify, describe, compare, and classify geometric figures.
- Identify, draw, and construct three-dimensional geometric figures from nets.
- Use mathematical language effectively to describe geometric concepts, reasoning, and investigations.
- Recognize and sketch views of three-dimensional objects.

Measurement

- Develop and use the formula for finding the surface area and the volume of a rectangular prism.
- Understand the relationship between the dimensions and the volume of a rectangular prism.

Key Words

polyhedron
prism
pyramid
net
surface area
volume